A

TEXT-BOOK

OF

ARITHMETIC

FOR

LOWER SECONDARY OR MIDDLE SCHOOLS

BY

S. RADHAKRISHNA AIYAR, B.A.,
Late Principal, Maharajah's College, Pudukotah

AND

K. VAIDYANATHASWAMI AIYAR, B.A.,
Retired Headmaster, Board High School, Tiruvalur.

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PREFACE TO THE 1st EDITION.

THE present work is intended to furnish the pupils of Lower Secondary Schools with a complete Text book of Arithmetic, prepared strictly in accordance with the Curriculum of Studies prescribed in the Madras Educational Rules. It is hoped, however, that the book will be found well adapted for Middle School students in other parts of India also.

The principles of the subject have been explained in the simplest and clearest form, and are illustrated throughout by copious model solutions. The examples given for practice have been framed or selected and arranged with great care and will be found to be sufficiently numerous for both class and home work. Oral exercises in concrete as well as abstract numbers are given everywhere, "anticipating, by means of rapid and varied practice with small numbers, the longer problems which have afterwards to be worked out in writing."

The first 46 pages are devoted to Numeration, Notation, and the First Four Simple Rules. Oral exercises in the fundamental processes of Arithmetic are given in these pages after the models given by *De Morgan* in his Appendix to his Arithmetic "On the Mode of Computing;" and it is believed that they are sufficiently numerous to enable the student to acquire rapidity and accuracy in computation. Teachers of the First Form who may use this book are

recommended to put their pupils* through these exercises before they begin the Compound Rules.

Great pains have been taken to make such problems as are not mere applications of the rules as attractive as possible, by classifying them into progressively arranged sets and presenting their solution in the simplest and most methodical form.

A novel feature of the book is that the student is shown at every step how he may verify his answers, a point the importance of which cannot be over-estimated, as the consciousness of the ability to verify his own answer imparts to the student much *self-confidence* and *self-reliance* and *checks the tendency to the vicious practice of copying.* Another novel feature is the occasional insertion of Notes to the Teacher on methods of teaching the subject.

It may also be noticed that oral exercises are given at an early stage on Arithmetical processes involving the fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, with a view to remove, in some measure, the complaint one often hears that boys educated in English Schools are unable to make even the smallest calculations involving these fractions, without the help of writing materials.

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January, 1898.

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* "If the student really wishes to become a ready computer, he should strictly follow the methods laid down in this Appendix; and he may depend upon it that he will thereby save himself trouble in the end, as well as acquire habits of quick and accurate calculation."—*Augusts De Morgan.*

"The teacher who follows the course recommended by De Morgan in training scholars quickly to count backwards and forwards, will carry his pupils forward with greater ease than one who fails to pursue this method"—*Cyclopædia of Education.*

PREFACE TO THE 2ND EDITION.

The rapid sale of about 4,000 copies of the First Edition in the short space of 40 days seems to indicate that the book has met a decided want.

We have availed ourselves of this opportunity for removing several errors which had crept into the first edition, and for making some improvements in the light of suggestions received from some experienced teachers. Nevertheless, there is nothing in this edition which will prevent it from being used with the first.

June, 1898.

PREFACE TO THE 15TH EDITION.

In this edition the book has been subjected to a careful and thorough revision in the light of modern requirements; and it is hoped that this revised edition will be found even more useful than its predecessors.

February, 1916.

PREFACE TO THE 21ST EDITION.

With a view to bring the book more or less into conformity with the *Syllabus in Elementary Mathematics* recommended by the Director of Public Instruction, Madras, for Forms I, II and III of all Secondary Schools under the Madras Educational Department, the matter of the previous editions has been carefully re-arranged with some slight but necessary omissions, alterations, elaborations and additions. And the Mysore Lower Secondary Examination Papers in Elementary Mathematics, which were inserted in the 20th Edition for the benefit of the students in the Mysore State, have been brought up-to-date.

2. The book has had to be issued till now in one single volume (unwieldy in the case of the last six editions); and it is a matter for gratification that the present revision has rendered it possible to issue it in two *handy* and *concentric* parts, of which Part I is intended for the pupils of Form I, and Part II for those of Forms II and III.

3. It is hoped that the book will, in its present form, be found even more suitable and satisfactory than its predecessors, as a *Text-Book of Elementary Mathematics* for Forms I, II and III of all Secondary Schools both in the Madras Presidency and in the adjoining Native States.

4. It may be added that the book which had long been approved by the Madras Educational Department for use only in schools under *private management*, may hereafter be used in schools under *public management* also, as the Director of Public Instruction, Madras, has recently ruled that "there is now no objection to the use in schools under Public Management of a book formerly approved for use in schools under Private Management."

April, 1924.

PREFACE TO THE 22ND EDITION.

In this the 22nd Edition, the number of examples in a few of the Exercises has been reduced at the suggestion of some teachers who have been using the book for a long time.

K. V. S.

August, 1926.

TABLE OF CONTENTS.

Note—Besides the usual Black-Board instruments including a graduated ruler one metre long, the teacher must provide himself with a black-board squared on one side, card-board models of the circle and of the chief rectilineal figures, and large wooden models of the chief geometrical solids. He must also have a measuring tape (50 ft. or 100 ft long), a tailor's tape, and a small box of Mathematical instruments

ARITHMETIC, PART I

CHAPTER I.

NUMERATION AND NOTATION—(*Integers*).

1. A unit is a *single* object taken as a whole, or a *portion* of any *continuous* magnitude taken as a standard for measuring that magnitude.

2. Number is that which denotes how many units there are in a group of objects of the same kind, or in a given magnitude.

For example, when we say *one* book, *three* trees, *four* yards of cloth, *half* a seer of milk, *one*, *three*, *four* *half* are numbers, a book, a tree, a yard a seer being the respective units.

3. Integers and Fractions.—A number denoting a *whole* (or unbroken) unit or a collection of *whole* units is called an *integer* (or *whole number*); a number denoting a *portion* of a unit is called a *fraction*.

For example, *one*, *three*, and *twenty* are integers · *half*, *three-fourths*, *one-eighth* are fractions

4. Names of some Integers.—First we have the numbers *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*, *ten*, each of which (except *one*) is one more than that which precedes it, then we have *hundred* (ten times ten), *thousand*, *ten thousand*, *hundred-thousand* (or *lakh*), *million* (or *ten lakhs*), *ten million* (or *crore*), *hundred-million* (or *ten crores*), etc., each of which (except hundred) is ten times the preceding one.

5. All other integers are expressed by naming in *descending* order the number of *units*, of *tens*, of *hundreds*, etc., which it contains.

NOTE.—A lakh (or lac) is *one hundred thousand*.
A million is *ten lakhs*
A crore is *one hundred lakhs*, or *ten millions*.

6. Digits.—The numbers *one two, three, four, five, six, seven, eight, nine,* are denoted respectively by the nine symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 called *figures* or *digits,* and the absence of number is denoted by a tenth digit 0, called *cipher, nought,* or *zero.*

7. All numbers greater than nine are denoted by placing in a row two or more of the *ten digits* 0 to 9, the value of each digit depending upon its position in the row, *i. e.,* the first digit on the right-hand which is said to be in the *units'* place denoting as many *units,* the second digit from the right which is said to be in the tens' place denoting as many *tens,* the third digit which is said to be in the *hundreds'* place denoting as many *hundreds,* and so on.

For example in the number 3333, each 3 has a different value: the first on the right-hand denotes simply 3 *units,* the second denotes 3 *tens* or thirty, the third 3 *hundreds,* and the fourth 3 *thousands,* so that 3333 denotes *three thousand three hundred and thirty-three* Each of the figures will be seen to denote ten times as much as that on its right.

Again, 3033 denotes a number composed of 3 units, 3 tens, *no* hundreds, and 3 thousands, *i e.,* the number *three thousand and thirty-three.*

NOTE—But for the cipher, 3033 would be mistaken for 333.

8. Local Value.—The value of a figure which depends upon the place it occupies in a number is called its *local* value, while its own value when standing alone is called its *absolute* or *intrinsic* value

For example, in the number 42025 the local value of 2 is two *tens* in one place and two *thousands* in another place, while its intrinsic value is only two *units.*

9. Significant Digits—The nine digits 1—9 are called *significant* digits; and 0 is called a *non-significant* digit, because, though it serves to show the place in which the accompanying figures in a number stand, it is not counted as anything itself. [See Art. 7.]

10. Affixing and Prefixing Ciphers to an Integer.—The value of a number is altered and increased by placing a cipher in the middle of it or on its right. Thus 24 is different from and less than 204 or 240.

Ciphers might be prefixed to a number without altering its value; thus 024 is the same as 24, since the cipher in 024 shows that there are no hundreds, which is evident from the number 24 itself. It is not usual, however, to prefix ciphers to a number.

To the Teacher.—The pupil may be informed that ciphers are prefixed to the numbers on *bank cheques, railway tickets, currency notes,* etc.

11. Numeration is the art of expressing in words any number denoted by figures.

For convenience in reading numbers expressed in figures, it is usual to divide them by commas into periods, beginning from the right.

For example, the number 143705489 may be divided into periods of three figures each, thus—

143,705,489

and read as 143 *millions*, 705 *thousand*, 489, and written wholly in words thus: *one hundred and forty-three millions, seven hundred and five thousand, four hundred and eighty-nine.*

The same number may also be divided thus—

14,37,05,489

and read as 14 *crores*, 37 *lakhs*, 5 *thousand*, 489: and written wholly in words thus: *fourteen crores, thirty-seven lakhs, five thousand, four hundred and eighty-nine*

Exercise 1.

(*a*) Express the following numbers in words using the denominations (*a*) *lakh* and *crore*, (*b*) *hundred-thousand* and *million*:—

1. 776705 2. 4175960 3. 2000000. 4. 1600009.

5. 18576000. 6. 64005004 7. 1650000000. 8. 111111011.

(*b*) Explain by examples what is meant by the *local value* of a digit.

(*e*) In what place is each 4 in 444444, and what is the local value of each ?

Exercise 2.

1. Write down the greatest and least numbers of *four* digits?

2. Write down the greatest and least numbers of 4 digits that can be formed with the digits 8, 6, 3, 5.

3. Write down the greatest and least numbers of 5 digits beginning with 6 and ending with 7.

4. Write down in order of magnitude the *three* numbers of 3 digits that can be formed with the figures 4, 3, 4.

12. Notation is the art of denoting by figures any number expressed in words.

In notation, as in numeration, it will be found convenient to consider all numbers as made up of periods and *place a comma after each period as soon as* it is written.

Example 1.—Express in figures *fourteen millions, four hundred and seven thousand, five hundred and five.*

Answer —14,407, 505.

Example 2.—Express in figures *fourteen crores, forty lakhs, seven thousand and seven.*

Answer.—14,40,07,007.

NOTE.—A lakh is denoted by 1,00,000 (1 followed by 5 ciphers).
A million...............1,000,000 (1 followed by 6 ciphers).
A crore...............1,00,00 000 (1 followed by 7 ciphers).

Exercise 3.

(*a*) Write down the following numbers in figures :—

1. One crore 2 Two lakhs. 3. Eleven millions.

4 Twelve lakhs eighty thousand six-hundred and six.

5. Three crores seventy-five lakhs and nine hundred.

6 Seven thousand and five millions four hundred and ninety thousand and six

7. One hundred and twenty crores forty thousand nine hundred and ninety.

8 Ninety millions ninety thousand and nineteen.

(*b*) 1 How many *lakhs* are there in a *crore* ? How many *millions* ?

2 How many *lakhs* are there in a *million* ?

3. How many *thousands* are there in a *lakh* ?

4. How many *lakhs* are there in 20 millions ? How many *crores* ?

5 Taking the population of India as *three hundred million*, express this population in *lakhs* and in *crores* ?

6 Taking the distance of the sun from the earth as *ninety millions* of miles, express this distance in *lakhs* of miles and in *crores* of miles.

CHAPTER II.

ARITHMETICAL TABLES.

13. Abstract and Concrete Numbers.

Numbers which refer to particular objects or magnitudes as *five apples*, *seven rupees*, *three inches* are called *concrete numbers*.

Numbers which are considered separately and without reference to particular objects or magnitudes are called *abstract* numbers. Thus when we say *five*, *seven*, *three* without at all thinking of objects like apples or magnitudes like inches, *five*, *seven*, *three* are called *abstract numbers*.

14. The term **quantity** is frequently, though not correctly, used for the word **number**.

15. Simple and Compound Quantities :—

A simple quantity or a **simple number** is either an abstract number or a concrete number expressed in one denomination, as 5, 5 rupees, 3 yards.

A compound quantity or a **compound number** is a concrete number expressed in several denominations, as 5 rupees 2 annas 3 pies, 4 yards 2 feet, 1 seer 3 palams.

16. Arithmetical Tables.—The several units employed for measuring magnitudes of various kinds (such as *length*, *area*, *volume*, *weight*, *capacity*, *time*, etc.), and their relations to one another are given in the following tables, and should be learnt thoroughly by heart.

I—MONEY.

1. Indian Money.

12 Pies (p.).........	make	1 Anna (a.)
16 Annas............	,,	1 Rupee (Re.)
15 Rupees...........	,,	1 Sovereign or Pound Sterling (£)

NOTE 1.—**The Indian currency now consists of (1) the gold sovereign, (2) the silver rupee, half-rupee, quarter-rupee, and one-eighth rupee, (3) the nickel anna, two-anna and four-anna pieces, and (4) the bronze quarter-anna and pie.**

NOTE 2.—The copper half-anna pieces which are now in circulation are those that were minted formerly; and no half-anna pieces are minted now.

NOTE 3.—The terms *gold mohur* and *pagoda* respectively indicate coins worth Rs. 16 and Rs. $3\frac{1}{2}$ which seem to have been in existence formerly but are no longer in existence. The term *pagoda* is often used even now in estimating the price of jewels and precious stones.

NOTE 4.—The diameter of a quater anna coin is 1 inch

NOTE 5.—The quarter-anna coin is sometimes called a *pice*.

§ 2. English Money.

4 Farthings (q)........................	make	1 Penny ($d.$).
12 Pence................................	„	1 Shilling (s).
20 Shillings............................	„	1 Pound (£ or $l.$)

NOTE 1.—The following are some of the coins met with or mentioned in England:—

A Groat	=	4 Pence.
A Florin	=	2 Shillings.
A Crown	=	5 Shillings.
A Sovereign	=	20 Shillings.
A Guinea	=	21 Shillings.
A Moidore	=	27 Shillings.

NOTE 2.—The English Pound is generally called a *pound sterling* to distinguish it from the *weight* called a pound and from foreign coins having the same name.

NOTE 3.—The letter q is now seldom used, 1 farthing, 2 farthings, 3 farthings being denoted respectively by the fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, annexed to the number denoting pence. Thus $3\frac{3}{4}d.$ means $3d.\ 3q.$

II.—WEIGHT.

1. Indian Weight.

(a) *Madras Weight.*

3 Tolas (tol.).....................	make	1 Palam (pal.)
8 Palams...........................	„	1 Seer (sr.)
5 Seers or 40 Palams	„	1 Viss (vis.)
8 Visses or 40 Seers	„	1 Madras Maund (md.)
20 Maunds	„	1 Candy (can.) (or Baram)

NOTE.—One Rupee weighs 1 tola or 180 grains Troy.

(b) *Indian Imperial Weight*

[Used in weighing goods sent by Railways]

80 Tolas.......................make 1 Imperial Seer.
40 Imperial Seers .... ,, 1 Indian Maund

2. English Weight

(a) *Avoirdupois Weight.*

[Employed in weighing all substances except gold and silver, precious stones, and the like.]

16 Drams (dr.)..........make 1 Ounce (oz.)
16 Ounces.............. ,, 1 Pound (lb.)
28 Pounds.............. ,, 1 Quarter (qr.)
4 Quarters or 112 Pounds ...... ,, 1 Hundredweight (cwt.)
20 Hundredweights ,, 1 Ton.

A Stone = 14 lb.

NOTE 1.—lb. stands for *libra*, Latin for *pound*, the *c* in cwt. stands for *centum*, Latin for *hundred*.

NOTE 2.—1 lb Avoirdupois = 39 8/9 tolas=13 palams roughly.

1 Madras Maund = $24\frac{34}{85}$ lb. Avoir. or 25 lb nearly.

1 Indian or Bengal Maund = 82 2/7 lb. Avoir.

(b) *Troy Weight.*

[Used for weighing gold and silver, precious stones, and the like.]

24 Grains (gr.)...........make 1 Pennyweight (dwt.)
20 Pennyweight or 480 Grains ... ,, 1 Ounce (oz.)
12 Ounces or 5,760 Grains ... ,, 1 Pound (lb.)

NOTE 1.—The letter *d* in dwt. stands for *penny*.
NOTE 2.—1 lb. Avoirdupois = 7,000 grains Troy.

(c) *Apothecaries' Weight*

(Apothecaries in compounding medicines sub-divide the Troy ounce into drams, scruples and grains, the pound, ounce and grain being the same as in Troy weight.)

20 Grains..................make 1 Scruple (sc. or ℈)
3 Scruples................ ,, 1 Dram (dr. or ʒ)
8 Drams or 480 Grains .... ,, 1 Ounce (oz. or ℥)
12 Ounces................ ,, 1 Pound (lb. or ℔.)

NOTE.—The pound, the ounce, and the grain in this table, it will be noted, are the same as in the table of Troy weights.

III.—CAPACITY.

1. INDIAN (*Madras*).

8 Ollocks (ol.)………make 1 Measure (mea.)
8 Measures………… „ 1 Marakkal (mar.)
5 Marakkals………… „ 1 Parah (par.)
80 Parahs or 400 Marakkals } =…. „ 1 Garce (gar.)

One Kalam = 12 Marakkals.

2. English

(*a*) *Corn and Liquid Measure.*

4 Gills (gil.)……make 1 Pint (pt.)
2 Pints…………… „ 1 Quart (qt.)
4 Quarts………… „ 1 Gallon (gal.) } For both liquids and dry goods.
2 Gallons………… „ 1 Peck (pk.)
4 Pecks………… „ 1 Bushel (bush.)
8 Bushels………… „ 1 Quarter (qr.) } For dry goods only.
5 Quarters……… „ 1 Load.
36 Gallons……… „ 1 Barrel of Beer.
63 Gallons……… „ 1 Hogshead of Wine.
2 Hogsheads or 126 Gallons } „ 1 Pipe of Wine.

(*b*) *Apothecaries' Liquid Measure*).

60 Minims……………make 1 Fluid Dram
8 Fluid Drams……… „ 1 Fluid Ounce
20 Fluid Ounces… …. „ 1 Pint

NOTE.—One tea spoon = 1 dram; one table spoon=½oz.)

IV.—LONG OR LINEAL MEASURE.

1. English

12 Inches (in.)………make 1 Foot (ft.)
3 Feet……………… „ 1 Yard (yd.)
5½ Yards…………… „ 1 Pole (po.)
40 Poles or 220 Yards } ……… „ 1 Furlong (fur.)
8 Furlongs or 1,760 Yards } …… „ 1 Mile (mi.)
3 Miles …………… „ 1 League (lea.)

An English Ell = 1 Yard 9 Inches.

22 Yards……make 1 Chain
100 Links… .. „ 1 Chain } Used in measuring land.

1 Furlong = 10 Chains.
1 Mile = 80 Chains.

2. Indian (*Madras*)

9 Inches	....	make	1 Span.
2 Spans or 18 Inches	....	„	1 Cubit or Mulam.
2 Cubits or 36 Inches	....	„	1 Yard.

1 Kadam = 10 English Miles.

3. Metric (*French*).

10 Milimetres (mm.)	......	make	1 Centimetre (cm.)
10 Centimetres	......	„	1 Decimetre (dm.)
10 Decimetres	......	„	1 Metre (m.)
10 Metres	......	„	1 Decametre (Dm.)
10 Decametres	......	„	1 Hectometre (Hm.)
10 Hectometres	......	„	1 Kilometre (Km.)
10 Kilometres	......	„	1 Myriametre (Mm.)

The metre = 39·4 inches or 40 inches nearly ;
1 decimetre is equal to 40/10 or 4 inches nearly ;
1 centimetre is equal to 4/10 or ·4 inch nearly ,
1 Kilometre is equal to 5/8 of a mile nearly.

Also 1 inch is roughly equal to 100/40 or 2½ or 2·5 centimetres.

V.—SQUARE MEASURE.

1 English

144 Square inches (sq. in.)		make	1 Square foot (sq. ft.)
9 Square feet	......	„	1 Square yard (sq yd.)
$30\frac{1}{4}$ Square yards	......	„	1 Square pole (sq. po.)
40 Square poles or 1,210 Square yards	......	„	1 Rood (ro.)
4 Roods or 4,840 Square yards	......	„	1 Acre (ac.)
640 Acres	......	„	1 Square mile (sq. mi.)

1 Sq. chain = 484 sq. yds. (= 22 yds. × 22 yds)
10 Sq. chains = 1 Acre.

2. Indian (*Madras*.)

2,400 Square feet	......	make	1 Ground.
24 Grounds or 6,400 Square yards	......	„	1 Cawni.
484 Cawnis	......	„	1 Sq Mile.

NOTE 1.—1 Cawni = 6400 Sq. yds.
But 1 Acre = 4840 Sq. yds
Hence . 1 Cawni is larger than 1 Acre.
Also · 1 Cawni = 6400/4840 Acres = 160/121 Acres
. 121 Cawnis = 160 Acres

NOTE 2 —100 Cents of land = 1 Acre

NOTE 3.—The following land measure is in use in portions of Southern India:—

144 Sq. feet..........................	make	1 Kuli.
100 Kulis..........................	,,	1 Mah.
20 Mahs..........................	,,	1 Veli.
$96\frac{4}{5}$ Velis..........................	,,	1 Square Mile.

1 Veli = 5 Cawnis or 6 & 74/121 Acres.

VI—CUBIC MEASURE.

1,728 Cubic inches (c. in.)	make	1 Cubic foot (c. ft.)
27 Cubic feet............	,,	1 Cubic yard (c. yd.)

NOTE.—A Cubic foot of water weighs nearly 1,000 oz. Avoir., *i.e.*, $62\frac{1}{2}$ lb Avoir.

VII.—MEASURES OF NUMBER.

12 Units..........................	make	1 Dozen.
12 Dozen or 144 Units	,,	1 Gross.
20 Units..........................	,,	1 Score.
24 Sheets of paper..........	,,	1 Quire.
20 Quires	,,	1 Ream.
10 Reams..........................	,,	1 Bale.

VIII.—MEASURE OF TIME

1 English.

60 Seconds (sec.)............	make	1 Minute (min.)
60 Minutes	,,	1 Hour (hr.)
24 Hours..................	,,	1 Day.
7 Days..................	,,	1 Week (wk.)
365 Days..................	,,	1 Common Year.
366 Days..................	,,	1 Leap Year

NOTE 1 —52 weeks are considered as equal to a year

NOTE 2.—A year is divided into 12 calendar months which contain unequal numbers of days. The number of days in each month can be readily known from the following lines.—

Thirty days hath September
April, June, and November,
February hath twenty-eight alone,
And all the rest have thirty-one,
But Leap Year coming once in four,
February then hath one day more

NOTE 3 —The names of the 4 months *April*, *June*, *September* and *November* (containing 30 days each) can be easily found from the *mnemonic* word **ajunesepno**

NOTE 4 —If the number denoting a year be divsible by 4, that year is a leap year For example, 1892 and 1896 are leap years. But the years which complete centuries are not leap years unless the numbers denoting the centuries are divisible by 4. For example, 1600 and 2000 are leap years, because 16 and 20 are divisible by 4; but 1700, 1800, and 1900 are not leap years, because 17, 18 and 19 are not divisible by 4

NOTE 5 —The symbols ″ and ′, which respectively denote *inches* and *feet* should *not* be used to denote seconds and minutes of *time*.

2 Indian.

60 Vinadis................make 1 Naligai.
$7\frac{1}{2}$ Naligais.... . ..... ,, 1 Jamam.
8 Jamams .... ,, 1 Day.

1 Jamam = 3 Hours.
1 Naligai = 24 Minutes.
1 Hour = $2\frac{1}{2}$ Naligais.

IX.—ANGULAR MEASURE.

60 Seconds (60″)....... ..make 1 Minute (1′)
60 Minutes............... ,, 1 Degree (1°).
90 Degrees................ ,, 1 Right Angle.

CHAPTER III.

LINES: RIGHT ANGLES · RECTANGLES: SQUARES.

17. Lines are either *straight* or *curved*; but the word *line* is often used for *straight line*.

18. Straight lines are either *horizontal*, or *vertical*, or *oblique* (slanting).

NOTE —*The following exercise is to be done free-hand and in pencil on plain paper, the straight lines being drawn as straight as possible.*

Exercise 4—(*Practical and Free-hand*).

1. Draw* two *straight* lines and two *curved* lines.

2. Draw (*a*) two *horizontal* lines. (*b*) two *vertical* lines. (*c*) two *oblique* lines slanting to the right, and (*d*) two *oblique* lines slanting to the left

3. Take a point A and from it draw three or more lines in different directions and name them AB, AC, AD, and so on How many lines can be drawn from any point ?

4. Take two points A and B at a small distance from each other and join them (*a*) by a straight line. (*b*) by curved lines. Which of these lines is the shortest ?

5. Take three points A, B C at random and join them successively by the straight lines AB, BC, CA

6. Take four points A, B, C D at random and join them successively by the straight lines AB BC, CD DA

NOTE —The space enclosed by three straight lines is called a *triangle*, by four lines a *quadrilateral*, and by *five* lines a *pentagon*.

7. Draw from any point P two straight lines PA and PB that shall be equal to each other as judged by the eye. Similarly draw three or more equal lines

8. From any point O draw a *horizontal* line OA and then a *vertical* line OB equal to OA.

9. Draw a short vertical line PQ, and from P and Q draw horizontal lines PR and QS that shall be equal to each other.

10. Draw a horizontal line AB, and from A and B draw vertical lines AC and BD that shall be equal to each other.

11 Draw a horizontal line AB and divide it roughly into 2, 3, 4, 5, etc. up to 10 *equal* parts Draw a vertical line and divide it likewise Check your division by means of a narrow strip of paper or a piece of fine thread equal to AB, folded into 2, 3, etc. equal parts.

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* **To the Teacher**—This and the next three Exercises are well calculated to train the hand and the eye of the pupil They are also intended to enable the pupil to draw on plain paper lines and squares *freehand* for *graphical proofs* They will be generally found *more suitable* for this purpose than the very small lines and squares on *squared paper*, even supposing that squared paper is always available.

It should also be borne in mind that the lines and squares, drawn freehand on plain paper, need not be very accurate for purposes of *graphical proofs*.

12. Take a straight line AB (horizontal or vertical) and produce it either way, so that the produced part may be *equal* to AB, or twice three times. etc. AB

13. Draw a long line AB and a short one CD, and find the number of times the latter is contained in the former by marking off on AB lengths equal to CD Produce the last remaining portion of the line, so as to make it equal to CD.

14 Take any point and through it draw half a dozen straight lines in different directions so that each of the lines passes from one side of the point to the other side. What do you infer from this regarding the number of straight lines that can pass through a point ?

15. Draw a short horizontal line AB, and produce it towards B to C, so that AC may be equal to twice AB, three times AB, and so on.

16 From a point A, draw a horizontal line AB and a vertical line AC, so that AB may be thrice another line PQ and AC twice PQ

---

19. Right Angle.—Where two lines meet, an *angle* is formed. Angles are of three kinds, *viz.*, *right*, *obtuse*, and *acute*.

Right angle. Obtuse angle. Acute angle.

Fig. 1.

20. Squares and Rectangles.—Draw *freehand* a *horizontal* line BC ; and from the points B and C draw *vertical* lines BA and CD so that each may be equal to BC. Join AD. It will be seen that the figure ABCD has its four *boundary lines* (called its four *sides*) equal to one another and that all its four angles are *right angles*. Such a figure is called a *square*.

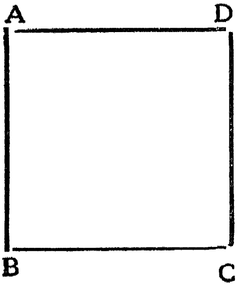


Fig. 2.

If we take the lines BA, CD *shorter* or *longer* than BC (but still *equal* to each other), the figure ABCD so formed will have only its *opposite* sides equal to each other though all the four angles *right angles* (as in a *square*). Such a figure is called an *oblong* or more commonly a *rectangle*,*

NOTE 1.—A figure is named by mentioning the letters at its corners *in order*, commencing at any corner and proceeding in any direction. Thus Fig. 2 may be named ABCD or ADCB.

NOTE 2.—Each of the two *longer* sides of a *rectangle* is called its *length* and each of the *shorter* sides is called its *breadth*.

NOTE 3.—Each of the four equal sides of a square is called its *length*.

Exercise 5.—(*Practical and Free-hand*)

(A) 1 (*a*) How many sides have a *rectangle* and a *square*? (*b*) What is the relation between the *opposite* sides of a rectangle? And between all the four sides of a *square*? (*c*) What kind of angles are the four angles of a *rectangle* and of a *sqaure*? (*d*) In which of the figures, *rectangle* and *square*, is the length *greater* than the breadth and the length *equal* to the breadth?

2 Draw *freehand* a *rectangle* and a *square*

3 Draw free-hand a rectangle whose length is *twice* its breadth, *three* times its breadth, and so on

4. What kind of figure is (*a*) the floor of a room, (*b*) the surface of a wall, (*c*) a tennis court, (*d*) the top of a common brick, (*e*) the top of a *square* brick, (*f*) a page of this book?

(B) 1 Describe a rectangle and divide it into two equal parts by joining the middle points (*a*) of its *longer* sides, (*b*) of its *shorter* sides

2 Describe a rectangle so that its length may be *three times* its breadth, and divide it into three equal *squares*

3 Draw a *long* rectangle and divide it into a number of equal squares of the largest size possible. What kind of figure is left at the end?

4. Draw a rectangle and divide it *first* into 3 equal parts by drawing 2 *breadthwise* lines and *then* into 6 equal parts by drawing another line *lengthwise*

---

* The term *rectangle* literally means a four-sided figure whose angles are all right angles, so that an *oblong* and a *square* are both *rectangles*. But the term *rectangle* is commonly (though not so correctly) used for *oblong*

To the Teacher.—It should be clearly pointed out to the pupil that by a *figure* is meant the *space* enclosed by its sides (boundaries) and not the sides only.

5 (*a*) Describe a rectangle ABCD so that its length may be 4 times a short line XY and the breadth 3 times the same line. Then divide ABCD into 12 equal squares by drawing 4 vertical lines and 3 horizontal lines (*b*) Similarly construct a rectangle which can be divided into 15 equal *squares* and divide it so.

6 Describe a number of *rectangles* and divide them into 2, 3, 4, 5, 6, 8 9, 10, 12, 15, 16 20, 24, 25, 30 equal parts respectively.

7. Describe several *squares* and divide them into 2 3, 4, 5, etc, equal parts respectively Note that in some cases the equal parts are *squares* and in other cases *rectangles*.

21. Paper-folding and Paper-cutting.—The following is a useful exercise and may be gone through at this stage.

Exercise 6 —(*Practical*).

1 Take a half-sheet of foolscap paper (which is *rectangular* in shape) and tear it into 2 equal rectangles after first folding it suitably either *lengthwise* or *breadthwise*, tear each of these two rectangles again into 2 equal rectangles thereby getting 4 equal rectangles Proceed in this manner till you have torn the half sheet into 16 equal rectangles

2. Take a paper rectangle and fold it into 4 equal rectangles then unfold and spread it out flat so as to show the creases which divide it into equal parts

Similarly take several paper rectangles and fold them into 6, 8, 9, 10 12 15, 16, 20, etc equal rectangles respectively and then spread them out flat

3 Learn from the teacher how to cut off from a rectangular piece of paper a paper square of the *largest* size.

4. Take a long narrow rectangular strip of paper and cut off from it as many squares of the largest size as possible.

22. Squared paper.—A sheet of paper whose surface is divided into equal small squares by means of lines drawn across it *vertically* and *horizontally* at *equal* distances, is called *squared paper*.

Exercise 7 —(*Practical*)

(To be done free-hand on squared paper divided into inches and tenths.)

1. Draw on squared paper *horizontal* and *vertical* lines of the following lengths:—4, 6, 8, 9, 10, 15, 20, 30 small divisions of the paper

2 On squared paper describe *rectangles* of the following dimensions and count the small squares in each:—

(*a*) Length 5 small divisions breadth 4 small divisions.
(*b*) Length 10 small divisions breadth 6 small divisions.

3. On squared paper draw *squares* of the following lengths and count the small squares in each :—

(*a*) 8 small divisions, (*b*) 10 small divisions, (*c*) 12 small divisions.

4. On squared paper construct a rectangle 30 small divisions long and 10 small divisions broad, and divide it into 3 equal squares. What is the length of each of these squares? And how many small squares are there in each?

5. On squared paper construct a rectangle 4 inches long and 3 inches broad, and divide it into 12 squares each 1 inch long.

CHAPTER IV.

GRAPHIC REPRESENTATION OF NUMBERS.

23. Graphic Representation of Numbers.—Any abstract number or concrete number of one denomination can be represented *graphically* by *lines*, by *rectangles*, and by *squares*

For example, if one small division of squared paper be taken to represent the abstract number 1 or the concrete quantity Re. 1, then 4 small divisions will represent 4 or Rs 4 5 divisions will represent 5 or Rs 5, and so on Again if 2 divisions be taken to denote 1 or Re 1, 8 divisions will denote 4 or Rs 4, 10 divisions will denote 5 or Rs. 5, and so on.

Similarly, if a *small square* be taken to represent 1, then 4 *small squares* will represent 4, and so on, and if 2 *small squares* which form a *rectangle* be taken to denote 1, then 8 small squares (forming 4 rectangles) will denote 4, and so on

Exercise 8 —(*Graphical*).

(*To be done on squared paper*)

1. Taking one small division of squared paper to represent the unit, draw lengths (horizontal or vertical) to represent 6, Rs 8 10 seers, and so on.

2. Draw a vertical or horizontal line on squared paper to represent the number 5 at 1, 2, 3 etc. small divisions per unit.

3. Draw on squared paper a rectangle to represent 7 feet at 2, 8, 4, etc. small squares per foot

4 Draw on squared paper a rectangle of length 5 sub-divisions and breadth 4 sub-divisions What number will this rectangle denote at 1 small square per unit, at 2 small squares per unit?

5. Draw a square of sides 10 sub-divisions, and note that it denotes the number 100 at 1 small square per unit.

6. Note that in the following figure the portions enclosed by thick lines denote respectively the numbers 100, 127 and 350 at the rate of 1 small square per unit

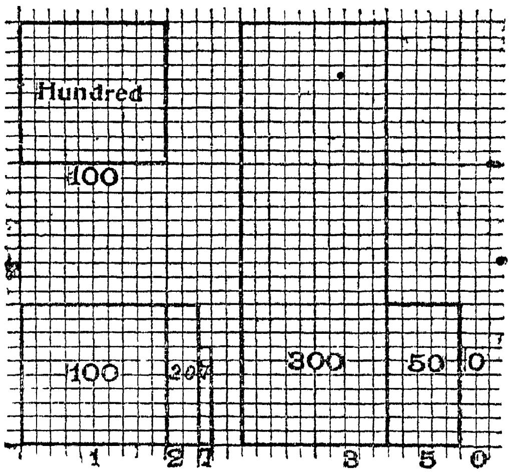


Fig. 2

7 Represent the numbers 200, 119 inches, Rs. 333, 211 on squared paper at 1 small square per unit.

8. Describe rectangles or squares on squared paper to represent the numbers 25, 30, 36, etc., at the rate of 2 small squares per *unit*

Exercise 9—(*Graphical* and *Free-hand*)

(*To be done on* plain *paper*.)

1 Draw a short line OA (*vertical* or *horizontal*) to represent a rupee, and produce it to X so that OX may represent 4 rupees 5 rupees, and so on

2. Draw a line OA to represent 2 yards and produce it to X so that OX may represent 4 yards, 6 yards and so on.

3. Draw a line AB and suppose it to denote 8 miles Into how many equal parts must you divide AB so that each part may denote 1 mile? Divide it so and thicken a portion denoting 3 miles

4 Describe a square and divide it into 16 small and equal squares. If the whole represent 1 rupee (or 16 annas), how many

of the small squares will represent 5 annas? And how many annas will be represented by 9 small squares?

5 Describe a *rectangle* or a *square* and divide it into 20 equal rectangles If the whole square or rectangle denote £1 (or 20 shillings), how many of the small rectangles will denote 4 shillings? Shade them. How many shillings will the remaining rectangles denote?

Exercise 10.—(*Practical*).

1 Take a paper rectangle and fold it into 16 small rectangles and spread it out flat, Taking the *whole* rectangle to represent one rupee (16 annas), scissor off from it a portion representing 6 annas How many annas will the remaining portion represent?

2 Fold a paper square into 36 small and equal squares, and taking the whole square to represent 3 yards or 36 inches, scissor off from it a portion to represent 9 inches, and another portion to represent 12 inches How many inches will be represented by the portion still remaining?

CHAPTER V.

THE FRACTIONS $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ & TENTHS.

24. Fractions—We have already seen (Art, 3) that a protion of a whole unit is called a *fraction* of it. A *fraction* may also be defined as one or more of the *equal* parts into which a unit is divided

For example, if the unit be divided into *two* equal parts, each of the two parts is called *one-half* (1/2) of it, if the unit be divided into 4 equal parts, any one of the four parts is called *one-fourth* (1/4) of the unit, any two are called *two-fourths* (2/4) or *half* of the unit, any three are called *three-fourths* (3/4) of the unit, and all the four parts which make up the entire unit are called *four-fourths* (4/4) of the unit.

Similarly, if the unit be divided into 8 equal parts, each of these parts is called *one-eight* (1/8) of the unit, any two are called *two-eighths* (2/8) of the unit, any three are called *three-eighths* (3/8) of the unit, and so on.

Exercise 11.—(*Graphical*).

1 Illustrate, by means of *straight lines*, *rectangles* and *squares* divided into a suitable number of equal parts the fractions 1/4, 1/2. 3/4, 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8.

2. Take some paper *rectangles* or paper *squares* and, after folding them suitably cut off from them or shade in them parts which represent the fractions 1/4, 1/2, 3/4, 1/8, etc.

25. Tenths of a unit.—For the sake of convenience the unit is often divided into 10 equal parts, and the fractions *one-tenth, two tenths, three-tenths, etc.*, which are ordinarily denoted by the symbols 1/10, 2/10, 3/10, etc., are also denoted for the sake of brevity and convenience by the symbols ·1, ·2, ·3, etc., where the elevated *dot* or *point* placed before the figures 1, 2, 3, etc., indicates that the numbers following the dot or point are those of parts that have been taken out of the *ten* equal parts into which the unit is divided. Thus we have the fractions ·1, 2, ·3, ·4, ·5, ·6, 7, ·8, ·9 which respectively denote (and may be read as) one-tenth, two-tenths, three-tenths, etc., or as *decimal* one, *decimal* two, etc., or as *point* one, *point* two, etc.

It will be easily seen that 10 tenths make the whole unit and may be written as 1·0 where 0 denotes that there are no odd tenths in the number And 1·0 may be read as 'one-decimal, nought,' or as 'one, point, nought.'

Exercise 12.

1 Supposing the unit to be divided into 10 equal parts, what is the name given to any *one* of those parts, to any *two*, to any *three*, etc., of those parts?

2. In what two ways may the fractions one-tenth, two-tenths, three-tenths, &c., be represented symbolically?

3. Read the following fractions :—

·1, ·3, ·5, ·9, 4/10, 7/10, 6/10. What portion of the unit does each of them denote?

4. In the subjoined diagram the whole rectangle ABCD is divided into 10 equal parts What fraction of the whole is each of the rectangles AEXD, GHUV, AFWD, AHUD, EMRX, BCTK, OPSL?

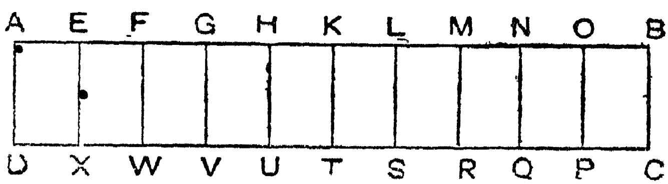


Fig. 3

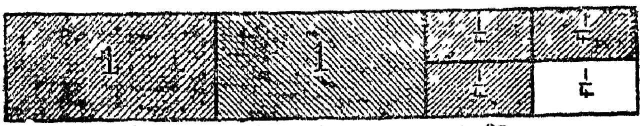
5 Fold some paper rectangles or paper squares suitably, and cut off from them or shade in them parts representing the fractions ·1, ·2, 3, ·4, 5, ·6, ·7, 8, 9.

26. Mixed Number.—A number consisting of a whole number or integer together with a fraction is called a *mixed number*.

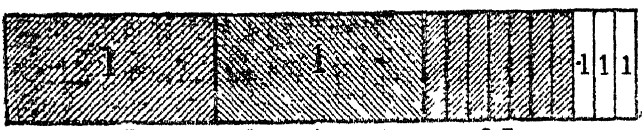
Explanation. Suppose you first take *two* oranges and then *half* an orange, the number of oranges you will now have will be two and a half, which is written $2\frac{1}{2}$ If you took three oranges first and then 1/4 of an orange, you would have in all $3\frac{1}{4}$ oranges (three and one-fourth oranges). Similarly if you take 1 orange and 3 of an orange, the total number of oranges will be 1·3 (one and three-tenths), 2 oranges and 7 of an orange will make 2·7 oranges (two and seven tenths of an orange), and so on.

The numbers $2\frac{1}{2}$, $3\frac{1}{4}$, 1 3 and 2 7 are *mixed numbers*.

27. Graphical representation of mixed numbers.



The shaded portion represents $2\frac{3}{4}$



The shaded portion represents 2·7

Fig. 4

Exercise 13.—(*Graphical*)

Represent *graphically* the following numbers :—

1. Two and a half.	2 $1\frac{1}{4}$	3, $2\frac{3}{4}$
4 One and three-tenths.	5 3 5	6 1·9,

CHAPTER VI.

SIMPLE ADDITION

28. Addition is the process of combining two or more numbers into one number. The numbers to be added must be either *all* abstract numbers, or *all* concrete numbers of the *same* kind. For example, we can add 3 and 5 without thinking of any particular object and say 8; or we can add

3 *books* and 5 *books* and say 8 books; but we cannot add 3 *books* and 5 *rupees*.

29. The result of the addition of two or more numbers is called their **sum**. Thus since 3 and 5 make 8, 8 is the *sum* of 3 and 5.

30. **Simple addition** is the addition of abstract numbers as 5, 6 and 7; or of concrete numbers of the same kind and denomination as 5 *rupees*, 6 *rupees* and 7 *rupees* or 5 *annas*, 6 *annas*, and 7 *annas*.

31. The sign + (called *plus*) is the sign of addition. Thus, 2 + 3 means that 3 is to be added to 2; 2 + 3 + 4 means that 3 is to be added to 2 and to their sum 4 is to be added.

32. The sign = (which is read '*is equal to*') is the sign of equality.

Thus, 4 + 5 = 9 means that 5 added to 4 is equal to 9.

NOTE.—The student may prove this *graphically* by taking on squared or plain paper a line ABC in which AB and BC represent 4 and 5 respectively.

Exercise 14.—(*Oral*).

(*a*) Write down a row of figures, thus—

7, 6, 5, 9, 6, 3, 4, 8, 6, 1, 4, 3, 6.

Add up the figures beginning from the left,

Thus—13 18, 27, 33. &c., without saying 7 and 6 make 13, 13 and 5 make 18, &c,

Again add up the same figures beginning from the right.

Thus—9, 13, 14, 20, &c., without saying 6 and 3 make 9, 9 and 4 make 13, &c.

(*b*) Write down a column of figures and add, first beginning from the top, then from the bottom.

(*c*) Find the sum of the 10 numbers from 1 to 10 (i) by counting from 1 forwards, (ii) from 10 backwards.

---

33. *The sum of two or more numbers is the same in whatever order we add them* For example, 8 + 5 and 5 + 8 are each equal to 13, and 4 + 5 + 6, 5 + 4 + 6, and 6 + 5 + 4 are each equal to 15.

Exercise 15.—(*Graphical*)

Show *graphically*, by means of suitable lengths taken on squared or plain paper that (*a*) $4 + 7 = 7 + 4$, (*b*) $8 + 3 + 6 = 8 + 6 + 3 = 6 + 3 + 8 = 3 + 6 + 8$

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34. *Mental addition of any two numbers of not more than three digits*—

Examples—Add together (1) 25 and 72.
(2) 137 and 22
(3) 239 and 426.

The mental process is as follows :—

(1) $25 + 70 = 95$;
$95 + 2 = 97$ *Ans*
(2) $137 + 20 = 157$;
$157 + 2 = 159$ *Ans.*
(3) $239 + 400 = 639$;
$639 + 20 = 659$;
$659 + 6 = 665$. *Ans.*

NOTE.—After some practice the student should, from a mere look at the given numbers, simply say
(1) 25; 95; 97.
(2) 137; 157 : 159.
(3) 239, 639; 659; 665.

Exercise 16—(*Oral*).

(*a*) Find the sum of (1) $72 + 109$, (2) $32 + 72 + 49$, (3) $128 + 99$, (4) $572 + 428$.

(*b*) In the following tables (called *magic squares*) add the numbers *vertically*, *horizontally*, and *from corner to corner*

(1)

41	34	37	46
36	47	40	35
42	33	38	45
39	44	43	32

(2)

267	268	263
262	266	270
269	264	265

(*c*) Find the sum of the 10 numbers from 11 to 20 (1) by counting from 11 forwards, (2) from 20 backwards

(*d*) Similarly find the sum of the 6 numbers from 26 to 31

(*e*) What number should be added to 350 to make 500 to 473 to make 700?

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35. The following is an example of *written* addition :—

Example.—Add together 40329, 36154, 3005, 36154, and 7343.

Solution.	Wording
40329	
36154	7, 12, 16, 25 (set down 5 and carry 2).
3005	2 6, 11, 16, 18 (set down 8 and carry 1).
36154	1 4, 5 6 9 (set down 9]
7343	7, 13, 16 22 (set down 2 and carry 2)
	2, 5 8 12 (set down 12)
122985 *Ans*	NOTE.—The words within brackets are not to be uttered.

36. Proof of Addition.—In every example in addition, after getting the answer in the usual way, that is, by adding the columns upwards, add the columns again downwards. If the same answer is got as before, it may be supposed that the result is correct, since it is very unlikely that the same mistake would occur in the answers obtained in different ways.

NOTE —The student is warned against the habit of checking work in addition by adding the figures in the same order several times For, when figures are added several times in the *same* order, mistakes committed in the first instance are likely to repeat themselves.

Exercise 17

(*a*) Add together the following groups of numbers and verify your answers :—

1.	2.	3	4
48693	9876	45	66666
6907	548	678	6777
77234	2123	9003	88888
9988	4567	765	9999
37178	89	999	32246

5. 56361 + 4905 + 870 + 83 + 7648 + 184534.

6 Five lakhs, fourteen thousand, four hundred and four, six crores, eight thousand and eighteen, thirty-four thousand, six hundred and five, forty-five lakhs, nine hundred and eighty.

(*b*) Find the sum of *five* numbers each equal to nine lakhs, nine thousand and ninety-nine

37. *Example.*—Add the numbers 4305, 72486, 1067, 44869, 536 *without writing them in a column.*

Solution.

6, 15, 22, 27, 32 ; set down 2, and carry 3 ,
3, 6, 12, 18, 26 26 , set down 6, and carry 2 ,
2, 7, 15, 15, 19, 22 , set down, 2, and carry 2 ;
2, 6, 7, 9, 13 ; set down 3, and carry 1 ,
1, 5, 12, set down 12.

NOTE.—It would be helpful to the student to put a dot at the *top* of each figure as soon as it is taken up. When verifying, the dots may be put at the *bottom* of the figures

Exercise 18.

(A) Find the sum of the following numbers without writing them one under another :—

1. (*a*) 467, 806, 1234, 7054 (*b* 1234, 456, 7891.
2. (*a*) 1009, 7054, 325, 105, 7054. (*b*) 987, 6543, 2105, 234
3. (*a*) 2457, 469, 7458, 12549. (*b*) 4038, 4608, 4806, 4680.

(B) Add the following numbers *vertically* and *horizontally*:—

	1	2.	3.	4.	5.
6.	4859	84035	235	5556	47774
7	23360	932	4386	63328	667
8.	7984	62380	739	4994	5480
9.	44075	7724	5528	840	735
10	6967	888	1736	12588	6372

(C) Find the sum of all the numbers given below by first writing down the totals of the *rows* and then adding up these totals for the grand total. Check this grand total by writing down the totals of the *columns* and adding them up :—

					Total of rows
	123	86	90	424	
	89	239	120	88	
	600	428	79	55	
	46	47	204	609	
	121	8	745	308	
Totals of columns					

38. Addition of the fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$—We shall now give some exercises and examples involving the addition of the fractions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.

Exercise 19.—(*Graphical*)

1 Describe a *square* or a *rectangle* to denote the unit, divide it into 4 equal parts, and show that $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$, $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$, $\frac{1}{4}+\frac{3}{4}=1$, $\frac{1}{2}+\frac{1}{2}=1$.

2 Describe two equal and contiguous *squares* or *rectangles* to represent *two units*, divide each into 4 equal parts, and shew that $\frac{1}{2}+\frac{3}{4}=1\frac{1}{4}$, $\frac{3}{4}+\frac{3}{4}=1\frac{1}{2}$, $1\frac{1}{4}+\frac{1}{2}=1\frac{3}{4}$.

Exercise 20 —(*Oral*).

Write down a row of numbers (integers and fractions), thus — $\frac{1}{2}$, 2, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{3}{4}$, $\frac{1}{2}$, 3, 0, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$.

Add up the numbers beginning from the left Thus $2^1/_2$, $2^3/_4$, $3^1/_2$, $4^1/_4$ &c

Again add up the same numbers beginning from the right. Thus—1, $1^1/_2$, $1^1/_2$, $4^1/_2$, 5, &c.

Examples,—Find the value of (1) $3\frac{1}{2}+2\frac{3}{4}$.

(2) $47\frac{3}{4}+39\frac{3}{4}$

The mental steps of the solution are as follows :—

(1) $3^1/_2+2=5^1/_2$; $5^1/_2+\frac{3}{4}=6^1/_4$ *Ans,*	(2) $47^3/_4+30=77^3/_4$; $77^3/_4+9=86^3/_4$; $86^3/_4+{}^3/_4=87^1/_2$ *Ans.*

NOTE —After some practice the student must say no more than

(1) $3\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{4}$, (2) $47\frac{3}{4}$, $77\frac{3}{4}$, $86\frac{3}{4}$, $87\frac{1}{2}$.

Exercise 21 —(*Oral*)

(*a*) Find the value of—

1. $4\frac{3}{4}+7\frac{3}{4}$. 2. $8\frac{1}{4}+9\frac{1}{2}$. 3. $1\frac{1}{2}+4\frac{3}{4}$.

4. $7\frac{1}{2}+6\frac{1}{2}$. 5 $8\frac{3}{4}+6\frac{1}{4}$ 6 $3\frac{1}{4}+8\frac{3}{4}$.

7. $25\frac{3}{4}+49\frac{1}{2}$. 8. $17\frac{1}{2}+79\frac{1}{2}$. 9. $120\frac{1}{2}+4\frac{1}{4}$.

(*b*) In the following *magic squares* add the numbers vertically, horizontally, and from corner to corner:—

(1)

$\frac{1}{4}$	$3\frac{3}{4}$	$5\frac{1}{2}$	$4\frac{1}{2}$	$2\frac{1}{4}$
$5\frac{3}{4}$	$4\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	3
$2\frac{1}{2}$	$\frac{1}{2}$	$3\frac{1}{4}$	6	4
$3\frac{1}{2}$	$5\frac{1}{4}$	5	$1\frac{3}{4}$	$\frac{3}{4}$
$4\frac{1}{4}$	2	1	$2\frac{3}{4}$	$6\frac{1}{4}$

(2)

$12\frac{1}{2}$	$3\frac{3}{4}$	$7\frac{1}{2}$	$18\frac{3}{4}$
$6\frac{1}{4}$	20	$11\frac{1}{4}$	5
$12\frac{3}{4}$	$2\frac{1}{2}$	$8\frac{3}{4}$	$17\frac{1}{2}$
10	$16\frac{3}{4}$	15	$1\frac{1}{4}$

Exercise 22.

(*a*) Add together—

1.	2	3.	4	5.
12 1/2	353/4	26 1/2	1506 1/2	2076
30 3/4	79 1/4	16 3/4	769 3/4	706 3/4
27 1/2	129 1/2	25 1/4	70 3/4	4654 1/2

(*b*) Find the sum of *four* numbers each equal to ninety lakhs, seven thousand nine hundred and eleven and three-fourths.

---

39. Addition of the fractions ·1, ·2, ·3, etc.—We shall now give some exercises and examples involving the addition of ·1, ·2, ·3, etc. up to ·9

Exercise 23—(*Graphical*)

1. Describe a *rectangle* or a *square*, to represent the unit, divide it into 10 equal parts and prove that 1 + ·3 = ·4, ·6 + ·4 = 1·0. ·5 + 2 = ·7

2. Describe two contiguous *squares* or *rectangles* to represent *two units* divide each into 10 equal parts, and show that ·4 + ·8 = 1·2, ·6 + ·9 = 1·5, 1·2 + ·8 = 2·0.

---

40. *Example*—Add together ·7, ·5, ·6, ·9, ·4.

Solution

(1) Beginning from the left we say 7 *tenths*, 12 *tenths*, 18 *tenths*, 27 *tenths*, 31 *tenths*, and set down 3·1 for the answer: *or* we may say simply 7, 12, 18, 27, 31 and set down 3·1 for the answer.

(2) Beginning from the right, we say 4, 13, 19, 24, 31, answer 3·1.

Exercise 24.—(*Oral*)

(*a*) Write down a row of *tenths* and add them up (i) beginning from the left, (ii) beginning from the right.

(*b*) Write down a column of *tenths* and add them up (i) beginning from the bottom, (ii) beginning from the top.

---

41. *Example.*—Add together 45·4, 88·6, 230·7, 109·0, and 29·8.

45·4 88·6 230·7 109·0 29·8 ——— 503·5	*Wording* 8 tenths, 15 tenths, 21 tenths, 25 tenths or 2·5; set down ·5 and carry 2 2, 11, 20, 28, 33, set down 3 and carry 3, and so on to the end.

Exercise 25

(*a*) Add together the following groups of numbers:—

1	2.	3	4
24·8	18·8	123·8	153·7
17·9	20·9	89·4	68·7
48·6	13·7	235·4	344·5
12·0	46·5	100·5	544·3
49·9	10·1	77·7	888·8

5. 88·6, 11·9, 55·6, 43·8, 22·0, 20·0

6. Thirty five and seven-tenths, forty and five-tenths, seventy-one and one-tenth, forty-nine and two-tenths.

(*b*) Find the sum of seven numbers of which *three* are each equal to 10·5 and *four* are each equal to 109·5.

CHAPTER VII.

SIMPLE SUBTRACTION.

42. Subtraction is the process of finding how much is left when a smaller number is taken from a greater number. The number to be subtracted (called the *subtrahend*) must be of the same kind as the number from which it is to be subtracted (called the *minuend*). For example, we can subtract 4 from 9, or 4 *rupees* from 9 *rupees*, or 4 *yards* from 9 *yards*, but we cannot subtract 4 *rupees* from 9 *yards*. The result of subtracting one number from another is called the **remainder**, or the **difference** of the two numbers. Thus, when we subtract 4 from 9, 5 is the *remainder*.

43. **Simple Subtraction** is the subtraction of an abstract number from another as 5 from 9, or of a concrete number from another of the same kind and denomination as 5 *rupees* from 9 *rupees*.

44. The sign — (called *minus*) is the sign of subtraction. Thus, 9—4 means that 4 is to be subtracted from 9; and 9—4=5 means that the *remainder* after subtracting 4 from 9 is 5, and is read 'nine *minus* four is equal to five.'

NOTE —The sign $\sim$ is sometimes used as the sign of subtraction. Thus 4 $\sim$ 9=5 means that the *difference* between 4 and 9 is 5.

Exercise 26—(*Graphical*).

1 Prove *graphically* by taking suitable lengths on squared or plain paper, that 14—9=5, 18—7—3=8.

---

45. *When three or more numbers are connected by the signs of addition and subtraction, the result will be the same in whatever order the processes of addition and subtraction are performed*

For example, 16—7—5 and 16—5—7 are each equal to 4: 9—4 +6 and 9+6—4 are each equal to 11. Similarly 15—9+4—3=15 +4—3—9=15—9—3+4=4+15—9—3 and so on.

Exercise 27 —(*Graphical*).

1. Draw on squared or plain paper a line AB 10 units long, produce it to C so that BC may be 5 units; then cut off from AC a length CD equal to 8 units hence shew that 10+5—8=7. Again take AB=10 units, cut off from it BD = 8 units, then produce AD to C so that DC=5 units, hence show that 10—8+5 = 7 Now note that you have thus proved that 10 + 5 — 8 = 10 — 8 + 5.

2. Similarly show that 12 — 4 + 6 = 12 + 6 — 4. 15 — 8 — 3 = 15 — 3 — 8. 17 — 4 — 8 + 5 = 17 — 8 + 5 — 4

Exercise 28 —(*Oral*).

(*a*) Write down a row of ten or twelve figures thus—

7, 2 1, 5 9, 6, 7, 9 1, 1, 6, 2,

and subtract these figures in succession from any number greater than 100, (1) beginning from the right (2) beginning from the left.

Thus, taking 120, say (1) 118, 112, 111, 110 101, &c.
(2) 113, 111 110, 105, &c

(*b*) From any number greater than 55, subtract in succession the first 10 numbers commencing first with 1, and then with 10.

46. *Mental Subtraction of numbers of two or three digits.*

Example—Subtract (1) 37 from 70, (2) 67 from 175. (3) 189 from 351.

Mental process—

(1) $70 - 30 = 40$,
$40 - 7 = 33$ *Ans.*

(2) $175-60=115$,
$115-7=108$ *Ans*

(3) $351-100=251$;
$251-80=171$;
$171-9=162$ *Ans.*

NOTE—After some practice, the student should, from a mere look at the given numbers, simply say—

(1) 70 40, *33*
(2) 175, 115, 1*08*
(3) 351, 251, 171, *162*

Exercise 29.—(*Oral*)

(*a*) Find value of—

1.	80—46	2.	200—74	3.	210—83.
4	350—155.	5	406—269	6.	354—175

(*b*) From any number greater than 150, subtract in succession the numbers in each row and each column of magic square (1) in Exercise 16 on page 23.

(*c*) From any number greater than 900 subtract in succession the numbers in each row and each column in magic square (2) in Exercise 16 on page 23

(*d*) From any number greater than 500 subtract in succession the numbers from 10 to 20, from 15 to 25, and so on.

(*e*) 1. At the Census of 1911 the total population of India was 315 millions of which 161 millions were males. Find the female population and the excess of the male population over the female population

2. The length of the earth's equator is 25 thousand miles, and the distance of the moon from the earth is 240 thousand miles, What is the difference between these two distances?

---

47. *Written subtraction* may be done by *three* methods—*viz*. (1) method of *decomposition* (commonly called method of *borrowing*). (2) method of *equal addition*. (3) method of *complementary addition*.

We shall here do a sum by the first two methods.

Example.—Subtract 296035 from 420604.

(1) *Solution by the method of decomposition.*

420604 296035 124569	**14—5, 9 . 9—3, 6 ; 5—0 5 ; 10—6, 4 ; 11—9, 2 , 3—2, 1.**

(2) *Solution by the method of equal addition,*

420604 296035 124569	**14—5, 9 ; 10—4, 6 ; 6—1, 5 ; 10—6, 4 ; 12—10, 2 ; 4—3, 1.**

Explanation of method (2)—Since 5 cannot be taken from 4, we add 10 to 4 which makes 14, and 5 taken from 14 leaves 9. Now setting down 9 we add 1 (*i e*, 1 ten to 3 tens) which makes 4. But since 4 cannot be taken from 0, we add 10 to 0 which makes 10. Then 4 taken from 10 leaves 6 which is set down Proceeding in this way, we get 124569 for the difference of the two numbers.

48. *Proof of Subtraction*—To prove subtraction add the *remainder* to the *smaller number*; if the result is the same as the *larger number*, the answer is right.

Exercise 30.

(*a*). Perform the following subtractions by *two* methods and verify your answers —

1		2		3		4.	
	683125 492816		290600 179605		444444 55555		502030 76567

(*b*) ***From*** one crore forty thousand four hundred and six *take away* ninety lakhs nine hundred and ninety.

(*c*) Find the value of—

1. $10085 - 9999$. 2. $2633 \sim 18765$
3. $5555 \sim 112344$. 4. $54322 \sim 100000$.

(*d*) Find the *difference* between—

1 4589 and 10254. 2. 200001 and 16666.

(*e*) Mount Everest in India is the highest peak in the world and is 29,012 feet high How much higher is it than Mount Blanc, the highest peak in Europe, which is 15 732 feet high ?

(*f*) Find the value of—

1. $22000 - 1450 - 2765$. 2. $12000 - 8888 - 1234$.
3. $7545 + 4051 - 9086$. 4. $10000 - 4444 + 5055$.
5 $9555 - 3888 - 1667 - 2223$

49. *Subtraction of the fractions* 1/4, 1/2, 3/4 —We shall now give some examples and exercises involving the subtraction of the fractions 1/4, 1/2, 3/4.

Exercise 31—(*Graphical*)

1 Describe a *rectangle* or a *square* to represent the unit, divide it into *four* equal parts, and show that 1/2 −1/4 =1/4 3/4 −1/2=1/4, 3/4−1/4=1/2 1—1/2=1/2, 1—1/4=3/4, 1 −3/4=1/4.

2. Describe two equal contiguous *squares* or *rectangles* to represent *two units*, divide each into 4 equal parts, and prove that $1\frac{1}{4}$—1/2 =3/4, $1\frac{1}{2}$—3/4 =3/4 2—1/2=$1\frac{1}{2}$ 2 −3/4=$1\frac{1}{4}$.

Exercise 32—(*Oral*).

Write down a row of numbers (integers and fractions) thus—1/2, 2, 1/4 1/2 3/4, 1 1/2, 1/4, 3, 3/4, and subtract the numbers in order from any number of two digits, (1) beginning from the right, (2) beginning from the left.

Thus taking the number 10, say

(1) 10, 9 1/4, 6 1/4, 6, 5 1/2, 4 1/2, &c.

(2) 10, 9 1/2, 7 1/2, 7 1/4 6 3/4 6 &c.

---

50. *Examples*—Subtract (1) $8\frac{1}{4}$ from 15, (2) $2\frac{3}{8}$ from $4\frac{1}{2}$, (3) $27\frac{1}{2}$ from $85\frac{1}{4}$.

The mental process is as follows:—

(1) $15 - 8 = 7$,

$7 - \frac{1}{4} = 6\frac{3}{4}$. *Ans.*

(2) $4\frac{1}{2} - 2 = 2\frac{1}{2}$,

$2\frac{1}{2} - \frac{3}{4} = 1\frac{3}{4}$. *Ans.*

(3) $85\frac{1}{4} - 20 = 65\frac{1}{4}$,

$65\frac{1}{4} - 7 = 58\frac{1}{4}$,

$58\frac{1}{4} - \frac{1}{2} = 57\frac{3}{4}$ *Ans.*

Note:—After some practice the student should, from a mere look at the given numbers simply say—

(1) 15, 7, $6\frac{3}{4}$.

(2) $4\frac{1}{2}$, $2\frac{1}{2}$; $1\frac{3}{4}$

(3) $85\frac{1}{4}$, $65\frac{1}{4}$; $58\frac{1}{4}$, $57\frac{3}{4}$

Exercise 33.—(*Oral*).

(*a*) Find the value of—

1. $15 - 9\frac{1}{2}$. 2. $14 - 8\frac{3}{4}$ 3 $5 - 1\frac{1}{4}$. 4 $10 - 3\frac{3}{4}$.
5 $5\frac{1}{2} - 3\frac{1}{4}$. 6. $6\frac{1}{4} - 2\frac{1}{2}$. 7. $12\frac{1}{2} - 9\frac{3}{4}$. 8. $15\frac{1}{2} - 2\frac{1}{4}$.
9. $25\frac{3}{4} - 16\frac{1}{2}$. 10. $121\frac{1}{4} - 85\frac{3}{4}$. 11 $135 - 86\frac{1}{4}$ 12. $100 - 56\frac{1}{4}$.

(*b*) From any number greater than 20 subtract in succession the numbers in each column and row of the *magic square* (1) in Exercise 21 at page 26.

(*c*) From any number greater than 50, subtract in succession the numbers in each row and each column of *magic square* (2) in Exercise 21 at page 26.

Exercise 34.

Perform the following subtractions and verify your answers.—

1.	$1508\frac{1}{4}$	2.	$1050\frac{3}{4}$	3	$2050\frac{3}{4}$	4.	$1614\frac{1}{3}$
	$886\frac{1}{4}$		$643\frac{1}{4}$		$1701\frac{1}{2}$		$765\frac{2}{3}$
5	$121\frac{3}{4}$	6.	$1009\frac{1}{2}$	7	86901	8.	1000
	$76\frac{1}{2}$		$909\frac{1}{4}$		$4094\frac{1}{3}$		$764\frac{1}{2}$

---

51. Subtraction of tenths.—We give below exercises and examples in the subtraction of the fractions ·1, ·2, ·3, etc.

Exercise 35—(*Graphical*)

1. Describe a *rectangle* or a *square*, and taking it to represent the unit, divide it into 10 *equal* parts, and prove that ·9 — ·4 = ·5, ·5 — ·3 = ·2, 1 0 — 4 = 6.

2. Describe two equal and contiguous *rectangles* or *squares* to represent *two units* divide each *rectangle* or *square* into 10 *equal* parts, and show that 1 5 — 6 = 9, 1 8 — 9 = ·9, 2 — 5 = 1 5, 2 — 1 3 = ·7.

Exercise 36 —(*Oral*)

Find the value of —

1. ·9 — ·4. 2. 1·5 — 6. 3. 1 0 — 7 4 ·8 — 2 5

5. ·8 + 4 — ·6 6 4 5 — 2 6 + 1 1. 7 5 — 2·4 — 1·6

Exercise 37.

(*a*) Perform the following subtractions and verify your answers:—

1,	24 8	2.	125·4	3	248 0	4	1000 6
	15·3		72 6		144 7		987 8

5. 454 3 — 2080 6 200 — 175 9. 7. 108 5 — 45·9

(*b*) Find the value of:—

1. 48 5 — 4·8 + 61 8. 2. 1000 — 125 5 — 720 6

(*c*) *From* one hundred *subtract* fifty and seven tenths and *from* the remainder *take away* fifteen and six-tenths

(*d*) *From* 1000 *subtract* 213·4 *three* times.

Exercise 38

1. Subtract the *largest* number of 4 digits from the *smallest* number of 6 digits.

2 From the *largest* number of 5 digits beginning with 6 and ending with 5, take away the *smallest* number of 5 digits beginning with 6 and ending with 5

3. From the *least* number of 7 digits subtract the sum of the *six* numbers of 4 digits that can be formed with the figures 3, 3, 2, 2

4 What is the difference between the *local* values of 83 and 54 in the number 2830546 ?

CHAPTER VIII.

MEASURING WITH THE TAPE.*

52. Measurement correct to a foot—If the distance between two points be found by measurement to consist of a whole number of feet together with a fraction of a foot, the fraction is often left out if it is *less than* 1/2 and taken as I if it is *exactly* 1/2 or *more than* 1/2. For example, $74\frac{1}{4}$ ft., $74\frac{1}{2}$ ft. and $74\frac{5}{8}$ ft. are taken respectively as 74 ft., 75 ft., and 75 ft. In these cases the measurement is said to be expressed *correct to a foot.*

NOTE 1.—For measuring with the tape lengths greater than, say, 3 or 4 feet, two persons are required.

NOTE 2.—In measuring with the tape, the outer edge of its ring may be placed at one end of the distance to be measured *or* the mark 1 (indicating a length of 1 ft) in the latter case 1 foot is to be deducted from the length denoted by the mark at the other end of the distance measured

NOTE 3.—If the distance to be measured be longer than the length of the tape, a string of sufficient length mey be tightly stretched between the ends of this distance and the string then measured in parts and the total length found out from these measurements

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* To the Taacher—Measuring tapes are available in *three* lengths namely 25 feet, 50 feet and 100 ft, and two or three of each of these kinds may be provided for the class Each of the pupils may also be asked to provide himself with some suitable length of ordinary tape and divide it into foot lengths by placing the foot-marks on slips of paper pasted on the tape at the proper places. In this case *fractions* of a foot may be judged by the eye, as to whether they are *less than*, or *equal to*, or *greater than* 1/2,

Exercise 39—(*Practical*).

1. With the *measuring tape* measure each of the following in feet and inches. Let the measurement be correct to 1 foot or 1 inch as the case may be:—

(a) The length, breadth and diagonal (*i e.*, distance from corner to corner) of the floor of a room, a tennis or badminton court, a black-board, a bench, a table, etc.

(*b*) The length of the shadow thrown by a post fixed in the ground at different hours of the day, a log of wood lying on the ground, a wall, your own shadow at different hours of the day, the circumference * of the curb of a circular well, of a tub at top and bottom, of a cask at the middle and at the ends, of a round table etc

2 Estimate the length and breadth of a hall, a table, etc., and check your estimate by measuring with the tape

---

53. Measuring with the Tailor's Tape†.—Some small measurements may be conveniently made with the *Tailor's Tape*, which is 5 feet or 60 inches long.

Exercise 40—(*Practical*).

Find, by measuring with th *Tailor's tape*—

1. The height of a chair, a stool, a box, *et cetera*.
2. The length, breadth, and diagonal of a sheet of card-board, an exercise-book, a sheet of foolscap paper, an atlas, a slate, etc.
3. The circumference of a bucket at the top and at the bottom.
4. The length of an umbrella, a walking stick, etc.
5. The chest, neck, head, etc of two or three boys.

CHAPTER IX

SIMPLE MULTIPLICATION.

54. Multiplication is a short method of finding the sum of a number *repeated or added to itself* any number of times. The number that is to be *repeated* is called the

---

* These may be first measured with a string and the string then measured with the tape

† The class may well be provided with half-a-dozen tailor's tapes, and each pupil may be asked to provide himself with one.

multiplicand, the number that denotes how *many times* the *multiplicand* is to be *repeated* is called the **multiplier**, and the *sum* thus found is called the **product** of the two numbers.

Explanation—Suppose we write down the number 5 *four* times and find by actual addition that the sum is 20 If we remember the result of this addition, and say at once that 4 *fives* added together make 20, or briefly 4 *times* 5 is 20, we are said to *multiply* 5 *by* 4 In this case 5 is the *multiplicand*, 4 the *multiplier*, and 20 their *product*.

55. The sign × (which is read *into* or *multiplied by*) is the sign of multiplication.

Thus 5×4 means that 5 is to be multiplied by 4, and $5 \times 4 = 20$ (which is read 'five into four is equal to twenty') means that the product of 5 multiplied by 4 is equal to 20 Again $2 \times 3 \times 4$ means that 2 is to be multiplied by 3 and the result by 4

Exercise 41—(*Graphical*)

1 Draw on squared or plain paper a rectangle containing 3 rows of 5 small squares each, and prove by counting these squares that 5×3 (*i e*, 3 times 5) $= 15$.

2. Draw two contiguous rectangles each containing 4 rows of 3 squares each, and show by counting the squares that $4 \times 3 \times 2 = 24$.

3. Prove that $4 \times 6 = 24$, $5 \times 5 = 25$, $3 \times 3 \times 2 = 18$, $2 \times 5 \times 3 = 30$.

---

56. Multiplication Table—It may be noted that the *Multiplication Table* which the student is presumed to have already learnt by heart up to 16 times 16 has been constructed by *actual addition*.

NOTE.—Since the sum of any number of *ciphers* is *cipher*, the product of 0 by any number is 0.

57. When the quantity to be multiplied (*i.e*, repeatedly added) is a simple quantity like Rs 5, or 5, the multiplication is called **simple multiplication**, and when it is a compound quantity like Rs. 5 4 as or 2 feet 3 inches, it is **compound multiplication** We shall in this chapter deal only with *simple multiplication*.

58. Continued Multiplication.—The multiplication of 3 or more numbers is called their **continued multiplication** and the product is called their **continued product.**

For example, the *continued product* of $2 \times 3 \times 4$ is 24, the *continued product* of $2 \times 2 \times 4 \times 5$ is 80.

59. It is evident that the *multiplicand*, *i.e.*, the number to be repeatedly added, may be any number, *abstract* or *concrete*: but the *multiplier*, which denotes only the number of *times* the multiplicand is to be repeated, must necessarily be an *abstract* number, since it would be absurd, for example, to speak of 5 rupees multiplied by 4 rupees

It is also clear that the multiplicand and the product must be *both* abstract quantities, or *both* concrete quantities of the same kind. Thus 5 multiplied by 4 gives 20, while 5 *rupees* multipiled by 4 gives 20 rupees.

60. Order of Multiplication.—From the multiplication table we find that $8 \times 4 = 32$ and $4 \times 8 = 32$; $3 \times 2 \times 5 = 30$, $2 \times 5 \times 3 = 30$; and so on. From these examples we may infer that *the product of two or more numbers is the same in whatever order they are multiplied.*

Exercise 42—(*Graphical*).

1. Show *graphically* that $5 \times 4 = 4 \times 5$ by describing a rectangle whose length is 5 times and breadth 4 times a small length, dividing it into 20 equal squares and considering these squares first as 4 rows of 5 squares each and then as 5 columns of 4 squares each.

2. Prove likewise that $3 \times 4 = 4 \times 3$, $6 \times 4 = 4 \times 6$.

3. Describe 4 contiguous rectangles 3 units of length long and 2 units of length broad, and divide each rectangle into 6 equal squares The large rectangle will denote the product of $3 \times 2 \times 4$ or $2 \times 3 \times 4$. Again consider the large rectangle as 2 rectangles of 12 squares each, (*i e.*, 4×3 or 3×4 squares), then the whole rectangle will denote $4 \times 3 \times 2$ or $3 \times 4 \times 2$. Hence show that $3 \times 2 \times 4 = 2 \times 3 \times 4 = 4 \times 3 \times 2 = 3 \times 4 \times 2$

4. Prove likewise that $2 \times 5 \times 3 = 5 \times 2 \times 3 = 5 \times 3 \times 2 = 3 \times 5 \times 2$.

61. Factors.—When several numbers are multiplied together, each of them is called a *factor* of the product.

Thus, since $3 \times 5 = 15$, 3 and 5 are the **factors** of 15; similarly 2, 3 and 5, or 15 and 2, or 10 and 3, or 5 and 6 are the **factors** or 30.

Exercise 43—(*Oral*).

1. Divide the following numbers into *any* two factors:—
   15, 25, 49, 48, 35; 77, 65, 91; 64; 63.
2. Divide the following numbers into *any* three factors:—
   30, 36, 72, 54, 48, 108, 64, 128.

---

62. The learner, who knows the multiplication table thoroughly well, can multiply *any* two numbers together by the application of the following principles:—

(1) *We multiply by the whole of a number, when we multiply successively by its factors.*

For example. $5 \times 12 = 60$, and $5 \times 3 \times 4 = 15 \times 4 = 60$. Thus $5 \times 12 = 5 \times 3 \times 4$, which can be proved *graphically* by taking a rectangle containing 12 columns of 5 squares each and dividing it into 4 rectangles containing 3 columns of 5 squares each.

Exercise—Prove *graphically* that $3 \times 16 = 3 \times 4 \times 4$. $4 \times 15 = 4 \times 5 \times 3$

(2) *We multiply one number by another, when we multiply each of the parts of the former by each of the parts of the latter and add the several partial products together for the entire product*

For example, we multiply 25 by 7, when we multiply 5 and 20 separately by 7 and add these two products, we multiply 25 by 37, when we multiply 10 and 5 first by 30 and then by 7 and add these four products together.

Exercise 1—Prove *graphically* that $25 \times 7 = 20 \times 7 + 5 \times 7$, by taking a rectangle 25 units in length and 7 units in breadth and dividing it into two rectangles 20 units by 7 units and 5 units by 7 units

Exercise 2.—Prove *graphically* that 25 by 37 $= 20 \times 30 + 20 \times 7 + 30 \times 5 + 7 \times 5$, by drawing a rectangle 25 units by 37 units and dividing it into 4 suitable rectangles.

(3) *We multiply a number by 10, 100, 1000, etc by affixing to it 1, 2, 3, etc ciphers respectively.*

Explanation.—It follows from the principle of our notation (Art 7) that to place a cipher to the right of a number has the effect of multiplying it by 10, because each figure, being removed one place to the left donotes 10 times as much as before In the same way by affixing 2 ciphers to a number we multiply it by 100, by affixing 3 ciphers we muliply it by 1000, and so on

63. The following results are deducible from the principles stated in the previous Article :—

(1) $8 \times 1000 = 8000$ (2) $200 \times 16 = 3200$. (3) $5000 \times 6 = 30000$; (4) $20 \times 30 = 600$, (5) $8000 \times 200 = 1600000$, (6) $500 \times 1200 = 60000$.

Exercise 44

Write down the following products :—

1. 1000×9. 2. 3000×15. 3 400×16. 4 300×40
5 200×500. 6. 1200×1200 7 800×160. 8 500×14.

---

64. The following are examples of multiplication by the numbers 2 to 16 and by the same numbers followed by one or more zeros ;—

(1)	(2)	(3)	(4)
2468	1206|0	245|	148|0
9	15|	7|0	13|00
22212	18080|0	1715|0	1924|000

Exercise 45.

(A) Porform the following multiplications :—

1. 3285×12. 2 1086×16 3 3320×400 4. 120500×800.

(B) Find the following products by multiplying by the factors of the multiplier :—

1. 123456789×54. 2 987654321×45 3. 251×144.
4 30800×72. 5. 7125×6400 6. 94500×1760.

---

65. To multiply (1) 4237 by 368, (2) 1325 × 408.

(1) Since $4237 \times 300 = 1271100$ ... (1)

$4237 \times 60 = 254220$ ... (2)

and $4237 \times 8 = 33896$ ... (3)

The entire product is equal to the sum of the three *partial* products (1), (2), and (3).

And since the order in which the figures of the multiplier are taken is indifferent, we may exhibit the partial products in the following *two* ways omitting the zeros at the end of the *partial* products by 3 and 6 care being taken that the first (right-hand) figure of each *partial* product is set down under that figure of the multiplier by which it is produced.

(1) *Method* (A)

```
   4237
    368
 12711
  25422
   33896
 1559216
```

Method (B)

```
   4237
    368
  33896
  25422
 12711
 1559216
```

Method (A) *again.*

			4	2	3	7
				3	6	8
1	2	7	1	1		
	2	5	4	2	2	
		3	3	8	9	6
1	5	5	9	2	1	6

(2) *Method* (A)

```
   1325
    408
 5300
  10600
 540600
```

Method (B)

```
   1325
    408
  10600
 5300
 540600
```

Method (A) *again.*

		1	3	2	5
			4	0	8
5	3	0	0		
	1		6	0	0
5	4	0	6	0	0

To the Teacher.—(1) The student must be made to see that it is often desirable to set down a multiplication sum *well to the right-hand side* of the paper, slate, or black-board (and a division sum *well to the left hand side*).

(2) Method (A) given above is rather difficult as it requires great care in writing down the figures of each partial product in their proper places But as this method is useful in abbreviated multiplication of integers and decimals, the student must be encouraged to learn this method, drawing lines, if necessary, as shown in the 3rd column of each solution till he can dispense with them

66-*A*. Short Multiplication.—Since multiplication by a number not greater than 16 requires only one line of multiplication, this kind of multiplication may be called *short* multiplication.

66-*B*. Long Multiplication.—Multiplication by a number greater than 16 is called *long multiplication*, since it requires several lines of multiplication.

Exercise 46.

Perform the following multiplication by *method* (A) of Art 65.

(A)	1. 448 × 212.	2. 705 × 127.	3 666 × 455
	4 324 × 324.	5 481 × 481.	6 456 × 456.
(B)	1. 364 × 108	2 620 × 208	3. 740 × 305
	4. 909 × 909	5 237 × 2008	6 4008 × 1006
	7. 1025 × 4002	8. 440 × 440	9 7505 × 7505.
(C)	1. 2400 × 4200	2. 1400 × 81000.	3 2030 × 23000.
	4. 3450 × 3450	5. 4500 × 4500	6. 80200 × 80200

Exercise 47.

Find the *continued products* of:—

1. 25 × 72 × 302.	2 750 × 50 × 25000.
3 45 × 300 × 105000.	4 480 × 700 × 25000.
5. 161 × 161 × 161	6 203 × 203 × 203.

Exercise 48

Find the following products by *short multiplication* and by *long multiplication*, and compare the two answers.—

1. 345 × 21 2. 7028 × 45. 3 2356 × 108. 4. 6504 × 121.

---

67. It is of the utmost importance that the student should be able to test the correctness of results in multiplication.

The simplest way of testing the accuracy of work in multiplication is the method known as "*casting out the nines*"

68. Casting out nines.—To cast out nines from a number, we add up its digits, omitting the nines and subtracting 9 whenever the sum exceeds 8.

For example, taking the number 796532, we have $7 + 6 = 13$, $13 - 9 = 4$, $4 + 5 = 9$, $9 - 9 = 0$, $0 + 3 = 3$, $3 + 2 = 5$, so that 5 is the result of casting the nines out of 796532.

NOTE —To cast out nines from a number, we may add up the digits of the number omitting the nines, then add up the digits of the sum, and proceed in this way until a single figure is obtained. The process in the case of the above number 796532 will be 7, 13, 18, 21, 23, 2 5 · so that 5 will be the result required.

Exercise 49 —(*Oral*)

Cast out nines from the following numbers :—

1. 234095.	2. 4569876	3. 41100232	4 6908452.
5. 5670420	6. 8888871.	7. 70701117	8 300101104.

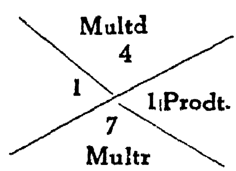
---

69 Proof of Multiplication.—To test the correctness of a result in multiplication, cast out the nines from the multiplicand and the multiplier; multiply the two results thus obtained and cast out nines from this product; if this last result is the same as the result of casting out nines from the answer to be verified, the answer is *most probably* correct; otherwise it is *certainly* wrong.

Example.—Multiply 8365 *by* 124.

```
   8385
    124
  33460
 100380
1037240   Ans.
```

Multd × Multr.



Explanation —Casting out nines from the multiplicand, we get 4.

Casting out nines from the multiplier, we get 7.

The product of 4 and $7 = 28$.

Again casting out nines from 28 we get 1.

Now casting out nines from the answer, we get also 1.

Hence the answer is *most probably* right.

Exercise 50

Find the following products by the method A of Art 65 and verify your answers by *casting out nines*.—

(A) 1. 185×33. 2. 409×57 3 945×94. 4. 679×59.
5. 18521×27, 6. 123451789×45 7 987654321×54.

(B) 1, 3245 × 3031, 2 1005 × 5001 3, 4041 × 4401.
4. 4321 × 1234 5 3041 ×3041 6 42010 × 42010.

70. To verify the continued product of three or more numbers, we have to multiply together the results of casting out nines from these numbers and examine whether the result of casting out nines from this product agrees with the result of casting out nines from the product to be verified.

Exercise 51.

Verify the following results —

1. 236 × 124 × 78 = 2282592. 2. 345×48×177 = 2931120.
3. 225 × 413 × 705 = 65512125. 4 28×37×71×29 = 2133124.

---

71. The following are examples of the **oral multiplication** of small numbers :—

Example (1) Find the value of 234 × 4.

Mental Steps

Indian Method	*English Method.*
200 × 4 = 800 30 × 4 = 120 ——— 920 4 × 4 = 16 ——— 936 *Ans.*	234 4 ——— 936 *Ans.*

Exercise 52.—(*Oral*).

A. Find *orally* by two methods the following products and verify them by *casting out nines*.—

1. 87 × 4. 2. 125 × 5. 3. 508 × 11. 4. 184 × 7.
5. 75 × 36. 6. 40 × 64. 7. 205 × 50. 8. 345 × 60.

B. Find the *continued* products of :—

1. $2 \times 3 \times 4 \times 5 \times 6$ 2 $3 \times 4 \times 5 \times 6 \times 7$. 3. $4 \times 5 \times 6 \times 7 \times 8$.
4. $3 \times 5 \times 7 \times 9$. 5 $2 \times 4 \times 6 \times 8 \times 10$. 6 $4 \times 7 \times 5 \times 11$.

Example (2). Find the value of 35×23.

Mental steps.

$$\begin{array}{r|r} 30 \times 20 = 600 & 20 \times 5 = 100 \\ 30 \times \ \ 3 = \ \ 90 & \overline{790} \\ \overline{690} & 3 \times 5 = \ \ 15 \\ & \overline{805} \quad \textit{Ans.} \end{array}$$

Exercise 53 —(*Oral*)

Find the following products and verify them by *casting out nines* —

1 25×23. 2 34×21 3 17×25. 4. 21×42.
5. $3 \times 8 \times 27$ 6 $12 \times 3 \times 24$ 7. $7 \times 7 \times 18$ 8, $6 \times 4 \times 35$

Example (3). Find the value of $25 \times 25 \times 40 \times 80$.

Solution

$25 \times 4 = 100$, $25 \times 8 = 200$; $100 \times 200 = 20000$: the answer is 2000000, affixing to 20000 the two ciphers in 40 and 80

Exercise 54.—(*Oral*)

Find the following *continued* products:—

1. $20 \times 60 \times 120$
2. $15 \times 150 \times 40$.
3 $600 \times 600 \times 600$
4. $40 \times 40 \times 40 \times 400$
5. $20 \times 30 \times 40 \times 50 \times 60$
6. $110 \times 100 \times 60 \times 50 \times 40$
7 $5 \times 50 \times 500 \times 40 \times 40$
8 $25 \times 250 \times 40 \times 400$

---

72. Square and Cube.—The product of a number multiplied by itself *once* is called the *square* of the number; and the product of a number multiplied by itself *twice* is called the *cube* of the number.

For example, since $5 \times 5 = 25$, 25 is the *square* of 5; and since $5 \times 5 \times 5 = 125$, 125 is the *cube* of 5.

73. Power.—The product of a number multiplied by itself any number of times is called a **power** of that number. Thus, 4 is called the *second power* of 2; 8 is called the *third power* of 2; 16 the *fourth power* of 2; 32 the *fifth power* of 2; and so on.

74. Examples.—Find (1) the *square* of 24, (2) the *cube* of 7, and (3) the 4th power of 5.

(1) 24×24

$$\begin{aligned}20 \times 20 &= 400\\ 20 \times 4 &= 80\\ &\overline{480}\\ 20 \times 4 &= 80\\ &\overline{560}\\ 4 \times 4 &= 16\\ &\overline{576}\end{aligned}$$

(2) $7 \times 7 \times 7$

$$\begin{aligned}7 \times 7 &= 49\\ &49 \times 7\\ 40 \times 7 &= 280\\ 9 \times 7 &= 63\\ &\overline{343}\end{aligned}$$

(3) $5 \times 5 \times 5 \times 5$

$$\begin{aligned}5 \times 5 &= 25\\ &25 \times 5\\ 20 \times 5 &= 100\\ 5 \times 5 &= 25\\ &\overline{125}\\ &125 \times 5\\ 100 \times 5 &= 500\\ 20 \times 5 &= 100\\ &\overline{600}\\ 5 \times 5 &= 25\\ &\overline{625}\end{aligned}$$

Exercise 55.—(*Oral*).

(*a*) Name the *square* of the numbers 1 to 16 inclusive.

(*b*) Find the *squares* of 25 and 21 and remember them.

(*c*) Find the *squares* of—

1. 18.	2. 22.	3. 23.	4. 19	5. 17.	6. 33.
7. 35.	8. 43.	9. 52	10. 28.	11. 47.	12. 64.

(*d*) Find the *cubes* of the numbers 1 to 10 inclusive.

(*e*) Find the 4th *powers* of the numbers 1 to 5 inclusive.

(*f*) Find the 5th *powers* of the numbers 1 to 5 inclusive.

(*g*) Find the 4th, 5th, 6th *powers* of 10.

Exercise 56.

(*Answers to be verified by casting out nines*)

(*a*) Find the *squares* of—

1. 145. 2 128 3 250. 4. 10400.

(*b*) Find the *cubes* of—

1. 25. 2. 102. 3. 220. 4 41. 5. 62.

75. Multiplication of the fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$,

(1) By repeated addition we can find the products of each of the fractions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ by the numbers 2 to 10.

For example, $\frac{3}{4} \times 3 = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 2\frac{1}{4}$, which can be shown *graphically* as in the margin.

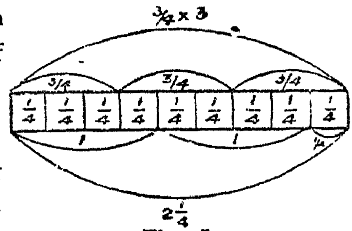


Fig. 5.

(2) By multiplying by 10, the product of these fractions by the numbers 2 to 10, we can find their products by 20, 30,......up to 100.

For example, since $\frac{1}{4} \times 7 = 1\frac{3}{4}$, $\frac{1}{4} \times 70 = 1\frac{3}{4} \times 10 = 1 \times 10 + \frac{3}{4} \times 10 = 10 + 7\frac{1}{2} = 17\frac{1}{2}$.

(3) By multiplying by 10, the products of these fractions by the numbers 20, 30......up to 100, we can find their products by 200, 300......up to 1000.

For example, since $\frac{1}{4} \times 70 = 17\frac{1}{2}$, $\frac{1}{4} \times 700 = 17\frac{1}{2} \times 10 = 17 \times 10 + \frac{1}{2} \times 10 = 170 + 5 = 175$.

The multiplication table given on the next page which has been obtained in the manner explained above is of much practical use, and the student is therefore strongly recommended to commit it to memory.

Multiplication Table of the Fractions ¼, ½, ¾ *

(To be learnt through the medium of the student's vernacular)

1	¼	¼	10	¼	2½	100	¼	25
2	,,	½	20	,,	5	200	,,	50
3	,,	¾	30	,,	7½	300	,,	75
4	,,	1	40	,	10	400	,,	100
5	,,	1¼	50	,,	12½	500	,,	125
6	,,	1½	60	,,	15	600	,,	150
7	,,	1¾	70	,,	17½	700	,,	175
8	,,	2	80	,,	20	800	,,	200
9	,,	2¼	90	,,	22½	900	,,	225
10	,,	2½	100	,,	25	1000	,,	250
1	½	½	10	½	5	100	½	50
2	,,	1	20	,,	10	200	,,	100
3	,,	1½	30	,,	15	300	,,	150
4	,,	2	40	,,	20	400	,,	200
5	,,	2½	50	,,	25	500	,,	250
6	,,	3	60	,,	30	600	,,	300
7	,,	3½	70	,,	35	700	,,	350
8	,,	4	80	,,	40	800	,,	400
9	,,	4½	90	,,	45	900	,,	450
10	,,	5	100	,,	50	1000	,,	500
1	¾	¾	10	¾	7½	100	¾	75
2	,,	1½	20	,,	15	200	,,	150
3	,,	2¼	30	,,	22½	300	,,	225
4	,,	3	40	,,	30	400	,,	300
5	,,	3¾	50	,,	37½	500	,,	375
6	,,	4½	60	,,	45	600	,,	450
7	,,	5¼	70	,,	52½	700	,,	525
8	,,	6	80	,,	60	800	,,	600
9	,,	6¾	90	,,	67½	900	,,	675
10	,,	7½	100	,,	75	1000	,,	750

* The above table is to be read thus—Once 1/4 is 1/4, 10 times 1/4 is 2½, 100 times 1/4 is 25; and so on.

Exercise 57—(*Graphical*).

(*a*) Let the pupils question one another on the above multiplication table.

(*b*) Let the pupils prove *graphically* that—

1. $\frac{1}{4} \times 7 = 1\frac{3}{4}$. 2. $\frac{1}{2} \times 9 = 4\frac{1}{2}$. 3. $\frac{3}{4} \times 8 = 6$.

4. $\frac{1}{2} \times 10 = 5$ 5. $\frac{3}{4} \times 5 = 3\frac{3}{4}$ 6. $\frac{1}{4} \times 9 = 2\frac{1}{4}$.

---

76. We add a few examples in the mental multiplication of the fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ by any number of two or three digits.

Example—(1) $\frac{1}{4} \times 35$, (2) $3\frac{1}{4} \times 300$, (3) $\frac{3}{4} \times 135$.

Solution.

(1) 30 times 1/4, $7\frac{1}{2}$
5 times 1/4, $1\frac{1}{4}$
$8\frac{3}{4}$ *Ans.*

(2) 300 times 3, 900
300 times 1/4, 75
975. *Ans.*

(3) 100 times $\frac{3}{4}$, 75
30 times $\frac{3}{4}$, $22\frac{1}{2}$
$97\frac{1}{2}$
5 times $\frac{3}{4}$, $3\frac{3}{4}$
$101\frac{1}{4}$ *Ans.*

Exercise 58—(*Oral*) *

Find the following products :—

1. 1/4×47 2. 3/4×62. 3 3/4×350 4 $3\frac{1}{4}$×70.

5. 3/4×255 6. $2\frac{3}{4}$×800 7. 1/4×705. 8. $5\frac{1}{4}$×300.

---

77. *To prove graphically that* $(\frac{1}{2})^2 = \frac{1}{4}$.

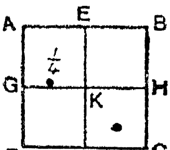


Fig 6

Now $(\frac{1}{2})^2$ means $\frac{1}{2} \times \frac{1}{2}$, *i.e.*, $\frac{1}{2}$ of half a unit.

Let us take a square ABCD to represent the unit, and divide it into 4 equal parts. Then AEFD represents $\frac{1}{2}$ of a unit, and AEKG represents $\frac{1}{2}$ of this half, or $\frac{1}{4}$ of a unit. Hence we see that $(\frac{1}{2})^2 = \frac{1}{4}$.

---

* **To the Teacher**—Since such products as 1/4×137, 1/2×259 can be easily found by dividing 137 by 4 and 259 by 2, this method is chiefly useful in finding the product of 3/4 by a whole number

78. To find the square of $4\frac{1}{2}$.—

Mental Steps.

$4\frac{1}{2} \times 4\frac{1}{2}$	4 *times* $1/2 = 2$
4 *times* $4 = 16$	20
4 *times* $\frac{1}{2} = 2$	$1/2$ *time* $1/2 = 1/4$
18	$20^1/_4$ *Ans.*

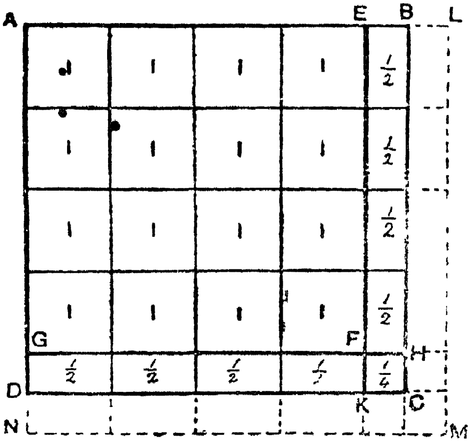


Fig. 7.

This may be proved *graphically* by describing a square $4\frac{1}{2}$ units in length and dividing it into *squares, half-squares* and *quarter-squares* as in the marginal figure and *counting* them.

NOTE —It will be seen that the entire figure ALMN including the portions enclosed by dotted lines represents 5

Exercise 59.—(*Graphical*).

Prove *graphically, by counting* squares,

$(1\frac{1}{2})^2 = 2\frac{1}{4}$; $(2\frac{1}{2})^2 = 6\frac{1}{4}$; $(3\frac{1}{2})^2 = 12\frac{1}{4}$, $(5\frac{1}{2})^2 = 30\frac{1}{4}$.

Exercise 60 —(*Oral*).

Find, as in the example under Art 78, the *squares* of the following numbers :—

$1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{1}{2}$, $9\frac{1}{2}$, $10\frac{1}{2}$.

---

79. The following is an example of the written multiplication of a mixed number containing one of the fractions, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.

Example.—Multiply $248\frac{1}{4}$ by 7.

Solution.	*Explanation.*
$248^1/_4$ 7 $1737^3/_4$ *Ans.*	$1/4 \times 7 = 1^3/_4$, set down 3/4 ; carry 1. 8 × 7 = 56 , 56 and 1 = 57; set down 7 and carry 5. And so on.

Exercise 61.

Find the value of—

1. $325\frac{1}{4} \times 6$. 2. $1056\frac{1}{2} \times 5$. 3. $420\frac{1}{2} \times 4$.
4. $2100\frac{1}{2} \times 7$. 5. $888\frac{3}{4} \times 10$. 6. $655\frac{1}{4} \times 8$.

80. Multiplication of the fractions ·1, ·2, ·3, etc., up to ·9

Example 1.—Find the value of ·6 × 4.

Solution

By addition we find that ·6 + ·6 + ·6 + ·6 = 2·4,
Therefore ·6 × 4 = 2·4 *Ans*

Example 2.—Find the value of ·4 × 10.

Solution

·4 × 10 = 4 tenths × 10 = 40 tenths = 4·0 or 4 *Ans.*

Exercise 62.

(*a*) Find the value of—

1. ·4×8. 2. ·2×4. 3. ·4×5. 4. ·7×9.
5. ·6×6. 6. ·2×5. 7. ·4×2. 8. ·5×6.

(*b*) Multiply each of the fractions ·1, ·2, ·3, ·4, ·5, ·6, ·7, ·8, and ·9 by (i) 10, (ii) 20, (iii) 40.

Exercise 63.—(*Graphical*).

1. Prove *graphically* that ·4 × 3 = 1·2 by drawing two equal and contiguous rectangles to denote *two units*, dividing each into 10 equal parts, and shading in them 4 small squares 3 times and counting them.

2. Prove likewise that ·6 × 3 = 1·8, ·5 × 4 = 2·0

---

81. The following are examples of the multiplication of mixed numbers containing one of the nine fractions ·1 to ·9 by the numbers 2 to 16.

Ex. 1.	*Ex.* 2.	*Ex.* 3.	*Ex.* 4.
15·2	246·3	2·8	408·6
4	7	10	13
60·8	1724·1	28·0	5311·8

4

Exercise 64.

Find the following products :—

1. 25·7 × 3.	2. 88·5 × 4.	3. 12 8 × 10.
4. 245·2 × 5.	5. 23 6 × 11.	6 808·8 × 15.

CHAPTER X.

SIMPLE DIVISION.

82. **Division** is the process of—

(1). finding how many times one number is contained in another; or,

(2) breaking up a given quantity into a certain number of equal parts

Example 1.—How many times is 5 contained in 20 ?

Ans. 4 *times.*

Example 2.—Divide 20 into 5 equal parts. *Ans.* 4.

Example 3 —How many times is 5annas contained in 20 annas?

Ans. 4 *times.*

Example 4.—Divide 20 annas into 5 equal parts. *Ans.* 4 *as.*

83. The number to be divided is called the **dividend** ; the number by which we divide is called the **divisor** ; and the result of the division is called the **quotient**

84. The sign ÷ (*divided by*) is the sign of division and denotes that the number preceding it is to be divided by the number which follows it. Thus 20÷5 means that 20 is to be divided by 5.

NOTE.—20 ÷ 5 is also written $\frac{20}{5}$ or 20/5

85. From the examples under Art. 82 above, we see that

(*a*) the *dividend* may be any number, *abstract* or *concrete* ;

(*b*) when the dividend is an *abstract* number, the divisor and the quotient must be both *abstract* numbers ; for example, 20 ÷ 5 = 4 ; and

(*c*) when the dividend is a *concrete* number, either the divisor or the quotient must be an *abstract* number, the other being a *concrete* number of the same kind as the dividend. Thus,

$$\frac{20 \text{ annas}}{5} = 4 \text{ annas}, \quad \frac{20 \text{ annas}}{5 \text{ annas}} = 4.$$

NOTE.—It is obvious that we cannot divide an *abstract* number by a *concrete* number.

86. Simple division.—When the dividend and the divisor are abstract numbers, or when the dividend is a concrete number of one denomination and the divisor an abstract number or a concrete number of the same kind and denomination as the dividend, it is *simple division.*

87. Questions in division are solved by the help of the Multiplication table.

For example, to divide 20 by 5 is to find how many times 5 is contained in 20 *i.e.*, to find the number by which 5 must be multiplied to produce 20. Now, we know from the multiplication table that $5 \times 4 = 20$, therefore we say that 5 is contained exactly 4 times in 20, *i e.*, $20 \div 5 = 4$.

Again, suppose we have to divide 23 by 5. Now, from the multiplication table we know that 5×4 is 20 (which is less than 23), and that 5×5 is 25 (which is more than 23) Thus we see that 5 is not contained an exact number of times in 23 or does not divide 23 exactly And hence we say that 5 divides 23 four times and leaves a remainder 3, *i e*, $23 \div 5 = 4\frac{3}{5}$.

NOTE.—Since the product of 0 multiplied by any number is 0, the quotient of 0 divided by any number is also 0.

Exercise 65.—(*Graphical*).

1 Prove *graphically* that $20 \div 5 = 4$ by drawing a line containing 20 sub-divisions and putting down the numbers 1 2, 3, 4 at the end of the 5th, 10th 15th, and 20th sub-divisions.

2. Similarly prove that $24 \div 4 = 6$, $30 \div 6 = 5$.

3 Prove likewise that in $23 \div 5$ the quotient is 4 and the remainder 3.

Exercise 66.—(*Oral*).

A. Find the *Quotient* in—

1. $25 \div 5$. 2 Rs $72 \div 9$. 3. 45 annas $\div$ 5.

4. $\dfrac{128}{16}$ 5 $\dfrac{144 \text{ feet.}}{12 \text{ feet.}}$ 6. $\dfrac{39}{13}$

B. Find the *Quotient* and *Remainder* in—

1. $30 \div 7$. 2. 50 yards $\div$ 8. 3. 100 ft. $\div$ 12 ft.

88. From the two examples in Art. 87 it will be easily seen (since $5 \times 4 = 20$ and $5 \times 4 + 3 = 23$) that—

(1) in *exact* division—

(*a*) quotient $\times$ divisor = dividened; and therefore

(*b*) divisor = dividend $\div$ quotient:

(2) in *inexact* division—

(*a*) quotient $\times$ divisor + remainder = dividend; and consequently

(*b*) quotient $\times$ divisor = dividend—remainder, and

(*c*) divisor $= \dfrac{\text{dividend} - \text{remainder}}{\text{quotient}}$

Exercise 67-A—(*Oral*).

Fill up the blanks in the following table :—

	Divisor.	Dividend.	Quotient	Remainder.
1	8 inches.	74 inches.	—	—
2	—	Rs 90	Rs 5	Rs 0
3	12 feet.	—	11	0 feet.
4	20	152 miles	—	—
5	—	102	12	6
6	9 annas	—	15	4 annas
7	—	120 yds.	7	8 yds.
8	—	100 apples.	11 apples	1 apple.

89. To divide any number we may divide its parts and add the partial quotients so obtained for the entire quotient.

For example, to divide 28 fruits into 4 equal parts, we may first take 20 fruits and divide them by 4 and then take the remaining 8 fruits and divide them also by 4. The sum of the partial quotients thus obtained, *viz.*, 5 fruits and 2 fruits, is 7 fruits, which is the same as the quotient obtained by dividing all the 28 fruits *at once* by 4. From this we infer

that $\dfrac{28}{4} = \dfrac{20}{4} + \dfrac{8}{5} = 5 + 2 = 7.$

We can similarly show that $\dfrac{144}{6} = \dfrac{120}{6} + \dfrac{24}{6} = 20 + 4 = 24,$

90. Short Division.—The student will now be able to understand the method of division by numbers not exceeding 16, which is called the method of *Short Division* and which is illustrated by the following examples :—

Example 1.—Divide 427 by 12

Solution.	*Wording.*
12)427 35—7. The quotient is 35 and the remainder is 7. *Ans.*	42: 12 times 3 (set down 3 under 2) is 36 ; remainder 6 : 67, 12 times 5 (set down 5 under 7) is 60 ; remainder 7 (set down 7 as the remainder).

Explanation —The dividend consists of 4 hundreds 2 tens and 7 units ; 4 hundreds divided by 12 cannot give hundreds in the quotient, so we take 4 hundreds and 2 tens or 42 tens as the first partial dividend, and dividing 42 tens by 12 we get 3 tens for the quotient which we set down under 2 (*i e.*, 3 in the tens' place) and 6 tens for the remainder, the 6 tens and 7 units give 67 units which being divided by 12 gives 5 units for the quotient. (which we show by setting down 5 under in the units' place) and 7 for the remainder which is shown as remainder, as it is less than the divisor and cannot therefore be divided by it.

Example 2.—Divide 96368 rupees by 8.

Solution.	*Wording.*
8)96368 rupees. 12046 rupees. The quotient is 12046 rupees and there is no remainder.	9 8 times 1 (set down 1 under 9) is 8 ; remainder 1). 16 ; 8 times 2 (set down 2 under 6) is 16, (no remainder). 3, 8 times 0 (set down 0 under 3) is 0 ; (remainder 3). 36, 8 times 4 (set down 4 under 6) is 32 ; (remainder 4). 48: 8 times 6 (set down 6 under 8) is 48 ; (no remainder).

Verification of Example 2.—12046 rupees × 8 = 96368 rupees.

" *Example* 1.—35 × 12 + 7 = 420 + 7 = 427.

NOTE 1 —The student should carefully note that in the above examples, the first figure of the quotient is set down under the last figure of the first partial dividend that is taken, and that for every remaining figure in the dividend there is a figure in the quotient.

NOTE 2.—He should also notice where the *cipher* occurs in the quotient.

NOTE 3.—The method of short division may be employed in the case of division by 20, 30, &c.

Exercise 67-B.

Perform the following divisions and verify your answers:—

1. 8037 ÷ 12 2. 576483 yds. ÷ 9 yds. 3 Rs. 24,271 ÷ Rs 8.
4 999999 pins ÷ 7. 5 £117455 ÷ 9. 6. 24100 mds. ÷ 12 mds.

---

91. Long Division.—When the divisor is too large for the several steps in the process of division to be done mentally, we employ the method of *long division* in which the work is written out in full. This method is best seen from the following example:—

Divide 754615 by 125.

Solution.

```
      6036—115
125)754615(
    750
     461
     375
      865
      750
      115
```

The quotient is 6036, and the remainder is 115.

Explanation.

Here 125 does not divide 7 or 75, so we take 754 as the first partial dividend. Now 125 is contained 6 times in 754, so we put down 6 *above* 4 as the first figure in the quotient (just as we would write 6 *below* 4 in *short division*), then we multiply 125 by 6 and write the product 750 under 754 Now subtracting 750 from 754, we get 4 for the remainder Then we bring down 6 from the dividend and place it to the right of 4 getting 46 Now since 46 is smaller than 125, we place a zero in the quotient above 6, and bring down the next figure 1, and so proceed with the division till we get 6036 for the quotient and 115 for the remainder.

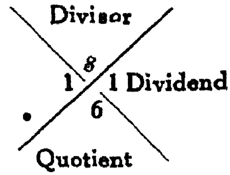
VERIFICATION.

Since, in *inexact* division, Quotient + Remainder = Dividend, we have to see if 125×6036 + 115=754615. This may be done

either by actual multiplication and addition, or by casting out the nines. Thus—

```
        6036
         125
       ------
       72432
       30180
       ------
      754500
plus     115
      ------
      754615
```

$8 \times 6 + 7^*$
$= 48 + 7$
$= 55$



* 7 is the result of casting the nines out of the remainder 115.

Exercise 68.

Perform the following divisions and verify your answers in *two* ways:—

1. 6384 ÷ 21. 2. 3040566 ÷ 5934 3. 706543 ÷ 203.
4. 53333333334 ÷ 54 5. 5555555505 ÷ 45.
6. 10000000000 ÷ 111. 7. 960000000 ÷ 186205.
8. 3674212542 ÷ 9999. 9. 2559375 ÷ 1625.

---

92. When the divisor terminates with ciphers, cut off all the ciphers at the end of the divisor and as many figures from the end of the dividend as there are ciphers at the end of the divisor. Divide the remaining figures of the dividend by the remaining figures of the divisor, and to the remainder thus obtained, annex the figures cut off from the dividend for the final remainder.

Example 1. 364903 — 1200

```
12φφ) 3649φ3
       304—103
```

The quotient is 304, and the remainder is 103. *Ans.*

Example 2. 7460596÷27000.

```
          276—8596.
27φφφ)7460 φφφ
      54
      ---
      206
      189
      ---
       170
       162
       ---
         8
```

The quotient is 276, and the remainder is 8596. *Ans.*

Exercise 69.

Perform the following divisions :—

1. 673204 ÷ 21000.
2. 753401 ÷ 500.
3. 570500 ÷ 700.
4. 395300 ÷ 1300.
5. 46280505 ÷ 201500.
6. 11111111111 ÷ 777000.
7. 33033033033 ÷ 567800.
8. 86543210 ÷ 123000.

---

93. Division by factors—We divide by the whole of a number when we divide successively by its factors.

For example, the result of dividing 72 at once by 12 is the same as that of dividing it by 3 and the quotient by 4. Thus $72 \div 12 = 6$; and $72 \div 3 \div 4 = 24 \div 4 = 6$.

This can be shown *graphically* by taking 72 squares in the shape of a *rectangle* containing 12 *columns* of 6 squares each. If we divide this rectangle by 12, we get for the quotient 1 column of 6 squares, so that $72 \div 12 = 6$. Again, if we divide the same rectangle by 3 we get for the quotient 4 columns of 6 squares each; and if we divide these 4 columns of 6 squares each by 4, we get for the quotient 1 column of 6 squares, *i.e.*, 6.

Thus $72 \div 12 = 72 \div 3 \div 4$.

NOTE.—We can also prove that $72 \div 12 = 72 \div 4 \div 3$. Hence it follows that *dividing by factors, the factors may be taken in any order.*

Exercise 70—(*Graphical*).

Prove *graphically* by means of suitable rectangles that—

(1) $72 \div 12 = 72 \div 6 \div 2$. (2) $30 \div 6 = 30 \div 2 \div 3$.

(3) $90 \div 15 = 90 \div 3 \div 5$. (4) $108 \div 9 = 108 \div 3 \div 3$.

94. *Example* (1).—*Divide* 210 *by* 30 by using *three* factors.

Solution.

$30 = 2 \times 3 \times 5$

2)210 *units.*
3)105 *twos.*
5)35 *three twos* or *sixes.*
   7 *six fives* or *thirties.*

Or more brifly thus:—

2) 210
 3) 105
  5) 35
    7

The quotient is 7 and there is no remainder.

Exercise 71.

Divide by *three* factors of the divisor, taking the factors in *different orders*.

1. $270 \div 30$. 2. $525 \div 105$. 3. $330 \div 66$.
4. $1155 \div 165$. 5. $693 \div 231$. 6. $3080 \div 385$.

Ex. (2) —*Divide* 99804 *by* 495 by means of *factors*.

Solution.

5)495
9)99
11

496 {
5) 99804
9) 19960 *fives* and 4 units over.
11) 2217 *forty-fives and* 7 *fives* or 35 units over.
201 *four hundred and ninety fives* and 6 *forty-fives* or 270 units over.

∴ The quotient is 201 and the total remainder is $4 + 35 + 270$ or 309.

NOTE 1 —The above sum may also be worked and the final remainder found thus:—

5)99804
9)19960—4
11) 2217—7
201—6

$6 \times 9 = 54$
$54 + 7 = 61$.
$61 \times 5 = 305$.
$305 + 4 = 309$, which is the final remainder.

NOTE 2 —The student may verify the answer both by long division and by casting out nines.

Exercise 72.

Perform the following divisions, employing short division, and verify each answer by long division and by casting out nines:—

1. 4634 by 45. 2. 146578 by 105. 3. 76540 by 125
4. 777777 by 1980 5. 40354 by 270. 6. 11111111 by 625

95. The following examples and exercises in *mental division* require a knowledge of the multiplication table of the fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.

Example 1.—Divide 34 by 8.

Mental Steps.

8 times 4 = 32

Remainder = 2

8 times $\frac{1}{4}$ = 2

$\therefore \frac{34}{8} = 4\frac{1}{4}$. *Ans.*

Example 2 —Divide $37\frac{1}{2}$ by 10.

Mental Steps.

10 times 3 = 30.

Remainder = $7\frac{1}{2}$.

10 times $\frac{3}{4}$ = $7\frac{1}{2}$.

$\therefore \frac{37\frac{1}{2}}{10} = 3\frac{3}{4}$. *Ans.*

NOTE.—When the quotient is a whole number or contains one of the fractions 1/4, 1/2, 3/4, its correctness may be proved by multiplying it by the divisor. Thus in *Example* 1. $4\frac{1}{4} \times 8 = 34$, and the answer is therefore correct.

Exercise 73 —(*Oral*).

Divide (verifying the quotient in each case) —

1. $36 \div 8$. 2. $50 \div 8$. 3. $32\frac{1}{2} \div 10$. 4. $57\frac{1}{2} \div 10$.
5. $52\frac{1}{2} \div 6$. 6. $97\frac{1}{2} \div 30$. 7 $112\frac{1}{2} \div 30$. 8. $162\frac{1}{2} \div 50$.

Example 3.—Divide 18 by 5.

Solution

5 times 3 = 15	∴ the quotient is $3\frac{1}{2}$ and the remainder is 1/2. *Ans*.
Remainder = 3	
5 times 1/2 = $2\frac{1}{2}$	
Remainder = $\frac{1}{2}$.	

VERIFICATION —Since this is a case of *inexact* division, we have to see if quotient × divisor + remainder = dividend. Now $3\frac{1}{2} \times 5 + 1/2 = 15 + 2\frac{1}{2} + 1/2 = 18$. Hence the answer is right.

Exercise 74.—(*Oral*).

Perform the following divisions *orally* and verify the answers:—

1. $48 \div 9$. 2. $86 \div 7$. 3 $34\frac{1}{2} + 6$. 4. $87\frac{1}{4} \div 9$.
5. $205\frac{1}{2} \div 20$. 6 $190 \div 50$. 7. $340 + 30$. 8. $581 \div 100$.

Ex. 4 $22^1/_2 \div 4^1/_2$.

Mental steps

$5 \times 4 = 20$.
$5 \times 1/2 = 2^1/_2$.
$22^1/_2$.

∴ The quotient is 5 with no remainder.

Ex 5. $15 \div 4^1/_2$.

Mental steps.

$3 \times 4 = 12$
$3 \times 1/2 = 1^1/_2$
$13^1/_2$

Remainder $1^1/_2$.

∴ the quotient is 3 and remainder is $1^1/_2$.

VERIFICATION.

(1) $4^1/_2 \times 5 = 20 + 2^1/_2 = 22^1/_2$. Hence the answer is right.
(2) $4^1/_2 \times 3 + 1^1/_2 = 13^1/_2 + 1^1/_2 = 15$. Hence the answer is right.

Exercise 75—(*Oral*).

Perform the following divisions *mentally* and verify the answers:—

1. $70 \div 3^1/_2$. 2 $34 \div 4^3/_4$. 3. $125 \div 6^1/_4$. 4. $143 \div 4^3/_4$.
5. $70 \div 7^1/_2$. 6. $76 \div 3^3/_4$. 7 $205 \div 10^1/_4$. 8. $77 \div 9^1/_2$.

96. In the following examples of written division, the dividend, or quotient, or both contain one of the fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$:—

Ex. 1. Divide 4686 by 8.

$$8)\underline{4686}$$
$$585^3/_4$$

The quotient is $585^3/_4$ and there is no remainder.

Ex. 2. Divide $1570^3/_4$ by 9.

$$9)\underline{1570^3/_4}$$
$$174^1/_2 - 1/4 \text{ remainder.}$$

The quotient is $174^1/_2$ and the remainder is 1/4.

VERIFICATION.

1. $585^3/_4 \times 8 = 4686$. Hence the answer is right.

2. $174^1/_2 \times 9 + 1/4 = 1570^1/_2 + 1/4 = 1570^3/_4$. Hence the answer is right.

Exercise 76

(*a*) Perform the following divisions and verify the answers:—

1. 634 by 8 2. $1288^1/_2$ by 6. 3. $1005^1/_4$ by 7.
4. $191^1/_4$ by 9. 5. $752^3/_4$ by 10. 6. $6206^1/_4$ by 7

(*b*) Perform the following divisions by using the factors of the divisors:—

1. $1478\frac{3}{4} \div 35$. 2. $1807\frac{1}{2} - 15$. 3 $2094\frac{3}{4} \div 49$. 4. $2062\frac{1}{2} \div 75$

---

97. The student will have no difficulty in understanding the following examples of division (by a whole number) of a *mixed number* containing *tenths*.

Ex. 1. 36 4 + 7.

$$7)\underline{36\ 4}$$
$$5\ 2$$

∴ the quotient is 5 2 and there is no remainder.

Verification.

$5{\cdot}2 \times 7 = 36\ 4$, the dividend.
∴ the answer is right.

Ex. 2. $4582{\cdot}4 \div 9$

$$9)\underline{4582{\cdot}4}$$
$$509{\cdot}1 - {\cdot}5 \text{ remainder.}$$

∴ the quotient is 509·1 and the remainder is ·5.

Verification.

$509{\cdot}1 \times 9 + {\cdot}5 = 4581\ 9 + {\cdot}5$
$= 4582\ 4$, the dividend.

∴ the answer is correct.

Exercise 77—(1—8 *Oral*).

(*a*) Perform the following divisions and verify your answers —

1. $72\ 8 \div 8$. 2. $24\ 0 \div 10$ 3. $3\ 5 \div 7$
4. $66666{\cdot}6 \div 11$. 5 $87654\ 0 \div 12$. 6. $1720\ 8 \div 16$.

(*b*) Perform the following divisions using the factors of the divisors:—

1. $55{\cdot}5 \div 15$. 2. $171{\cdot}5 \div 35$. 3. $1014\ 3 \div 49$ 4. $3010 \div 25$.

(*c*) Add together 325·8, 46·4 and 7·6, and divide the sum by 9.

98. Remainder after Division.—In *inexact* division the quotient is often written as a *mixed number*.

For example, in $23 \div 4$, $258 \div 12$, $217 \div 15$ the quotients are written respectively as $5^3/_4$, $21^6/_{12}$, $14^7/_{15}$, in which 3, 6, and 7 are the *remainders* and 4, 12 and 15 are the *divisors*.

99. Quotient correct to the unit.—A quotient in the form of a *mixed number* is often expressed as a *whole number*, by taking its *fractional part* as 1 when it is *equal* to or *greater* than $\frac{1}{2}$, and omitting it when it is *less than* $\frac{1}{2}$.

For example, the above quotients $5^3/_4$, $21^6/_{12}$, $14^7/_{15}$ are taken as 6, 22, 14 respectively, and when this is done the quotient is said to be expressed correct to the unit.

Exercise 78—(*Oral*).

State whether the following fractions are *equal to*, or *greater than*, or *less than* 1/2,—3/8, $^{17}/_{34}$, $^{45}/_{91}$, $^{100}/_{215}$, $^{77}/_{180}$, $^{48}/_{96}$, $^{212}/_{424}$.

Exercise 79—(1—4 *Oral*).

Find *correct to the unit*, the quotient in—

1. $35 \div 9$. 2 $108 \div 20$. 3 $67 \div 10$. 4. $104 \div 16$.
5. $1743 \div 42$. 6 $9465 \div 31$. 7. $2750 \div 39$. 8 $8060 \div 40$.

---

100. The student knows that, if we divide a number into 4 equal parts, each of these parts is called *one-fourth* (¼) of the number. Hence to find ¼ of a number we have to divide it by 4. Similarly, to find 1/5, ⅛, etc. of a number, we have to divide it by 5, 8, etc. respectively.

Example.—1/5 of 375 = 75, 1/7 of 1424 = $203^3/_7$, 1/8 of a lakh = 1/8 of 100000 = 12500.

Exercise 80.

Find the value, correct to the unit, of—

1. 1/4 of 720. 2. 1/5 of 24552. 3 1/6 of 7205. 4. 1/7 of ten-thousand. 5. 1/9 of a lakh. 6 1/12 of a million. 7. 1/16 of a crore. 8. 1/15 of two lakhs 9. 1/5 of 1235—1/9 of 1356. 10 1/4 of 7211 + 1/10 of $1202^1/_2$.

Exercise 81—(*Oral*).

(*To be done through the medium of the vernacular of the student*)

Take any integer, name its half, then 1/2 of the result, then 1/2 of this result, and so on as far down as you can.

For example, taking 124, say 62, 31, $15\frac{1}{2}$, $7\frac{3}{4}$, $3\frac{7}{8}$, &c ;
taking 122, say 61, $30\frac{1}{2}$, $15\frac{1}{4}$, $7\frac{5}{8}$, $3\frac{13}{16}$, &c.

CHAPTER XI.

USE OF BRACKETS.

101. Brackets.—The symbols ( ) called *brackets* denote that the quantities within them or (enclosed by them) are to be taken together and treated as one quantity.

For example,

(1) $60-(35+10)$ means that the *sum* of 35 and 10 is to be subtracted from 60, whereas $60-35+10$ would mean that 35 is to be subtracted from 60, and to the result 10 is to be added, so that

$$\left.\begin{array}{l} 60-(35+10)=60-45=15. \\ 60-35+10=25+10=35. \end{array}\right\}$$

Similarly.

(2) $$\left.\begin{array}{l} 60-(35-10)=60-25=35. \\ 60-35-10=25-10=15. \end{array}\right\}$$

(3) $(4+3)\times 5=7\times 5=35.$

(4) $(14-2)\div 3-2=12\div 3-2=4-2=2.$

Note 1 :—Other forms of brackets are [ ]. { }.

Note 2 —( ) are called *simple brackets*, [ ] are called *rectangular brackets*, and { } are called *double brackets*.

Exercise 82—(*Oral*).

(*a*) Find the difference in value between—

(1) $21-(10+4)$ and $21-10+4$.

(2) $100-(70-10)$ and $100-70-10$.

(*b*) Find the value of—

1. $25-(15-8)$.
2. $25-(15+8)$.
3. $42-(23+7)$.
4. $81-(35+21)$.
5. $100-(70-32)$
6. $200-(85-30)$.
7. $48-(25-10+8)$.
8. $48-(25+10-8)$
9. $48-(25-10-8)$.
10. $45\div(8-3)$.
11. $(8-4)\times5-3$.
12. $4\times(14-3)-8\frac{1}{2}$.
13. $7{\cdot}5\div(8{\cdot}2-3{\cdot}2)+5{\cdot}5$.
14. $(1^3+2^3+3^3)\div(1+2+3)$.
15. $(12^2-5^2)+(12-5)$.

Exercise 83.

Find the value of—

1. 2050—(1547+358).
2. 2050—(1547—358).
3. 10000—(4530—1099)
4. 10000—(4530+1099).
5. 2576—(1000—377).
6. 2576—(1000+377).
7. 4444—(444+155—66).
8. 4444—(444+155+66).

---

102. Multiplication of quantities within brackets is often expressed by writing the quantities side by side *without the sign of multiplication between them.*

For example,

(*a*) $4\,(8-3)=4\times5=20$;

(*b*) $(5+4)\,(6-2)=9\times4=36$;

(*c*) $(8+7)\,(8-6)\,(5+4)=15\times2\times9=270$.

Exercise 84—(*Oral*)

Find the value of—

1. $8\,(16-10)$.
2. $(8+7)\,(7-3)$.
3. $8\,(3-2\cdot5)$.
4. $(7^1/_2+3)\,(10-4^1/_2)$.
5. $8\,(8-3)\,(8+2^1/_4)-50$.
6. $(48-23)\,(29-25)\,(1^1/_2+3)$.
7. $(125+25)\,(125-75)$.
8. $(8+4)\,(8-3)\,(18+2)-10^3$.
9. $225-(7+8)\,(10-5)\,(7-4)$
10. $(10^3-6^3-4^3)\div(10+6+4)$.

---

103. Insertion of Brackets.—The pupil will do well to learn the following method of simplifying quantities involving *addition* and *subtraction* by **inserting brackets.**

(1) Rs. 4 + Rs. 5. = Rs. (5 + 4) = Rs 9.

(2) 8 lakhs—5 lakhs + 10 lakhs=(8—5+10) lakhs = 13 lakhs.

Exercise 85.

Find the value of the following by inserting brackets as in the above examples :—

1. £18 + £7.
2. 49 pins + 7 pins—20 pins.
3. $20^1/_2$ miles—$7^3/_4$ miles + $3^1/_4$ miles
4. 72·5 lakhs—60 lakhs—4 8 lakhs.

CHAPTER XII.

EXPRESSIONS AND TERMS.

104. Expressions and terms.—Quantities like 3, 5 + 8, 7—6 + $4\frac{1}{2}$, 3 × 5 — 6, 1·6 ÷ 2, 12 (6 + 3) — 40 are called *expressions*, and the parts of an expression connected by the signs *plus* and *minus* are called its *terms*. For example,

(1) 7 — 6 + 4½ is an expression of *three* terms, *viz.*, 7, — 6, + 4½;

(2) 3 × 5 — 6 is an expression of *two* terms, *viz.*, 3 × 5, — 6.

(3) 1·6 ÷ 2, and 3 are expressions containing only *one* term.

(4) 12 (6 + 3) — 40 is an expression of *two* terms, *viz.*, 12 (6 + 3), — 40.

NOTE.—When the 1st term of an expression is preceded by no sign, the sign + is understood before it. Thus in 4 + 8 — 3 the *first* term is + 4, in 4 (8—5) + 7, the *first* term is + 4 (8 — 5).

Exercise 86.

(*a*) Write down or state the *terms* of each of the following expressions:—

1. 15¼ + 9 — 4. 2. 8(6 + 4) ÷ 12). 3. 12 — 3 + 4(8 + 2).
4. — 5 + 3 (6 + 5). 5. (4 + 8) (3 — 1½) 6. — (4 + 1) (4 — 1) + 100.

---

105. Order of Operations.—In the simplification of an expression containing several terms, the process indicated by the signs × and ÷ must be done *before* those indicated by the signs + and —*

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* **To the Teacher.**—The student must from the very first be trained to the habit of correct reasoning and accurate expression. He must *not* be allowed to use such faulty and slovenly modes of expression as the following —

(1) Rs 4 + Rs. 5 = Rs 4 + 5 = Rs 9.

(2) 4 oranges + 5 oranges = 4 + 5 oranges = 9 oranges.

(3) A and B together earn £5 + £4 = £9.

Examples.

(1) $16-3\times5+28\div4=16-15+7=8$.

(2) $8\frac{1}{2}(4+1)-12\frac{1}{2}\div5=8\frac{1}{2}\times5-2\frac{1}{2}=42\frac{1}{2}-2\frac{1}{2}=40$.

(3) $40-(4+1)(8-2)+6(8-4)(6-5)=40-5\times6+6\times4\times1=40-30+24=34$.

Exercise 87.

Find the value of the following expressions :—

1. $144-72\div8+5$.
2. $18^1/_2-3^3/_4\times4-2^1/_4\div3$.
3. $18-4(3+1^1/_2)+6$.
4. $1{\cdot}2\times3+(4+1)(4-2)-17{\cdot}5\div5$.
5. $6(17-3)-(14+8)\div4+21^1/_2$
6. $10\times9-9\times8+8\times7-7\times6$
7. $(15+5)(15-5)-6(4-1^1/_2)(4+11)+25$.

---

106. Vinculum.—A horizontal line like — called a *vinculum* is often used for the sign of division. Thus $(15+4)\div(8-5)$ is written as $\frac{15+4}{8-5}$.

107. The following are examples of the simplification of expressions involving the *vinculum* :—

(1) $\frac{5\times8-4}{10+2}=\frac{40-4}{12}=\frac{36}{12}=3$.

(2) $\frac{4(11+12^1/_2)}{99-7\times13}=\frac{4\times23^1/_2}{99-91}=\frac{94}{8}=11^3/_4$.

(3) $\frac{6{\cdot}3+4{\cdot}5}{3\times5{\cdot}2-11{\cdot}6}=\frac{10{\cdot}8}{15{\cdot}6-11{\cdot}6}=\frac{10{\cdot}8}{4}=2{\cdot}7$.

Exercise 88.

Simplify the following expressions :—

1. $\frac{150+80}{60-30}$. 2. $\frac{2\times15-18}{12+3\times4}$. 3. $\frac{15+48}{^1/_2(5+9)}$.

4. $\frac{205-2\times15}{6\times4\frac{1}{2}+8}$. 5. $\frac{16{\cdot}4\times4-58{\cdot}4}{5\times24-116}$. 6. $\frac{15+4\times13\frac{3}{4}}{8\times2\frac{1}{2}-12}$.

7. $\frac{8(16-4\frac{1}{2})}{5\times4{\cdot}6}$. 8. $\frac{135}{3(7+5-2)}$. 9. $\frac{6(8+5)-22}{6(8-5)-4}$.

CHAPTER XIII.

USE OF THE SCALE AND DIVIDERS.

1. Tests of a straight line and a straight edge.

108. Tests of a straight line —

(i) Apply the straight edge (one which is known to be straight) to the line if the straight edge coincides with the entire length of the line, then the line is straight

(ii) Draw a straight line AB On tracing paper make a neat tracing of it and mark A and B at the ends of the tracing Then turn the tracing paper over (face downwards) and put it again over the line so that the reversed A falls on A. and the reversed B falls on B If the traced line falls exactly over the line AB throughout its length, then the line is straight

(iii) Take a fine piece of thread and stretch it tightly along the line If the thread lies exactly along the entire line, then the line is straight.

(iv) Hold the paper on which the straight line is drawn up to the eye and look along the line If you do not notice any unevenness, then the line is straight.

Exercise 89.

How does a gardener make a straight furrow for a hedge ? How will you use a string or rope for fixing pegs in a straight line ?

---

109. Tests of a straight edge —

(i) Rule a line AB on a piece of paper with the straight edge. Then reverse the paper, apply the straight edge to the reversed line BA, and draw the line BA again. If the second line coincides exactly with the first, then the straight edge is straight

(ii) Draw a straight line with the straight edge and examine whether this line is straight by the methods explained above.

110 Measuring with the scale—In measuring lines with the *inch scale* or *centimetre scale*, care should be taken to place the scale quite close to the line, as otherwise there would be difference in the reading for slight alterations in the position of the eye.

111. Measuring with the dividers and scale.—

To measure any line AB with the *dividers and scale*, open the legs of the dividers just wide enough for the two pin-points to be applied to the two ends of the line. Keeping the legs in this position, put one point on the *zero* mark (or any other mark) of the *scale* and note the mark on which the other point falls, and read off the length accordingly.

NOTE 1.—In measuring lines it is advisable *not* to use the *zero* mark always as the scale would be worn out at that mark.

NOTE 2.—In applying the pin-points of the dividers to the scale care should be taken *not* to apply them *vertically*, as this will injure the scale The dividers might be held *horizontally* or *slantingly*.

---

2. Measurement of Lines.

112. Measurements with the *inch scale* may be made *correct* to 1 inch, or $\frac{1}{4}$ inch, or $\frac{1}{2}$ inch, or $\frac{1}{10}$ ($\cdot1$) of an inch; measurements with the *centimetre scale* are made *correct* to a *centimetre* or *correct to* a millimetre ($\frac{1}{10}$ or $\cdot1$ of a centimetre).

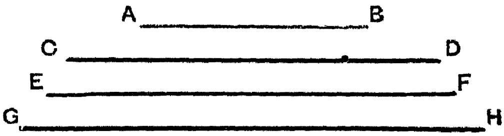
NOTE 1.—The symbols ' and " are used respectively for feet and inches. For example a length of 4 ft 6 inches is written briefly as 4' 6".

NOTE 2.—A length of 8 centimetres 4 millimetres is briefly written as 8 cm. 4 mm. or as 8·4 cm.

NOTE 3.—In expressing the length of a line *correct* to a quarter of an inch, any fraction of a quarter of an inch that is left after taking a whole number of inches and a whole number of quarters of an inch, must be considered as one quarter of an inch if it be exactly half or greater than half of a quarter of an inch, and must be omitted if it be less than half of a quarter of an inch. A similar rule is to be followed in expressing lengths *correct* to an *eighth* of an inch, a *tenth* of an inch, a *tenth* of a centimetre, etc.

Exercise 90—(*Practical*).

(A) 1. Measure the following lines with scale *correct* to (*a*) 1/4 inch, (*b*) 1/2 inch, (*c*) 1/10 of an inch, (*d*) 1 mm.



2. Measure the above lines with dividers and scale.

3. Draw some *horizontal*, *vertical* and *oblique* lines and measure them *correct* to (*a*) 1 of an inch, (*b*) 1 of a cm, by using the *scale* only. Also compare these readings with those obtained by using the *dividers and scale*.

4. You have seen that a line may be measured either by applying the scale directly to it or by means of the dividers. Which of these two measurements is likely to be more accurate? And why?

5. Mark two points on paper and measure the distance between them *correct to* (*a*) 1 inch, (*b*) 1 cm. (*To be done by two methods*)

6. Find by measurement (*a*) *in inches and tenths* (*b*) *in centimetres and tenths*, the *length*, *breadth* and *diagonal* of the subjoined rectangles.

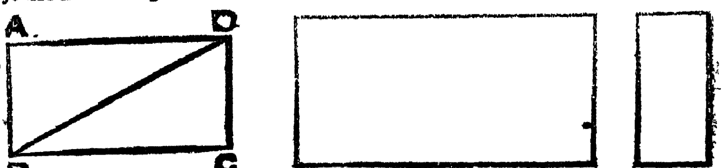


Fig. 9.

7. The points P, Q, R and S marked below are in one and the same straight line. Measure in inches and centimetres the lengths PQ, QR, RS, arrange your measurements one below another and add them together.

X P   X Q   X R   X S

PQ = ——in. = —— cm.
QR = ——in. = —— cm.
RS = ——in. = —— cm.

PQ + QR + RS = ——in. = —— cm.

Check the length thus obtained by measuring PS.

8. The points A, C and B marked below are in one and the same straight line Measure the lengths AB and BC, write them down one below another, and find the length of AC.

X	X	X
A	C	B

AB = —— in. = —— cm.
BC = —— in. = —— cm.

AB − BC = —— in = —— cm.

Check the above result by measuring AC.

(B) 1. Guess the lengths of the following lines in inches and centimetres and then measure them.

A ———————————————— B

C ———————————————————————— D

Fill up the following table with the lengths guessed and measured:—

Line to be measured.	Length guessed	Length measured	Length guessed	Length measured.
AB	—— in	—— in.	—— cm.	—— cm.
CD	—— in	—— in	—— cm	—— cm.

2 Draw some straight lines in different directions and guess their lengths (*a*) in inches and tenths of an inch, (*b*) in cm and mm. Then find the length of each of these lines by actual measurement.

3 Draw *free hand* straight lines of the following lengths,— 2"; $3\frac{1}{4}$", $1\frac{1}{2}$", 4", 6 cm, 4 5 cm.; etc. Then check the length of each of these lines by actual measurement.

(C) 1. Draw a *long line* with the straight edge and measure it by stepping along it the arms of the dividers opened to a suitable width Measure the same line with the *scale* only. Which of these two methods is the better one ?

2 Measure (i) in feet and inches and (ii) in decimetres* and centimetres, the length and breadth of (*a*) a page of this book, (*b*) a sheet of foolscap paper, (*c*) a page of your Exercise book, etc. Let the answer be *correct* to an inch or a centimetre.

* A decimetre = 10 centimetres.

3. Measure (i) in *feet* and *inches* and (ii) in decimetres and centimetres the length and breadth of (*a*) a black-board, (*b*) a wall map, (*c*) the top of a table, (*d*) the top of a bench, (*e*) the floor of the class-room, etc. Express the answer to the nearest inch or centimetre. Use the measuring tape where necessary.

4. Measure in feet and inches the diagonal of a black-board, of the top of a table, of the floor of your class-room, etc., using the measuring tape if necessary.

---

3. Joining two points and producing straight lines.

113. To join *two* points or produce a straight line, place the *straight edge* as close to the two points or the line as possible and so as to just admit of the pencil point being inserted between the points and the straight edge, or between the line and the straight edge.

Exercise 91.—(*Practical*).

1. How many *straight lines* can be drawn between two points? How many *curved lines*?

2. Make two pencil marks on paper and join them by a *straight line*.

3. Make two *pin-pricks* on paper and *join them*.

4. Draw a *horizontal line* with the straight edge and produce it some distance *both ways*.

5. Draw with the straight edge a *vertical line* and a *slanting line*, and produce each some distance *both ways*.

6. Mark 3 points at random, join them, two and two, and measure these three lines.

7. Take 4 points at random, and form a *quadrilateral* by joining the points in succession. Measure the 4 sides of the quadrilateral.

---

4. Drawing lines of required lengths.

114. Lines of any required length may be drawn (1) by using the *scale only*, (2) by using *both scale and dividers*. For example, a line 2·5 inches long may be drawn—

(1) By placing the scale on paper, marking two points on it just below the outer ends of any two divisions of the scale which are 2·5 inches apart, and joining these two points, or

(2) By opening with the help of the scale the legs of the dividers so that their ends may be 2·5 inches apart, applying them to the paper, and pressing them *slightly* on it so that two pin-pricks may be made on it, and finally *joining* these two *pin-pricks*.

Exercise 92.—(*Practical*)

(A) 1. Draw, in *two* ways, *horizontal* lines of the following lengths :—

(*a*) 2 inches (*b*) 3½ inches. (*c*) 4·3 inches. (*d*) 5cm. 2 mm.
(*e*) 4·6 cm. (*f*) 7·5 cm. (*g*) 5¼ inches (*h*) 10·1 cm.

2. Draw *vertical* lines of the following lengths :—
(*a*) 3″. (*b*) 8 cm (*c*) 1 8″ (*d*) 1¾.

3. Draw an *oblique* line 6 6 cm long slanting to the left, and another 2¼″ long slanting to the right.

4 Draw a *horizontal* line, a *vertical* line, and an *oblique* line each equal to the line——— ——— ——

(B) 1. (*a*) Draw a line 4 inches long and measure it in centimetres. Hence show that an inch is equal to about 2·5 cm.

2. Repeat the above exercise, taking straight lines of lengths 3 inches, 5 inches and 6 inches.

3 Draw a line 6·5 cm. long, and measure it in inches and enths of an inch.

4. Repeat the above Exercise taking 12 cm and 15 cm.

(C) 1. Draw a *horizontal* line AB 6 inches long and with the help of the scale take in AB points C, D, E, etc, so that AC, AD, AE, etc, may be respectively equal to—

2 inches, 2½ inches, 3 inches, 3¼ inches, 3¾ inches, etc.

Mark the positions of the points C, D, E, etc. by drawing a short vertical line across AB at the position of each of these points.

2. Draw a *horizontal* line AB 15 cm long and with the help of the dividers and scale mark in it points C, D, E, etc., so that BC, BD, BE, etc, may be respectively equal to—

1 cm., 1 7 cm., 5 cm., 7·8 cm. 9 cm, 10·6 cm., etc,

3. Draw (i) *vertical* lines, and (ii) *oblique* lines, and deal with them as you did with the horizontal lines drawn in Questions 1 and 2 above.

(D) 1. Use the dividers for drawing lines of a given length.

(*a*) in different directions from different points, and (*b*) in different directions from one and the same point.

2. Draw a horizontal, vertical, or slanting line AB 5 inches long, and commencing from the point A, step along AB (with the dividers) lengths equal to (*a*) 1 3 inches, (*b*) 3/4 inch, (*c*) 17 cm. Measure the portion of the line AB that remains after this

3 Draw a line PQ 5 6" long and cut off from it a length PR equal to 3 8". Measure RQ Check this measurement by calculation

4 Draw a line PQ 12·4 cm long and find a point R in it so that QR may be equal to 5 9 cm Measure PR. Check this measurement by calculation.

5 Repeat Questions 3 and 4 above, taking lines of various other lengths.

6. Draw a line OX equal to 11 cm And in it take lengths OA, AB, BC, CD equal to 8 cm, 2 5 cm 1 2 cm and 3 1 cm respectively. Measure the length DX. Check this last measurement by calculation.

7. Repeat Question 6 above, taking lines of various other lengths

(E) 1 Take a line XY equal to 3 2". Produce it to Z so that YZ may be equal to 2 3" Measure the whole line XZ and check the measurement by calculation.

2 Repeat the above Question, taking lines of various other lengths

3 Draw a *horizontal* line AB 8 cm. long and produce it towards B to C, so that BC may be (*a*) equal to AB, (*b*) 1½ times AB; (*c* 3/4 of AB,

4 Draw a *vertical* line XY 7·4 inches long and produce it towards X to Z, so that XZ may be (*a*) 1/2 of XY, (*b*) *twice* XY.

CHAPTER XIV

AVERAGES.

115. Average.—The *average* of two or more quantities of the *same* kind is the *sum* of the quantities divided by the *number* of the quantities.

Suppose two boys have 9 marbles and 5 marbles respectively. If they put their marbles together and divide them equally be-

tween themselves, each of them will get $\frac{9+5}{2}$ marbles or 7 marbles And the *average* of 9 marbles and 5 marbles is 7 marbles

Similarly the *average* of Rs 10, Rs 6, and Rs $9^1/_2$ is $\frac{\text{Rs } (10+6+9\frac{1}{2})}{3}$ or $\frac{\text{Rs } 25\frac{1}{2}}{3}$ or Rs. $8^1/_2$.

NOTE 1 —From the above examples we may infer that the *average* of the *abstract* numbers 9 and 5 is $\frac{9+5}{2}$ or 7 and of the *abstract* numbers 10, 6 and $9^1/_2$ is $\frac{10+6+9^1/_2}{3}$ or $8^1/_2$.

NOTE 2—It must be noted that the average of two or more numbers is *greater* than the *least* and *less* than the *greatest* of the numbers For example the average $8^1/_2$ of 10, 6 and $9^1/_2$ is *greater than* 6 and *less than* 10

Exercise 93—(*Oral*).

(A) 1 If Rama has 7 plantains and Krishna 11 plantains, how many plantains has each of them *on an average*?

2 Can the average of Rs. 6, Rs. 9, and Rs 12 be Rs. 5 or Rs. 14? If not, why not?

(B) Find the *average* of—

1. Rs. 5 and Rs 11.
2. 8 as, 6 as, and 11 as
3. 15, 17. 0, 10.
4. $8\frac{1}{2}$, $12\frac{3}{4}$, $5\frac{3}{4}$, 10, $6\frac{3}{4}$.
5. 8 miles, 9 miles, 10 miles, 11 miles, and 12 miles
6. 7·5 lakhs 9 2 lakhs 3 0 lakhs, 8·3 lakhs, 10 5 lakhs.

(C) 1 A is 15 years old, B 12 years, C 13 years, and D 10 years. What is the average age of the 4 boys?

2. A certain line as measured by 4 boys is 8 4 cm, 8·7 cm, 8 5 cm. and 8 4 cm respectively What is the average of the 4 measurements?

3. There are 3 lines measuring $7\frac{1}{2}$ ft, 12 ft, and 1 ft. What is the *average* length of the three lines?

Exercise 94.

(A) Find the *average* of—

1. 100, 205, 304.
2. 209 4, 78·0 100 2.
3. $333\frac{1}{2}$, $66\frac{3}{4}$, $79\frac{1}{4}$, $109\frac{1}{2}$.
4. 370 0, 18, 499

(B) 1. The number of fruits gathered from 4 mango trees was 375, 487, 555 and 399 respectively. What was the average number of fruits gathered from each of the trees?

2. In a certain school the numbers present during a certain week were:—On Monday 165, on Tuesday 169 on Wednesday 180, on Thursday 156, on Friday 172. What was the *daily average* attendance during the week?

3. The income of a Railway Company during the first six months of a certain year was as follows:—

Month	Income.	Month	Income.
January ...	Rs. 32,420	April ...	Rs. 35,869
February ..	Rs 48,307	May .	Rs 40,048
March ...	Rs 39 564	June .	Rs 38,122

Find the *average* monthly income of the company for (*a*) the 1st quarter of the year, (*b*) the 2nd quarter, (*c*) the half-year.

4. Find *correct to a foot* the *average* of the following lengths:—

1000 ft., 278 ft., 135 ft., 903 ft., 345 ft

CHAPTER XV

SHORTENED METHODS OF MULTIPLICATION AND DIVISION

116 To multiply a number by 5, 25, and 125

Since $5 = 10 \div 2$, to multiply a number by 5, we may multiply it by 10, *i.e.*, affix a cipher to it, and then divide by 2. Again, since $25 = 100 \div 4$, to multiply a number by 25, we may multiply it by 100, *i.e.*, affix *two* ciphers to it, and then divide it by 4. Similarly, since $125 = 1000 \div 8$, to multiply a number by 125, we may affix *three* ciphers to it, and then divide by 8.

Examples.

(1) 379 × 5.

2)3790

1895 *Ans.*

(2) 478 × 25.

4)47800

11950 *Ans.*

(3) 1681 × 125.

8)1681000

210125. *Ans.*

117. To multiply a number by 9, 99, 999, &c.

Since 9 = 10 — 1, to multiply a number by 9, we may multiply it by 10 and from the product subtract the multiplicand.

Example.—Multiply 853 by 9.

$$10 \textit{ times } 853 = 8530$$
$$\textit{once } 853 = 853$$
$$\therefore\ 9 \textit{ times } 853 = 7677. \quad \textit{Ans.}$$

Similarly, to multiply a number by 99, 999, &c., we may multiply it by 100, 1000 &c., respectively, and from the product subtract the multiplicand.

118. To multiply by 98, 997, &c.

Since 98 = 100—2, to multiply by 98, we may multiply by 100 and from the product subtract *twice* the multiplicand.

Example—Multiply 646 by 98.

$$100 \textit{ times } 646 = 64600$$
$$2 \textit{ times } 646 = 1292$$
$$\therefore\ 98 \textit{ times } 646 = 63308. \quad \textit{Ans.}$$

Again, since 997 = 1000 — 3, to multiply a number by 997, multiply it by 1000 and from the product subtract 3 times the number.

Example—Multiply 1466 by 997.

$$1000 \textit{ times } 1466 = 1466000$$
$$3 \textit{ times } 1466 = 4398$$
$$\therefore\ 997 \textit{ times } 1466 = 1461602. \quad \textit{Ans.}$$

119. In multiplying one number by another we generally employ as many lines of multiplication as there are significant digits in the multiplier. We shall here show how, in certain cases, the number of partial multiplications can be reduced.

Example 1.—Multiply 3459 by 124.

```
  3459
   124
 13836
41508
428916
```

Here multiply first by 4 and then by 12, *taking care to place 6 (the right-hand figure of the product from 4) under 4, and 8 (the right-hand figure of the product from 12) under 2.*

In the foregoing example, to obtain 12 times 3459, we may either multiply 3459 by 12, *or* take 3 times 13836 (which is equal to 4 times 3459), since 12 = 4 times 3 Thus it will be seen that **when a multiplier can be divided into parts each of which is a multiple or a factor of another, the multiplication can be shortened.**

Example 2.—Multiply 1045 by 364.

```
  1045 (i)
   364 (ii)
  4180 (iii)
 37620 (iv)
380380 (v)
```

36 = 9 times 4.

First we take 4 times 1045 and put 0 (the right-hand figure of the product) under 4 in (ii) Then for 36 times 1045, we take 9 times (iii) and place 0 (the right-hand figure of the product) under 6 in (ii)

Example 3.—Multiply 203414 by 964812.

```
      203414   (i)
      964812   (ii)
     2440968   (iii)
    9763872    (iv)
 19527744      (v)
196250268168   (vi)
```

$48 = 12 \times 4$, $96 = 48 \times 2$.

First we take 12 times (i), placing the 8 in (iii) under the 2 in (ii): then for 48 times (i), we take 4 times (iii), placing the 2 in (iv) under the 8 in (ii), and lastly for 96 times (i) we take twice (iv), placing the 4 in (v) under the 6 in (ii).

Example 4.—Multiply 135405 by 56749.

```
    135405   (i)
     56749   (ii)
  947845     (iii)
   6634845   (iv)
7582680      (v)
7684098345   (vi)
```

$49 = 7 \times 7 \cdot 56 = 7 \times 8$.

First we take 7 times (i) placing the 5 in (iii) under the 7 in (ii): then for 49 times (i) we take 7 times (iii), placing the 5 in (iv) under the 9 in (ii), lastly for 56 times (i) we take 8 times (iii), placing the 0 in (v) under the 6 in (ii).

Example 5.—Multiply 19863 by 972216.

```
      19863  (i)
     972216  (ii)
178767       (iii)
 1430136     (iv)
    4290408  (v)
19311126408  (vi)
```

$72 = 9 \times 8$: $216 = 72 \times 3$

First we take 9 times (i) placing 7, the right-hand figure of (iii), under the 9 in (ii): again for 72 times (i) we take 8 times (iii), placing the 6 in (iv) under 2 the right-hand figure of 72 in (ii) · lastly for 216 times (i) we take 3 times (iv) and place the 8 in (v) under the 6 in (ii).

120. To divide a number by 25, 125, and 625.

Division by 25 may be effected by multiplying the dividend by 4 and dividing the product by 4 times 25 or 100. In

this case the remainder, if any, will be 4 times the true remainder.

Example—Divide (1) 785, (2) 875 by 25.

(1)
$$\begin{array}{r} 785 \\ 4 \\ \hline 1\cancel{0}\cancel{0})\overline{31\cancel{4}\cancel{0}} \\ 31 \end{array}$$
31—remainder 40.

∴ 31 is the quotient and $\frac{40}{4}$ or 10 is the remainder.

(2)
$$\begin{array}{r} 875 \\ 4 \\ \hline 1\cancel{0}\cancel{0})\overline{35\cancel{0}\cancel{0}} \\ 35 \end{array}$$

∴ 35 is the quotient and there is no remainder.

Similarly, division by 125 and 625 may be effected by multiplying the dividend by 8 and 16 respectively, and dividing the product by 1000 and 10000 respectively. In the former case the remainder, if any, will be 8 times and in the latter 16 times, the true remainder.

Example—Divide 16379 by 125, and by 625.

$$\begin{array}{r} 16379 \\ 8 \\ \hline 1\cancel{0}\cancel{0}\cancel{0})\overline{131\cancel{0}\cancel{3}\cancel{2}} \\ 131 \end{array}$$
131—32 rem.

∴ 131 is the quotient and $\frac{32}{8}$ or 4 is the remainder

$$\begin{array}{r} 16379 \\ 16 \\ \hline 1\cancel{0}\cancel{0}\cancel{0}\cancel{0})\overline{26\cancel{2}\cancel{0}\cancel{6}\cancel{4}} \\ 26 \end{array}$$
26—2064 rem.

∴ 26 is the quotient and $\frac{2064}{16}$ or 129 is the remainder.

Exercise 95.

(*a*) Multiply by the methods of Arts. 115—117 :—

1.	849 by 5.	2	483 by 25	3	6998 by 25
4.	10869 by 5	5.	6048 by 125	6	1776 by 125.
7.	1234 by 994.	8.	6076 by 9999.	9	4853 by 625.
10.	935 by 984.	11.	796 by 988.	12.	4231 by 894.

(*b*) Find the following products in 2 lines of multiplication:—

1.	864 × 126	2	987 × 9612.	3	445 × 12111
4.	4651 × 648	5.	10244 × 6513.	6.	90045 × 1296.

(*c*) Find the following products in 3 lines of Multiplication :—

1.	41234 × 36124.	2.	61043 × 18624.
3.	14325 × 12111	4.	96057 × 721236.

5. 37765×168112.　　6. 34699×1166132
7. 348712×1431166　　8. 806944×131296.
9. 345671×15225105.

(*d*) Divide by the method of Art. 119 :—

1. 1785 by 25.　2 1606 by 125　3. 8690 by 25.
4. 10625 by 125.　5. 76050 by 25　6 12345 by 625.
7. 12725 by 125　8. 189875 by 125.　9. 54321 by 625.

CHAPTER XVI.

PROBLEMS ON THE SIMPLE RULES.

121. Problems *—A problem is a question demanding some thought in discovering the rule or rules to be applied in its solution

122. To the Teacher.—In the following exercises there are 28 groups of *problems*; and in each group the first question marked (*a*), which is to be done *orally*, is followed by two or more questions of the same *type*, which are to be done in writing.

The pupils must be made to write out *in full* all the steps of the solution of the written problems.

Exercise 96.

[*Questions marked (a) are to be done orally, the steps being repeated aloud, and questions marked (b), (c), etc, are to be done in writing all the steps of the solution being written out in full.*]

1. *(a) What number added to 17 will make 40?*

(*b*) What number added to 4059 will make 8000? $7765\frac{1}{2}$?

* To the Teacher —" Too often children are dismayed by the sight of a sum in problematic form, however easy it may be The teacher should, therefore, encourage them before attempting to solve a problem, always to substitute for the hard numbers given easy ones which can be easily dealt with *mentally*, they thus have a ready test whether the method they propose to adopt is the correct one or not."

(c) What number must be added to the *smallest* number of 6 digits that can be formed with the figures 4, 6, 8, 3, 4, 1 to make the *largest* number of 6 digits that can be formed with the same figures ?

2. (a) *What number subtracted from 40 will give* 25? $17\frac{1}{2}$? 30·4 ?

(b) What number must be subtracted from 4890 to give a remainder $3109\frac{3}{8}$? 1008 8 ?

3. (a) *The sum of two numbers is* 50 *If one of them is* 24, *what is the other* ? *If* $17\frac{1}{4}$? *If* 32·4 ?

(b) The sum of two numbers is 7054 : if one of them is 3654 what is the other ? If $2999\frac{3}{4}$? If 1876·5 ?

4. (a) *The* difference *of two numbers is* 15. *If the* greater *number be 32 find the smaller, if the* smaller *number be* 26, *find the* larger.

(b) The *difference* of two numbers is 2054. If the *greater* number is 3333, find the *smaller* If $5402\frac{1}{2}$. If 4444 4.

(c) The *difference* of two numbers is 1214·8, find the *greater* number if the *smaller* be 1785 2. If 848·5.

5. (a) *If you add* $14\frac{1}{2}$ *to a certain number, the sum is* 30. *Find the number*.

(b) If you add $205\frac{3}{4}$ to a certain number you get 1000. What is the number ?

(c) What number increased by 174·8 will give 759·6 ?

(d) If you add 141 to 5 times a number you get 1151. What is the number ?

6 (a) *If you subtract 7 from a certain number you get* 30. *What is the number* ?

(b) Work the above sum, substituting for 7 and 30 the numbers (i) $699\frac{1}{2}$ and 2568, ii) 104·8 and 2722 6.

(c) What number decreased by 214·7 will give 385 ?

7. (a) *After spending Rs. 25, I had Rs. 35. What sum had I at first* ?

(b) After spending Rs 412, I have Rs. 795 left. What sum had I at first ?

(c) After losing Rs. $355\frac{3}{4}$, I had Rs. $759\frac{1}{2}$. What sum had I at first ?

8. (*a*) *The product of two numbers is 120. If one of them is 15, find the other. If 40. If 16.*

(*b*) The product of two numbers is 19635. If one of them is 77, find the other. If 153. If 119. If 1/4 of 396.

(*c*) The product of two numbers is 6665666606. If one of them is 54, find the other.

9. (*a*) *What number divided by 15 will give 12 for the quotient?*

(*b*) What number divided by 201 will give for the quotient (i) 185, (ii) 1/5 of 2540, (iii) 12^3, (iv) 1/4 of 18^2.

10. (*a*) *Find the* dividend *when the* divisor *is 8,* quotient *12, and* remainder *5.*

(*b*) Find the dividend when the divisor is 205, the quotient 148, and the remainder 135.

(*c*) The dividend is 6,260, the quotient 59, and the remainder 65, find the divisor.

(*d*) The quotient being 65, the remainder twice the quotient, and the dividend 13,390, what is the divisor?

(*e*) The quotient obtained by dividing a certain number by 151 is 133, and the remainder is 37. Find the number.

11. (*a*) *How many times can 8 be subtracted from 75 and what will be the remainder?*

(*b*) How many times can 23 be subtracted from 1000, and what will be the remainder?

(*c*) How often can Rs. 45 be taken away from a purse containing Rs. 800? How many rupees will be left in the purse after this?

12. (*a*) *What is the* least *number that must be added to 18 to make the sum exactly divisible by 5? By 7?*

(*b*) Work out the above sum, substituting for 18 and 5 the numbers (i) 13456, 123 (ii) 20005, 79.

13. (*a*) *What is the* least *number that should be subtracted from 35 so that the remainder may be exactly divisible by 8? By 10?*

(*b*) Work out the above sum, substituting for 35 and 8 the following numbers.—(i) 2222; 15 (ii) 7000, 120.

14. (*a*) *The product of two numbers increased by 10 is 106. If one of the numbers is 12, what is the other?*

(*b*) The product of two numbers increased by 708 is 193,008. If one of the numbers is 2564, find the other.

(*c*) The product of two numbers diminished by 875 is 970,000. If one of the numbers is 375, find the other.

(*d*) The product of two numbers increased by 14·8 is 1420 4, if one of the numbers is 8, what is the other?

15. (*a*) *The product of* three *numbers is 192; if* two *of the numbers are 8 and 4, what is the* third?

(*b*) The product of three numbers is 134400, and the product of two of them is 525 What is the third number?

(*c*) The product of three numbers is 1 86,06,500, if two of the numbers are 8,7.6 and 17, find the third.

16 (*a*) *The product of 3 numbers is 144, if the product of the 1st and 2nd be 48 and of the 1st and 3rd 24, what are the three numbers? Verify your answer*

(*b*) Solve the above sum after substituting the following numbers for 144, 48 and 24, and verify your answers:—

(i) 30870, 735; 882. (ii) 386325, 7575, 510

17. (*a*) *An umbrella costs Rs. 5, a box costs 4 times as much as an umbrella, and a gramophone 8 times as much as a box. What is the cost of a gramophone?*

(*b*) A cow costs Rs 75, a horse 5 times as much as a cow, and an elephant 24 times as much as a horse What is the cost of an elephant?

(*c* The radius of the earth is 4000 miles, the distance of the moon from the earth is 60 times this radius, and the distance of the sun from the earth is 40 times the distance of the moon. Find the distance of the sun from the earth

(*d*) A has Rs $140\frac{1}{2}$; B has 5 times as much as A; and C has 4 times as much as A and B together. How much has C?

18. (*a*) *What number divided by 15 will give the same quotient as $80 \div 5$?*

Solution

$80 \div 5 = 16$.
$\therefore$ required number $\div 15 = 16$.

$\therefore$ required number
$= 15 \times 16 = 240$. *Ans*

(*b*) What number divided by 72 will give the same quotient as 5280 divided by 176?

(*c*) What number divided by 30 will give the same quotient as $302\frac{1}{2}$ divided by 5.

(*d*) What number divided by 29 will give the same quotient and remainder as 3,259 divided by 16?

(*e*) What number divided by 40 will give the same quotient and remainder as $200 \div 15$?

19. (*a*) *What number multiplied by 8 will give the same product as 20 multiplied by 12?*

(*b*) What number multiplied by 15 will give the same product as 21 multiplied by 50?

(*c*) What number multiplied by 36 will give the same product as 48 multiplied by 63?

(*d*) What number multiplied by 85 will give the same product as 51 multiplied by 55?

20 (*a*) *The sum of 3 numbers is 24 the sum of the 1st and 2nd is 14 and of the 2nd and 3rd 18, find the three numbers and verify your answer.*

(*b*) Substitute the following numbers for 24, 14 and 18 above and work out the sum

(i) 817, 604, 336 (ii) $168\frac{1}{2}$, 148, $125\frac{3}{4}$ (iii) 710 5, 300 8; 610·0

21. (*a*) *The sum of two numbers is 20 and their difference is 12 Find the two numbers and verify the answer.*

Solution.

If the *smaller* number were made *equal* to the *larger* number, then the sum of the two numbers would be $20 + 12$ or 32.

That is *twice* the larger number $= 32$

$$\left.\begin{aligned} \therefore \quad \text{the larger number} &= 32 \div 2 \text{ or } 16 \\ \text{And the smaller number} &= 16 - 12 \text{ or } 4 \end{aligned}\right\} \textit{Ans.}$$

[VERIFICATION $16+4=20$; $16-4=12$.]

(*b*) The sum of two numbers is 66666, and their difference is 2222. Find the two numbers. [Verify your answer].

(*c*) The sum of two numbers is 1720 8 and their difference is 666 6. Find the numbers [Verify your answer].

(*d*) The sum of two numbers is 1700, if one of the numbers is greater than the other by 235, find the two numbers

22. (*a*) *A has Rs. 4 B has Rs. 5 more than A, and C has Rs 10 more than A and B together How much money have A, B, and C together?*

(*b*) Substitute the following numbers for 4, 5 and 10 in the above sum and work it out

(i) 304, $412\frac{1}{2}$, $89\frac{3}{4}$ (ii) 100·1, 89·5, 380 6.

(c) A has Rs. $150\frac{1}{2}$, B has Rs 15 more than A, and C has twice as much as B How much money have all the three together?

23. (a) *A has Rs 49 and B has Rs. 30. If A give B Rs. 11 and B gives A Rs. 15, what sum has each after this?*

(b) Substitute for 49, 30, 11, 15 in the question above the following numbers and work it out:—

(i) 388, 100, $28\frac{1}{2}$, $30\frac{1}{4}$. (ii) 149·1, 208·0, 59·1, 20·6.

24. (a) *A has Rs. 50, B has Rs. 20 less than A, and C has Rs 40 less than A and B together How much have they all together?*

(b) Work out the above sum after substituting for 50, 20, 40 the following numbers:—

(i) 729, 125, $402\frac{1}{2}$. (ii) 1000·8, 300·9, 255·5

(c) The population of a town A is 25,007, that of a town B is 5009 less than that of A, and that of a town C is less than that of A and B together by 17030 Find the population of the three towns together.

25. (a) *3 books and 4 slates together cost Rs. $9\frac{3}{4}$. If a book costs Rs $2\frac{1}{4}$, what is the cost of a slate?*

(b) If 9 horses and 20 cows cost Rs. $8,227\frac{1}{2}$, find the price of a cow, given that the price of a horse is Rs $752\frac{1}{2}$.

(c) A merchant takes Rs 10,000 to the market, and buys 725 bags of rice at Rs 8 a bag and some tins of ghee at Rs 28 a tin. How many tins of ghee does he buy, supposing that he spends all his money?

(d) A Railway train runs 360 miles in 13 hours If during the first 6 hours its speed is 25 miles an hour, what is its speed per hour during the remaining 7 hours?

(e) A man walked 4,290 miles in 275 days If his rate of walking was 18 miles a day during 1/5 of the number of days, find his rate of walking per day during the remaining days.

(f) 7 men and 10 women together earn Rs $300\frac{1}{4}$. If each man earns Rs. $20\frac{3}{4}$, how much does each woman earn?

(g) 7 times one number *plus* 11 times another number is 7886. If the first number is 333, find the second number.

26. (a) *I gave 4 cows worth Rs. 45 each and received in exchange 20 sheep. What was the cost of a sheep?*

(*b*) A man gives 87 horses worth Rs 525 each and receives in exchange 145 mules. What is the cost of each mule ?

(*c*) I exchange 42 horses worth Rs. 520 each for bulls worth Rs 364 a pair How many pairs of bulls do I receive ?

(*d*) A man gives 12 acres of wet land worth Rs. 1,210 an acre in exchange for 110 acres of dry land and a house worth Rs. 2,750 What is the value per acre of the dry land ?

(*e*) I exchanged 24 yds of velvet worth Rs 5 a yard for Rs. 20 and a certain number of photos worth Rs. $1\frac{1}{4}$ each. How many photos did I get ?

27. (*a*) *Multiply 1/8 of 72 by 30 and divide the product by 40.*

(*b*) Multiply 1728 by 1/5 of 560 and divide the product by the square of 48

(*c*) Divide the square of 240 by 18 and from the quotient subtract 16 times the square of 5.

(*d*) Divide 34786 by one-sixth of the square of 72.

(*e*) Divide the difference of the squares of 48 and 56 by the sum of these two numbers

(*f*) Multiply 1/16 of a lakh by 10 and divide the product by the cube of 5.

(*g*) To 2345 add 1000 times itself and divide the sum by 143.

28. (*a*) *I take a certain number, add 8 to it, and divide this sum by 4 and get 6. What is the number taken ?*

Solution.

We must proceed *backwards* beginning from 6. Thus :

A certain number ÷ 4 = 6.

∴ this number is 4×6 or 24.

Again a certain number + 8 = 24

∴ the number = 24 − 8 or 16,

That is, the number taken is 16. *Ans,*

[VERIFICATION · $16 + 8 = 24 : 24 \div 4 = 6$.]

(*b*) I multiply a certain number by 8, subtract 23 from the product, and divide the remainder by 143. If the quotient is 7, find the number.

(*c*) I multiply a certain number by 215, add 24 to the product and divide the sum by 12 If the quotient is 217, find the number.

(*d*) I take a certain number, divide it by 5, subtract 216 from the quotient, and multiply the remainder by 11. If the product is 1265, find the number taken.

.29. (*a*) *Divide Rs 90 between A and B, so that A may have 2 shares and B 3 shares.*

(*b*) Divide Rs. 93 among A, B, and C, so that A may have 1 share, B 2 shares, and C 3 shares.

(*c*) Divide Rs. 420 between A and B, so that A may have 2 1/2 shares and B 3 1/2 shares.

(*d*) Divide Rs 105 among A, B and C, so that A may have 3 shares, B 2 shares C 1 share.

(*e*) Divide £ 723 among A, B and C, so that A may have 1 1/4 shares, B 1/4 share more than A and C 1/2 share more than A and B together.

CHAPTER XVII.

USE OF THE SET SQUARE.

123. To draw a straight line perpendicular (or at right angles) to another straight line from a point in it or outside it.

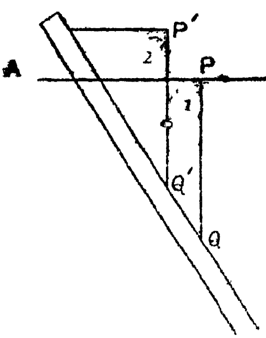


Fig. 10.

Suppose a *perpendicular* has to be drawn to AB. Place the *set square* with a side containing its right angle along AB as shown in position (1) in Fig 10. Place the *straight edge* along the longest edge of the set square Then, pressing the straight edge firmly, slide the set square to the position (2) that is to say, till the edge P′Q′ corresponding to PQ passes through the point from which the perpendicular is to be drawn Now draw the perpendicular required.

Exercise 97.—(*Practical*)

1. Draw a line PQ and from a point R *in it*, draw a line RS at *right angles* to PQ.

2. Draw a line AB and from one of its extremities draw another line at right angles to it.

3. From a point C *outside* a line AB, draw a straight line CD perpendicular to AB,

4. Draw rectangles of the following dimensions :—

(*a*) 4 inches by 3 inches. (*b*) 10 cm by 5 cm.
(*c*) 3 2 inches by 2 4 inches

5. Describe :—

(*a*) a 2 inch *square*, (*b*) a 3 inch square.
(*c*) a 10 cm square. (*d*) a $2\frac{3}{4}$ inch square.

6. Describe rectangles of the following length and breadth and measure their *diagonals* :—(*a*) 12 cm., 9 cm. (*b*) 4 8 in. 2·0 in. (*c*) 6 in., $4\frac{1}{2}$ in.

7. Show by measurement that in the set square in which the edges containing the right angle are equal to each other, the third side is about 1·4 times as long as either of them.

8 Show by measurement that in the other set square, the longer of the two edges containing the right angle is about 1·7 times the shorter, and that the *longest* edge is exactly *twice* as long as the *shortest*.

9. Draw, on card-board or paper, *rectangles* and *squares* of various sizes and cut them out

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CHAPTER XVIII.

AREA OF RECTANGLES AND SQUARES.

124. Area.—By the *area* of a rectangle or a square is meant the *space* occupied by it, *i.e*, the space enclosed by its four sides; and it is measured by means of units like *square inch*, *square foot*, *square centimetre*, and so on.

125. To find the area of a rectangle 3 inches long and 2 inches broad :—

If we describe a rectangle 3 inches long and 2 inches broad and divide it into 5 rows of 3 sq inches each. we see, that the area of a rectangle 3'' long and 2'' broad is (3×2) sq inches or 6 sq. inches Similarly the area of a rectangle 4'' by 3'' is (4×3) sq. inches or 12 sq inches, and the area of a 3'' square is (3×3) sq. inches or 9 sq inches

Exercise 98.—(*Practical.*)

1. Show by a diagram that (1) the area of a rectangle (1) 10 cm. by 4 cm is 40 sq cms 2) the area of a 6 cm. square is (6×6) sq. cms. or 36 sq cms

2. (*a*) Draw on *squared paper* a square 12 small divisions long to represent a square 1 foot long, and show from it that 1 sq. foot = 144 sq. inches. (*b*) Show the same by a square 12 cm. long drawn on *plain paper* (*c*) Similarly show that a sq. cm. = 100 sq. mm.

3. Cut out a paper rectangle 3″ long and 2″ broad. And show by folding it suitably that its area is 6 square inches.

4 Find by paper folding the area of rectangles of the following dimensions in square inches or square centimetres as the case may be —(*a*) 4″ by 3″ : (*b*) 6 cm by 4 cm : (*c*) 3″ by $2\frac{1}{4}$″.

5. Find by paper-folding the area in square inches or in square centimetres of (*a*) a 2″ square, (*b*) a 3 cm. square, (*c*) a 5″ square, (*d*) a $4\frac{1}{2}$ cm. square

Exercise 98-A.—(*Oral*)

1. What is the area of the following rectangles?

(*a*) 8″ by 6″. (*b*) 5 cm by 4 cm. (*c*) $4\frac{1}{2}$″ by 4″

2. What is the area of. (*a*) a 5″ square, (*b*) a 4 cm square. (*c*) a $5\frac{1}{2}$ inch square ?

CHAPTER XIX.

CUBOID AND CUBE.

126. Cuboid—If you examine a model of a *cuboid**, you will find (1 that it is bounded by 6 *flat* surfaces which are *rectangles*, (2) that it has 8 corners (or *solid angles*), and (3) that it has 3 dimensions, *viz.*, *length*, *breadth* and *thickness*.

* A common brick and a dealwood box are examples of a *cuboid*.

NOTE.—Since a *cuboid* is bounded by rectangular faces, it is otherwise called a *rectangular solid*.

127. Cube.—If the *length*, *breadth* and *thickness* (or *height*) of a cuboid be equal to one another, it is called a *cube*.

NOTE 1.—The boundaries of a *cube* are 6 *equal squares*.

NOTE 2.—A *cuboid* and a *cube* have each 12 edges, each of which is the *common* boundary between two adjacent faces.

NOTE 3 – The opposite faces of a cuboid are equal.

Exercise 99 —(*Practical*).

1. Give some examples of a *cuboid*

2. How many *faces*, *edges*, *corners* (*solid angles*), and *plane angles*, has (*a*) a cuboid, (*b*) a cube ?

3 Name the three dimensions of the following:—

(*a*) a dealwood box, (*b*) a common brick, (*c*) a wall, (*d*) a black board, (*e*) a rectangular cistern.

4. (*a*) Make clay models of a cube and a cuboid of different sizes. (*b*) Make clay models of a cuboid or cube with the following dimensions. (1) 3 in. by 2 in by 1 in (2) 4 in. by 2 in. by 2 in. (3) 2 in by 2 in by 2 in (4) 10 cm by 10 cm by 10 cm.

5. Draw *freehand* in pencil a model of a *cube* and of a *cuboid* similar to the following —

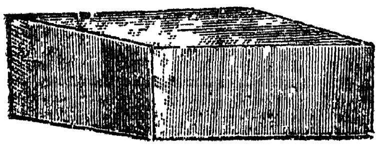

Fig 11.

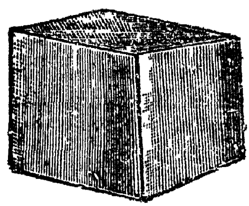

Fig. 12.

6 Draw *freehand* a *rough* outline sketch of *a cube* and a *cuboid* similar to the following —

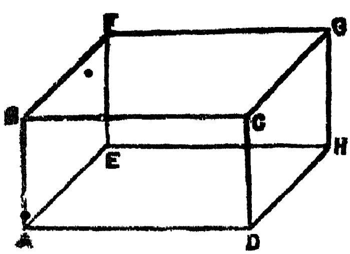

Fig. 13.

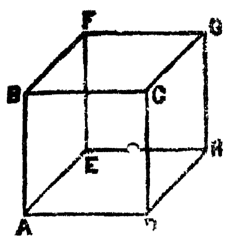

Fig. 14.

CHAPTER XX.

VOLUME OF RECTANGULAR SOLIDS.

128. Volume.—By the *volume* of a rectangular solid is meant the space occupied by it, *i.e.*, the space enclosed by its 6 boundaries or faces; and it is measured by means of units like *cubic inch*, *cubic feet*, *cubic centimetre*, and so on.

NOTE.—A *cube* whose edge is 1 inch is called *an inch cube*, and the amount of space it includes is called a *cubic inch*. Similarly for a *cubic foot*, a *cubic centimetre*, and so on.

129. To find the volume (or cubical content) of a rectangular solid—

Let ABCD be a rectangular solid whose length AB is 4 inches, breadth BC 3 inches, and height BD 2 inches First cut the block into two equal flat plates, each 1 inch thick (as in Fig. 15) Then cut each of these plates into 3 square rods each one inch broad and one inch thick (as in Fig 16), the number of rods will be in all 2 × 3 or 6, and after this, cut each of these six rods into 4 cubes, each 1 inch long, 1 inch broad and 1 inch thick as in Fig. (16) The total number of inch cubes will be 6 × 4 or 24 And as the cubical content of each of the inch cubes is 1 cubic inch, the volume of the rectangular solid is 24 cubic inches.

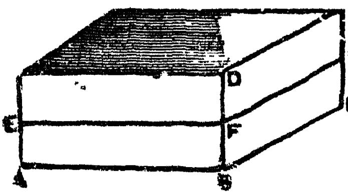

Fig. 15.

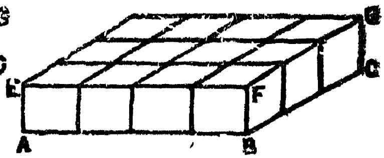

Fig. 16.

By similar reasoning, if the length be 5 cm, breadth 4 cm. and height $3\frac{1}{2}$ cm, the volume would be $5 \times 4 \times 3\frac{1}{2}$ or 70 *cubic* cms.

Hence we see that the volume of a rectangular solid is found by multiplying together the length, breadth and height. This fact is briefly expressed thus —

Volume of a rectangular solid = **length × breadth × height.**

130. To find the volume of a cube.—Since the three dimensions of a *cube* are equal, its *volume* is found by cubing any one of its edges

For example, the volume of a 2-inch cube = 2 inches × 2 inches × 2 inches = (2 × 2 × 2) cubic inches = 8 cubic inches, the volume of a cube 5 cm. long is 125 cubic cms. : and so on.

Exercise 100 —(*Oral and Practical.*)

(A) 1 Find the *volume* of cuboids of the following dimensions:—

(*a*) 5 inches by 2 inches by 1 inch. (*b*) 6 cm. by 4 cm. by 2 cm. (*c*) 4″ by 3″ by 3″.

2 Find the *volume* of (*a*) a 4-inch. cube (*b*) a 3-inch cube. (*c*) a 6 cm. cube.

3. Show (*a*) that a *cubic foot* = 1728 *cubic inches*, (*b*) that a *cubic yard* = 27 *cubic feet*. (*c*) that a cubic centimetre = 1000 cubic mms,

(B) Make, out of wet clay, models of *cubes* and *cuboids*, and find their volume by cutting them into small cubes

CHAPTER XXI.

REDUCTION.

131 Reduction is the process of expressing concrete numbers of one denomination in units of a lower or higher denomination.

132. The reduction of a quantity from a higher denomination to a lower denomination is called *descending reduction*, and is performed by multiplication and addition. The reduction of a quantity from a lower to a higher denomination is called *ascending reduction* and is performed by division.

Example 1.—Reduce Rs. 15-4 6 to pies.

```
Rs  a. p.
15  4  6
16
240 as.        2928 p.
  4 as.           6 p.
244 as.        2934 p. Ans.
 12
```

Example 2.—Reduce 8166 half-pence to £ s. *d.*

```
 2)8166 half-pence
12)4083 pence
20) 340 shillings 3d.
  £ 17 0s

  £ 17-0-3.  Ans.
```

NOTE 1.—At each successive step of the process, the learner must be careful to insert the respective denomination.

NOTE 2 —Since the processes of *ascending* and *descending* reduction are each the converse of the other, the correctness of a result obtained by either process may be tested by working the result back again by the other. The above examples may be proved thus:—

Proof of *Example* 1

12)2934 pies.
16)244 as 6 p.
Rs. 15 4 as

That is Rs. 15 4 as. 6 p.

Proof of *Example* 2.

£	s.	d.
17	0	3
20		
340 s.		
12		
4080 d.		
3 d.		
4083 d.		
2		
8166 half-pence.		

Exercise 101.

Reduce (verifying each result)—

English and Indian Money.

1. Rs 410 8 as. 1 p , Rs. 126 6 as. to pies
2. Rs 220 7 as : Rs. 100 4 as, 3 p. to quarter annas.
3. 10 pagodas to annas. ; 3 pagodas to pies.
4. 4,170 pies : 3.151 pies. : 10,000 pies to Rs as p.
5. £34 , £16-9-10 , £150-17-6 to pence.
6. £17 ; 27 moidores to farthings.
7. 20 guineas 10s. to pence : £50 to four pence.
8. 18,105*d*. : 7,242*d* to £ *s*. *d*.
9. 170,692 farthings : 151 crowns to £. *s* *d*
10. 40,605*d*. to gui. *s* *d* , 100 gui 4*s*. 6*d*, to farthings.

Avoirdupois Weight.

11. 24 lb.; 2 qrs 18 lb. ; 1 cwt. 8 lb. to oz.
12. 12 lb. to drams ; 2 tons 10 cwt, 7 lb to oz.
13. 1 cwt. 3 qrs 26 lb. to oz. , 1 ton to lbs.
14. 2 tons 2 cwt to lb. , 7 stones 12 lb to oz.
15. 964 oz. to qrs. : 4,224 oz. to cwt
16. 3,001 lb. ; 39,680 oz. to tons.

Apothecaries' Weight.

17 5 lb. 3 oz. 2 dr. 2 scr to grains, 576 scr to lbs
18. 2 lb. 7 oz. 5 dr. 1 scr 1 gr to grains, 1,445 scr to lb.
18 (*a*) 3 lb. 5 oz. 4 drms, 5 lb. 6 oz. to grains*.
18 (*b*). 12602 grains, 6404 grains, to lb., oz., drams, and grains.*

Troy Weight.

19. 54 oz. to dwt, 1 lb, 1 lb 10 oz. 5 grs to grains.
20. 2,113 grs. to oz.; 480 dwt; 725 dwt to lb.

Apothecaries' Liquid Measure

20 (*a*). 5 oz 5 dr. 20 minims, 10 oz 6 drams, 18 minims, to minims
20 (*b*). 1000 minims, 1440 minims, 2400 minims, to oz, etc.
20 (*c*) How many tea-spoons to an ounce? To 2 oz.? To 5½ oz?
20 (*d*). How many table-spoons to 1 oz? To 3 oz? To 6½ oz?
20 (*e*). Reduce 420 minims, 1230 minims, 525 minims, 375 minims, to tea-spoons

Madras Weight

21. 1 maund to tolas, 2 candies to seers.
22. 2 mds. 1 vis. 3 pals to tolas.
23. 1 can. 3 vis. 5 pals, 5 mds. 4 vis 3 srs to palams.
24. 7 mds. 5 vis. 3 seers 3 pals to tolas.
25. 3,840 half-tolas, 369 pals to maunds, etc.
26. 4,094 tolas; 5,150 tolas to maunds, etc.

Madras Measure—Capacity.

27. 8 gar 33 mar., 2 gar to measures
28. 2 gar. 28 mar. 1 mea. 6 ol. to ollocks.
29. 431 mea. to kalams, 832 ol to parahs.
30. 5,740 ol to mar; 8,623 mea. to garces

English Liquid Measure.

31. 30 gals 2 qts. to gills, 5 gals 3 qts 1 pt to gills.
32. 500 gills to gal, qts, etc.; 121 gills to gal, &c.

English Dry Measure

33. 4 qrs. 3 bush. 2 pks to pints; 1 load to gills.
34. 12,345 gills to pecks; 345,678 gills to quarters

* 60 grains make 1 dram.

English Lineal Measure.

35. 1 mile 3 fur. 200 yds. to ft ; 2 miles to ft.
36. 2 miles 1,700 yds. to inches , 1 mile to inches.
37 100,000 inches , 60,004 ft to miles, fur , yds., &c.
38 2,456,789 inches , 123 456 ft. to miles, yds., &c
38-A 1 mile 5 chains to inches.
38-B. 10952 ft to miles, fur, chains, etc.

English Square Measure.

39. 5 ac. 1,720 yds to sq. ft , 3 ac. 4,000 yds to sq. ft.
40 1 ac 2 ro 1,065 yds. 1 ft to sq. ft ; 2 ro. to sq in.
41 200,000 sq inches , 165,705 sq ft to acres, &c.

Madras Square Measure.

42. 4 caw 4,865 sq yds to sq yds , 1 caw to sq ft.
43. 1 caw 20 gr. 120 sq ft to sq ft., 2 caw. to sq ft.
44 100,000 sq. ft. to caw , grounds and sq. ft
45 806,565 sq. yds. to acres and yards.

English Cubic Measure

46. 2 c yds 20 c ft to c. in , 1,867,065 c. in to c. yds., &c. ; 180,000 c in. to c yds , &c ; 10 c yds 17 ft 64 in. to c. in.

English Time

47 1 day to sec. ; 2 days 2 hrs. 8 min to min.
48 2 wks 10 hrs 15 min to min , 1 wk 3 hrs to sec.
49 100,806 sec to days ; 37,807 min to weeks.
50. 36,356 sec. to hrs . 3,024 000 sec to weeks.

Angular Measure.

51. 4 deg 5 min. 30 sec , 1 right angle 8 deg. to seconds.
52. 2 rt. angles $8^\circ 4'$; 3 rt angles $5'$ to minutes.
53. 1 rt. angle 30° , 2 rt angles 17° ; 4 rt. angles to degrees.
54. 125° ; 348° , 400° ; 270° to right angles.
55. 21,000 sec., 2548 min. to degrees, etc.

Exercise 102—(*Oral*).

Reduce—

1. Rs 3-12-5 to pies
2. Rs. 4-6-6 to pies
3. £4-0-10 to pence.
4. 3 gui 17*s*. 6*d* to pence.
5. Rs 2-8-8 to pies
6. 3 pagodas to annas.

7.	Rs 5-5-2 to pies	8	£5-10-5 to pence
9.	4 viss 20 pal 2 tol to tolas	10	1 cwt 1 qr to lb.
11.	846 pies to Rs	12	1,210 pies to Rs
13.	485 pence to £.	14	606 pence to £
15	1,443 pence to £.	16	1,448 pies to Rs
17.	4 acres 25 cents to cents.	18	1,670 cents to acres
19.	1° 2' to min.	20	1 rt angle 20° to deg.
21.	2 rt. angles to deg.	22	300° to rt angles.
23.	1 fur. 4 ch to ft	24	396 ft to chains

CHAPTER XXII.

COMPOUND ADDITION.

133. Compound Addition is the addition of *compound* quantities of the same kind

134. The process of *compound addition* is as follows:—

Add together Rs. 25-15-11, Rs. 7-12-5, Rs. 450-8-9, Rs. 516-4-0, and Rs. 89-6-6.

Rs.	As	P.		*Wording*
25	15	11		6, 15, 20, 31 pies, 2 as 7 p. (set down 7 p. and carry 2 as).
7	12	5		2, 8, 12, 20, 32, 47 annas Rs 2 15 as. (set down 15 as and carry Rs 2);
450	8	9		2, 11, 17, 24, 29, (set down 9 and carry 2), and so on
516	4	0		*N B*—The words within brackets are not to be uttered
89	6	6		
Rs. 1,089	15	7	*Ans*	

Note—The method of proof in *compound addition* is the same as that in *simple addition*.

Exercise 103

(A) Add the following quantities *vertically* and *horizontally*:—

	1. Rs.	as.	p.	2. Rs.	as	p	3. Rs.	as	p	4. Rs	as.	p.
5.	209	7	6	6	13	7	0	12	11	869	0	8½
6.	1,666	9	7	14	12	8	100	14	2	1,764	13	9½
7.	409	15	0	109	15	6	78	13	7	369	12	0¾
8.	76	0	11	8	12	10	3,256	11	1	754	0	8¼
9.	69	13	9	166	13	8	409	10	0	64	9	2¾

(B) • Fill up the following table with the totals of the *rows* and of the *columns*, and check the grand total of the former by finding the grand total of the latter:—

	1.			2			3.			Totals of rows.		
	Rs	as	p.	Rs.	as.	p	Rs	as.	p.	Rs	as	p.
4	429	14	8	829	15	0	10	10	10			
5	728	13	9	49	0	9	78	12	8			
6	84	9	4	120	9	8	0	15	11			
7	94	8	11	359	12	11	329	0	6			
8.	870	0	7	8	13	0	80	14	0			
Totals of columns												

(C) Add the following quantities *horizontally* and *vertically*:—

(i)

	1.			2.			3.			4.		
	£	s	d	£	s	d	£	s	d	£	s.	d.
5.	849	0	11	35	17	9¼	220	0	6	760	8	2¼
6.	1,809	19	9	16	0	8½	0	10	8	464	9	3¾
7.	605	8	10	30	15	11¾	112	18	0	608	5	6¼
8.	9	13	8	9	18	0½	0	6	9	66	9	11½
9.	4,695	6	0	19	17	6¾	711	9	6	100	6	0

(ii)

	1.			2.			3.		4.			
	Hrs.	Min	Sec.	Hrs.	Min	Sec.	Min.	Sec.	Days	Hrs.	Min	Sec.
5.	10	8	16	12	16	17	13	14	4	3	10	17
6.	14	0	25	23	20	30	14	13	9	4	30	42
7.	13	9	36	15	30	40	25	35	6	4	0	6
8.	4	0	7	20	45	55	35	25	9	9	9	9
9.	6	3	2½	7	8	30¼	6	9	4	5	3	2½

(iii)

	1.				2.				3.				4.			
	Vis	Sr	Pal.	Tol.	Vis	Sr.	Pal	Tol	Vis	Sr.	Pal.	Tol.	Vis	Sr.	Pal	Tol
5.	20	4	7	2	10	0	2	1½	0	3	5	2	39	2	7	2¼
6.	15	3	0	1	9	4	7	2½	7	2	7	1	0	4	6	2½
7.	16	4	6	2	19	0	6	1¾	15	1	0	2	18	3	6	1¾
8	27	3	5	2	0	2	5	2¼	12	1	4	1	6	4	0	0½
9.	39	0	2	0	28	3	4	0½	9	4	6	2	129	0	4	1¾

(iv)

	1			2.			3.		4.			
	Kal	Mar	Mea.	Mar	Mea.	Ol	Mea.	Ol	Kal	Mar	Mea.	Ol
5	5	10	5	5	7	7	5	7	0	4	0	5
6.	7	0	4	7	5	5	7	5	9	7	7	7
7.	3	2	0	0	3	0	3	4	6	6	5	5
8.	9	10	7	6	4	4	4	3	4	0	7	3

(v)

	1.				2.			3.			4.		
	cwt.	qrs	lb.	oz	cwt	qrs.	lb.	tons	cwt.	qrs	cwt.	qrs.	lb.
5.	3	2	27	0	1	2	3	0	7	3	19	3	24
6.	1	3	19	5	2	3	8	4	3	2	16	2	17
7.	2	2	24	9	17	0	10	3	2	3	18	0	15
8.	8	3	9	15	12	2	18	1	9	1	15	1	7
9.	0	1	5	12	14	1	20	0	8	0	19	3	0

(vi)

	1.				2.			3.			4.		
	lb.	oz	dwt	grs.	lb.	oz.	dwt.	oz.	dwt.	grs	lb.	oz	dwt.
5.	4	10	10	20	1	0	6	5	16	12	4	10	0
6	3	11	7	19	7	8	4	6	15	10	5	0	7
7	5	10	0	15	0	9	5	7	17	18	6	8	0
8.	0	7	9	21	4	5	8	4	0	19	0	9	6

(vii)

	1.			2.			2.			4.		
	lb.	oz.	dr.	oz	dr	scr.	dr	scrs	grs.	oz	dr.	scr.
5.	12	10	7	11	7	2	7	2	15	11	6	0
6.	10	8	6	10	6	0	6	1	18	8	7	2
7	8	7	5	11	4	1	5	0	17	7	5	0
8.	16	0	4	10	3	0	4	1	9	6	4	1
9.	5	5	3	9	0	2	3	0	19	9	0	2

(viii)

	1.			2.			3			4.		
	mls	fur	yds	mls	fur	yds.	fur	yds	ft.	yds.	ft	in.
5.	4	6	100	0	0	5	4	100	2	30	2	10
6.	8	7	200	6	7	55	5	75	1	4	0	11
7.	5	4	205	3	3	75	1	10	2	1	1	11
8.	7	5	106	4	6	0	0	77	1	6	1	9
9	6	4	15	3	4	10	7	19	0	14	2	3

(ix)

	1.			2.			3.			4.	
	ac	ro.	sq. yds.	ac	ro	sq yds	ro	sq yds.	sq. ft.	ro	sq yds.
5.	3	3	1000	5	3	186	3	0	0	2	86
6.	4	2	265	3	2	1200	4	1004	2	1	49
7.	5	2	79	4	0	765	2	865	1	3	65
8.	6	1	85	5	2	609	2	320	2	3	34
9.	7	0	169	6	1	321	1	159	3	3	60

(x)

	1.			2.			3.			4.		
	c.yd	ft	in.	c. yd.	ft.	in.	c.yd.	ft	in	c yds.	ft	in.
5.	3	20	1700	0	2	204	5	5	169	4	21	610
6.	4	20	609	5	7	495	0	6	0	6	22	612
7	3	0	708	8	6	899	6	7	789	7	23	614
8.	5	6	54	0	0	145	7	8	964	9	24	896

(xi)

	1. rt angles.	deg	min	sec.	2. rt angles	deg.	min	3 deg	min	sec.
4.	3	35	50	30	5	40	20	40	20	35
5.	2	12	7	12	0	18	42	32	18	17
6.	1	40	0	0	4	0	15	60	8	9
7.	4	0	20	30	0	15	26	75	11	0

CHAPTER XXIII.

COMPOUND SUBTRACTION.

135. Compound Subtraction is the process of subtracting one *compound* quantity from another of the same kind.

136. The process of *compound subtraction* is as follows:—

Example 1 —*Subtract* Rs. 651-10-8 *from* Rs. 716-5 4

Rs.	as	p.	
716	5	4	
651	10	8	
64	10	8	*Ans*

Example 2.—From £2000 take away £1724-8 9.

£	s.	d	
2000	0	0	
1724	8	9	
275	11	3	*Ans*

137. The method of proof in compound subtraction is similar to that in simple subtraction.

Exercise 104.

(*a*) Perform the following *subtractions* and verify the results:—

1.
Rs.	as.	p
43	7	6
39	6	4

2.
Rs.	as.	p.
10	12	4
8	7	10

3.
Rs.	as.	p
60	0	0
54	6	9

4.
£	s	d
7,001	3	$5\frac{1}{4}$
6,694	18	$4\frac{1}{4}$

5.
£	s.	d.
200	0	0
79	6	5

6.
£	s.	d.
571	4	$4\frac{1}{2}$
40	10	6

7.
£	Rs	as.	p.
28	8	12	5
15	14	13	11

8.
£	Rs.	as	p.
100	0	4	8
40	10	10	10

	tons	cwt.	qrs.	lb.		tons	cwt	qrs	lb.
9	21	4	2	$9\frac{1}{4}$	10	9	7	2	11
	4	3	3	5		6	3	2	15

	Rt. angles.	deg.	min		deg	min	sec.
11.	4	54	6	12	35	20	40
	2	49	25		20	36	31

(*b*) Find the *difference* between—

1. 315 lb 6 oz. 1 dwt. and 19 lb 3 oz. 21 grs.
2. 7100 lb and 4695 lb. 7 oz 13 dwt. 17 grs.
3. 2 oz 1 dr. 1 scr 18 grs and 1 oz 2 scr 19 grs.
4. 9 mds. 6 viss 3 seers 2 pals and 10 mds 4 viss.
5. 24 can 7 mds and 19 can 7 mds 6 vis. 2 seers.
6. 3 gar 7 mar 5 ol and 15 gar. 2 mar. 3 mea.
7. 356 mar. 5 mea. and 349 mar 4 mea 3 ol.
8. 2 miles 6 fur. 2 ft and 7 miles 5 fur 16 yds.
9. 2 days 3 hrs. 17 min and 4 days 1 hr 10 sec.
10. 4 ac. 2840 sq yds 7 sq. ft and 5 ac 4 sq yds.
11. 7 rt. angles and 3 rt angles 48° 50'

Exercise 105.

Fill in the blanks in the following *subtraction* sums —

	Rs	as.	p.		Rs	as	p.		Rs.	as.	p.
1.	486	15	8	2	100	0	0	3.	...	...	...
	...	...	..		.	8	9		108	0	5
	324	10	9		48	..	.		200	15	$8\frac{1}{4}$

CHAPTER XXIV.

COMPOUND MULTIPLICATION.

138. Compound Multiplication is the multiplication of *compound* quantities.

In compound multiplication, the multiplier must, as in simple multiplication, be an abstract number, since it denotes the number of *times* the multiplicand is repeated. For example, we may multiply Rs. 4-6-5 by 4, but not by 4 *pies*.

(xi)

	1				2.			3		
	rt angles.	deg	min	sec.	rt angles	deg.	min	deg	min	sec.
4.	3	35	50	30	5	40	20	40	20	35
5.	2	12	7	12	0	18	42	32	18	17
6.	1	40	0	0	4	0	15	60	8	9
7	4	0	20	30	0	15	26	75	11	0

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COMPOUND SUBTRACTION.

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Example 1 —*Subtract* Rs. 651-10-8 *from* Rs. 716-5 4

Rs.	as	p.	
716	5	4	
651	10	8	
64	10	8	*Ans*

Example 2.—From £2000 take away £1724-8 9.

£	s.	d	
2000	0	0	
1724	8	9	
275	11	3	*Ans*

137. The method of proof in compound subtraction is similar to that in simple subtraction.

Exercise 104.

(*a*) Perform the following *subtractions* and verify the results:—

	Rs.	as.	p		Rs.	as	p.		Rs.	as	p
1.	43	7	6	2	10	12	4	3.	60	0	0
	39	6	4		8	7	10		54	6	9

	£	s	d		£	s.	d.		£	s.	d.
4.	7,001	3	$5\frac{1}{4}$	5.	200	0	0	6.	571	4	$4\frac{1}{2}$
	6,694	18	$4\frac{1}{2}$		79	6	5		40	10	6

	£	Rs.	as	p.		£	Rs.	as	p.
7.	28	8	12	5	8.	100	0	4	8
	15	14	13	11		40	10	10	10

		tons	cwt.	qrs.	lb.
9		21	4	2	$9\frac{1}{4}$
		4	3	3	5

	tons	cwt	qrs.	lb.
10.	9	7	2	11
	6	3	2	15

	Rt. angles	deg.	min
11.	4	54	6
	2	49	25

	deg.	min	sec.
12	35	20	40
	20	36	31

(*b*) Find the *difference* between—

1. 315 lb 6 oz. 1 dwt. and 19 lb 3 oz. 21 grs.
2. 7100 lb and 4695 lb. 7 oz 13 dwt. 17 grs.
3. 2 oz 1 dr. 1 scr 18 grs and 1 oz 2 scr 19 grs.
4. 9 mds. 6 viss 3 seers 2 pals and 10 mds 4 viss.
5. 24 can 7 mds and 19 can 7 mds 6 vis. 2 seers,
6. 3 gar. 7 mar 5 ol and 15 gar. 2 mar 3 mea
7. 356 mar. 5 mea. and 349 mar 4 mea 3 ol,
8. 2 miles 6 fur. 2 ft and 7 miles 5 fur 16 yds.
9. 2 days 3 hrs, 17 min and 4 days 1 hr 10 sec.
10. 4 ac 2840 sq. yds 7 sq. ft and 5 ac 4 sq yds.
11. 7 rt. angles and 3 rt angles 48° 50'.

Exercise 105.

Fill in the blanks in the following *subtraction* sums —

	Rs	as.	p.
1.	486	15	8
	...	...	.
	324	10	9

	Rs	as	p.
2	100	0	0
	..	8	9
	48	..	.

	Rs	as.	p.
3,	...	..	...
	108	0	5
	200	15	$8\frac{1}{4}$

CHAPTER XXIV.

COMPOUND MULTIPLICATION.

138. Compound Multiplication is the multiplication of *compound* quantities.

In compound multiplication, the multiplier must, as in simple multiplication, be an abstract number, since it denotes the number of *times* the multiplicand is repeated. For example, we may multiply Rs. 4-6-5 by 4, but not by 4 *pies.*

139. The process of *compound multiplication* is as follows :—

Multiply (*a*) Rs. 4-6-8 by 4, (*b*) £7-7.5½ by 36, (*c*) Rs. 5-7-4 by 127.

(*a*)

Rs.	as	p.
4	6	8
		4
17	10	8

(*b*)

£	s.	d
7	7	5½
		6
44	4	9
		6
265	8	6

(*c*)

Rs	as	p
5	7	4
		127
693	3	4

Explanation

(*a*) 8 p. × 4 = 32 p. = 2 as 8 p.
6 as. × 4 = 24 as., 24 as. + 2 as.
= 26 as. = Re. 1 10 as.
Re 4 × 4 = Rs. 16, Rs. 16 + Re. 1
= Rs 17.

(*b*) Since 36 = 6 × 6, we first multiply by 6 and the product again by 6

(*c* Since 127 has no factors, we multiply at once by 127.

The steps ara as follows.—

4 p × 127 = 508 p., 7 as. × 127 = 889 as : Rs. 5 × 127 = Rs. 635

12)508 p.
42 as. 4 p

42 as
889 as
16)931 as
Rs. 58-3 as

Rs.
58
635
Rs. 693

NOTE —Since 127 = 12 × 10 + 7, instead of multiplying by 127 in *Example* (*c*). we may multiply successively by 12 and 10 and to this product add 7 times the multiplicand The work will then stand as shown below.

Rs.	as.	p.	
5	7	4	
		12	
65	8	0	= 12 times the multiplicand
		10	
655	0	0	= 12 × 10 or 120 times the multiplicand.
38	3	4	= 7 times the multiplicand.
693	3	4	= 127 times the multiplicand.

Exercise 106.

Multiply—

(A) 1. Rs. 5-7-3 by (*a*) 4, (*b*) 11
2. Rs. 257-6-9 by (*a*) 12, (*b*) 40
3. Rs. 2 796-14-9 by (*a*) 24, (*b*) 63.
4. £1,706-12-11 by (*a*) 123, (*b*) 323
5. £60-17-10 by (*a*) 240, (*b*) 415.
6 7 cwt. 2 qrs 10 lb 4 oz by (*a*) 12, (*b*) 32.
7. 2 tons 3 qrs 12 oz by (*a*) 9, (*b*) 24.
8 3 mds 4 viss 3 pal by (*a*) 11, (*b*) 31
9 1 can. 5 viss 2 pal 1 tol, by (*a*) 4. (*b*) 24
10. 3 lb 10 oz 15 dwt. by (*a*) 20, (*b*) 34.
11 4 days 5 hrs. 10 sec by (*a*) 30 (*b*) 14.
12 3 miles 4 fur 120 yds. 2 ft. by (*a*) 4, (*b*) 16
13. 3 fur. 200 yds 7 in. by (*a*) 12, (*b*) 14.
14. 4 gar 5 par 3 mar. 5 mea by (*a*) 120, (*b*) 94.

(B) 1 Rs 15-4-6$\frac{1}{2}$ by (*a*) 4 (*b*) 10
2 £5 0-7$1/4$ by (*a*) 4, (*b*) 20.
3. £100-7-10$3/4$ by (*a*) 4, (*b*) 8,
4. Rs 17-15-4$1/2$ by (*a*) 3. (*b*) 8
5. £964-19-5$1/2$ by (*a*) 81, (*b*) 40
6. £6-4-5$1/2$ by (*a*) 1036, (*b*) 4005.
7, Rs 1,345-8-3$1/2$ by (*a*) 100000, (*b*) 48506.

CHAPTER XXV.

COMPOUND DIVISION.

140. Compound Division is the division of a *compound* quantity either by an abstract number or by a concrete number of the same kind as itself.

141. When we divide a *compound* quantity by an *abstrct* number, the quotient is a *concrete* quantity. [See Art. 145.]

142. The process of dividing a compound quantity by an abstract number is as follows:

Example 1.

Divide £17 6*s*. 8*d*. by 5.

```
   £   s   d.
5)17   6   8
 £3    9   4   Ans.
```

Here, dividing £17 by 5, the quotient is £3 and the remainder is £2. Reducing £2 to shillings and adding 6*s* we get 46*s*., which being divided by 5, gives 9*s*, for the quotient and 1*s*. for the remainder. Now reducing 1*s* to pence and adding 8*d*. we have 20*d*, which being divided by 5 gives 4*d*. for the quotient.

Example 2.

Divide Rs.137-9-6 by 42.

```
   ( 7)137   9   6
42 { 6) 19  10   6
   ( Rs  3   4   5     Ans.
```

Here, since $42 = 7 \times 6$, we divide the given quantity by 7, and the quotient again by 6.

The same result will be obtained if we divide first by 6 and then by 7.

143. When the divisor is a large number, and has no factors or cannot be easily separated into factors each of which is less than seventeen, we employ *long division*.

Example.—Divide £173 8*s*. 8*d*. by 43.

```
     £    s.  d.   £   s.  d.
43)173    8   8   (4   0   8   Ans.
   172
     1
    20
    28s.
    12
    33 d.
     8d.
   344d.
   344d.
```

Verification.

£4-0-8 × 43 = £173-8*s*. 8*d*. Hence the answer is correct.

Exercise 107.

Divide (verifying the quotient)—

1. Rs. 18-6-4 by 4.
2. £320-18-4 by 5.
3. £79 by 6.
4. £3,675-12-3 by 21.
5. 126 cwt. 1 qr. by 28.
6. 31 mls. 2 fur. 125 yds. by 25.
7. 62 dys. 7 hrs. 45 min. by 31.
8. 128 mds. 2 viss by 48.
9. Rs. 96-6-7 by 121.
10. 43 yds 2 in. by 25.
11. 7 fur. 98 yds. 2 ft 9 in. by 11.
12. Rs 35 by 60.
13. Rs. 37-6-2 by 74.
14. 20 rt. angles 8° 6 by 9.
15. £12 Rs. 7-9 as. 4 p. by 8.
16. 5 mls. 4 fur. by 121.

17. Rs. 176-13-2 by 350. 18 Rs. 88-6-7 by 85.
19. 3 gui. 6s. 4d by 32. 20 1 cwt 3 qrs by 56.
21. Rs. 1,205-10-0 by 120. 22. Rs 86 590-3-7 by 143.
23. Divide Rs 62,56,256-8-2 by (a) 625 (b) 1001, (c) 3575.
24. Divide £16,816 875-1-6 by (a) 168, (b) 4 29, (c) 616.

---

144. The following are examples of the division of a compound quantity by an abstract number, where the quotient is *not* exact.

Example 1.

Divide £25 5s. 10d. by 11.

$$\begin{array}{r|rrr} & £ & s. & d. \\ 11) & 25 & 5 & 10 \\ \hline & 2 & 5 & 11 \end{array} \; - 9d$$

∴ the quotient is £2-5-11 and the remainderr is 9d.

Verification —£ 2-5-11 × 11 +9d.=£25-5-1+9d.=£25-5-10.

Example 2.

Divide Rs. 240-13-11 by 56.

$$\begin{array}{r|rrrl} & Rs. & a. & p & \\ 8) & 240 & 13 & 11 & \\ \hline 7) & 30 & 1 & 8 & - 7 \text{ pies} \\ \hline & 4 & 4 & 9 & - 5 \times 8 \text{ pies.} \end{array}$$

∴The quotient is Rs 4-4-9 and the remainder is (7 + 40) pies or 47 pies or 3 as 11 pies.

Verification —Rs 4-4-9×56 + 3 as 11 pies=Rs. 34-6-0×7 + 3 as 11 pies=Rs 240-10.0 + 3 as. 11 pies=Rs. 246-13 as. 11 pies.

Exercise 108.

Perform the following *divisions* and verify your answers :—

1 Rs 4-9-2 by 10. 2 Rs 85-10-0 by 23. 3 £134-6-5 by 56.
4. Rs. 257-9-10 by 53. 5. 100 yds 1 ft 11 in. by 5.
6. Rs 103-10-5 by 49. 7 £80-6-4 by 8.
8. Rs 298-9-3 by 235. 9 3 tons 8 cwt 5 lb by 43.

---

145. Suppose we have to divide Rs. 15-3-2 equally among 10 men. If we divide Rs. 15-3-2 by 10, we get for quotient Re. 1-8-3 and for remainder 8 pies. This means that we can give Re. 1-8-3 to each of the 10 men and there will be a remainder of 8 pies. Now, if we divide this remainder of 8 pies among the 10 men, each of them will get *more* than ½ a pie. And so, the answer to the sum *correct* to a pie will be Re. 1-8-4.

NOTE.—If the remainder be *exactly* 5 pies so that we can give exactly $\frac{1}{2}$ a pie to each, then too the answer *correct* to a pie will be Re. 1-8-4. But if the remainder be less than 5 pies, so that we cannot give so much as $\frac{1}{2}$ pie to each, then the answer will be only Re. 1-8-3. the remainder being ignored.

Exercise 109.

Perform the following *divisions* and let the answer be *correct* to a *pie* or to a *penny* as the case may be :—

1. Rs 25-3-9 ÷ 8. 2. Rs. 13-5-9 ÷ 6 3. £7-18-4 ÷ 7.
4. £60-8-10 ÷ 12 5. Rs. 3-14-3 ÷ 23 6. £18-13-10 ÷ 31.
7. £3765-17-8 ÷ 325. 8. £10154-6-5 ÷ 240

146. When we divide one *compound* quantity by another *compound* quantity, the quotient is an *abstract* number, because it tells us how many *times* the divisor is contained in the dividend. In this case, we reduce both the dividend and the divisor to the same denomination and then divide. The following are examples of this kind of division :—

Example 1.—Divide Rs. 2 8 as. by 2 as. 6 p.

Reducing Rs. 2 8 as and 2 as. 6 p. to pies, we have 480 pies and 30 pies respectively. Now dividing 480 pies by 30 pies, we have 16 for the quotient This means that 2 as. 6 p is contained exactly 16 times in Rs. 2 8 as

Example 2.—Divide £3-9-11 by £1-7-6.

Reducing the dividend and the divisor to pence we have 839*d.* and 330*d.*, from which we have 2 for the quotient and 179*d.* or 14*s.* 11*d.* for the remainder This means that £1-7-6 can be taken from £3-9-11 *twice* and that the remainder after this is 14*s.* 11*d.*

Exercise 110

(*a*) Divide (verifying your answer)—

1. Rs. 26 4 as by Rs 8-12 as. 2. 15 as by 2 as 6 p.
3. Rs. 12-8-0 by Re 1-0-8. 4. £3 0*s* 6*d*. by 5*s* 6*d.*
5. Rs. 525,2-6 by Rs. 52-8-3. 6. £240 by 12*s.*
7. £3, 675 12*s* 6*d*. by £175 0*s*. 7*d* 8. Rs 11 by 5 as 6 p
9. Re. 1-8-5 by 2 as 6 p. 10. £3-9-0 by £1-0-3
11. Rs 150 by Rs 2-4-6. 12. £121-8-0 by £2-10*s*.
13. 35 mds 5 viss 30 pal by 2 mds. 3 viss 2 pal
14. 2 tons 13 cwt 1 qr. 14 lb. by 3 cwt. 3 qrs. 7 lb.
15. 5 guineas by 3 crowns. 16. £4 by $7\frac{1}{2}d$

17. 21 mls. 2 fur. 150 yds. by 2 mls. 1 fur. 15 yds.
18. 1 fur. 163 yds. 6 in. by 5 yds 6 in.
19. 1 day 14 hrs. 32 min 45 sec. by 1 hr. 10 min 5 sec.

(b) 1. How many payments of 3 as. 6 pies each can be made out of Rs 2-10-0 and what sum will remain over?

2 How many lengths of 2 ft 8 inches each can be cut off from a length of 10 yards and what length will remain over?

147. *To find a fraction (like $\frac{1}{6}$) of a Compound Quantity*—

Example—Find the value of $\frac{1}{6}$ of Rs. 19-3-7.

Solution.

$\frac{1}{6}$ of Rs 19-3-7 $= \frac{\text{Rs. } 12\text{-}3\text{-}7}{6}$ See Art 100) = Rs. 3-3-3$\frac{1}{6}$

= Rs. 3-3-3, omitting $\frac{1}{6}$ pie which is less than $\frac{1}{2}$ a pie.

Exercise 111.

Find, *correct* to a pie or a penny, the value of—

1. $\frac{1}{5}$ of £15-7-6
2. $\frac{1}{7}$ of Rs 48-6-6.
3. $\frac{1}{13}$ of Rs. 100-10-10.
4. $\frac{1}{8}$ of £89-0-0.
5. $\frac{1}{10}$ of £49-6-10.
6. $\frac{1}{9}$ of Rs. 43.
7. $4\frac{1}{3}$ of Rs. 420-5-6.
8. $3\frac{1}{7}$ of Rs. 72-0-0

CHAPTER XXVI.

PROBLEMS ON THE COMPOUND RULES.

Model 1.—A has Rs. 396-7-8, B has Rs. 48-9-6 less than A, and C has Rs. 50-4-3 more than B. How much money has C? And how much have A, B and C together?

Solution.

A has		Rs 396- 7-8
B has Rs 396- 7-8	*minus* Rs 48- 9-6	Rs 347-14-2
C has Rs 347-14-2	*plus* Rs. 50- 4-3	Rs. 398- 2-5
A, B and C have		Rs. 1,142- 8-3. *Ans*

Exercise 112.

1. Samuel has Rs 125-5-6 John has Rs 75-8-9 more than Samuel, and Williams has Rs 26-9-4 less than John. How much has Williams? And how much have all the three together?

2. A has £273-14-6. B has £49-14-9 less than A. and C has *twice* as much as B How much has C?

3. A has Rs 500-8-6. B has Rs. 39-0-6 less than A, C has as much as A and B together, and D has twice as much as C, how much money has D?

4. James has £817-8-9, Joseph has £720-6-3. If Joseph gives to James £30-8-6, and then James gives to Joseph £48-6-3, how much money has each of them now?

5. A property consists of Rs. 820-0-6 in ready cash. (2) jewels and (3) a house. If the jewels are worth 4 *times* as much as the ready cash, and the house *twice* as much as the jewels, what is the value of the whole property? And what is the difference between the value of the house and the ready cash?

6. A purse contains £347-13-4, if 1/5 of it belongs to A, ¼ to B and the remainder to C, what sum does each possess?

7. I have half a lakh of rupees. I give away 1/15 of this to my nephew and 1/7 of the remainder to my niece. How much remains with me after this?

8. There are 4 quantities (1) Rs 409-8-7 (2) Rs 860-7-8. (3) Rs 964-5-6. (4) Rs 1000-0-0 Subtract (3) from (4). (2) from (3). (1) from (2) add the remainders together and compare the sum with the remainder got by subtracting (1) from (4).

9. A B and C have shares in an estate worth 2 lakhs of Rupees. A's share is Rs. 543-6-9 less than ⅛ of the estate B's share is Rs 10000 4-6 more than *twice* A's, C has all the rest. What is the value of the share of each?

Model 2.—If one ton of sugar costs £74-13-6, what is the cost of 4⅓ tons?

Solution.

Cost of 1 ton of sugar is £74-13-6.

∴ cost of 4 tons of sugar is £74-13-6 × 4 or £298-14- 0.

And cost of ⅓ ton of sugar is $\frac{\text{£74-13-6}}{3}$ or £ 24-17-10.

∴ cost of 4⅓ tons of sugar is £323-11-10.

Exercise 113.

1. If one maund of sugarcandy costs Rs 21-8-0, find the cost of (*a*) $3\frac{1}{4}$ maunds, (*b*) $4\frac{1}{6}$ maunds, (*c*) $3\frac{1}{5}$ maunds.

2 A has £75-6-9, B has $2\frac{1}{3}$ times as much as A and C has $3\frac{1}{7}$ times as much as B How much has C?

3 A has £840-8 6, B has $4\frac{1}{4}$ times as much as A and C has £84-6-9 less than $2\frac{1}{3}$ times B's, How much money has C?

Model 3.—$5\frac{1}{4}$ yds. of silk and 8 yds. of mull together cost Rs. 47-15-9. If 1 yd. of silk costs Rs. 7-3-0, find the cost of mull per yard.

Solution.

Cost of $5\frac{1}{4}$ yds. of silk and 8 yds of mull = Rs 47-15-9.

But cost of $5\frac{1}{4}$ yds. of silk = Rs. 7-3-0 $\times 5\frac{1}{4}$
= Rs 37-11-9

$\therefore$ Cost of 8 yds. of mull = Rs. 10- 4-0

$\therefore$ Cost of 1 yd of mull = Re. 1- 4-6. *Ans.*

Exercise 114.

1. $4\frac{1}{8}$ yards of silk and 6 yards of mull together cost Rs, 34-12-6. If 1 yd of silk costs Rs 6-12-0, what is the cost of mull per yard?

2 12 men and 15 women together earn Rs 19-3-6. If a man earns 12 as. 6 pies, how much does a woman earn?

3. 120 apples are bought for Rs 21-14-0. $\frac{1}{4}$ (of the number) at $3\frac{1}{2}$ as each and $\frac{1}{3}$ at 3 as each What is the price of each of the remaining apples?

4 I buy 120 mangoes and some cocoanuts for Rs 8-2-0, the mangoes at 8 pies each and the cocoanuts as 10 pies each. How many cocoanuts do I buy?

5 2 mds. 4 vis. of sugar and 3 mds 2 vis of sugarcandy together cost Rs 89-2-0 If sugarcandy costs Rs. 2-4-0 a viss, what is the cost of sugar per viss?

Model 4.—What quantity multiplied by 12 will give the same product as Rs 250-7-6 multiplied by 18.

Solution.

Rs. 250-7-6 $\times$ 18 = Rs 250-7-6 $\times 6 \times 3$
= Rs 1502-13-0 $\times$ 3 = Rs. 4508-7-0.

$\therefore$ the required quantity = Rs $\frac{4508\text{-}7\text{-}0}{12}$ = Rs. 375-11-3.. *Ans.*

Exercise 115.

1. What quantity multiplied by 15 will give the same product as 8 miles 4 fur. multiplied by 12 ?

2 What quantity divided by 8 will give the same quotient as Rs 29-4-5 divided by 11 ?

3. What quantity divided by 13 will give the same quotient as £ 128-7-6 divided by 10 ?

4. What quantity muitiplied by 21 will give the same product as. 4 mds, 6 viss. 4 pal multiplied by 12 ?

5. What sum multiplied by 13 will give a product which is Rs. 1-2-0 more than the quotient obtained by dividing Rs. 2,409-6-0 by 12 ?

Model 5.—A man gives 40 sheep worth Rs. 3-4-6 each, and receives in exchange goats worth Rs. 5-7-6 each ; how may goats does he receive ?

Solution.

Cost of 40 sheep at Rs. 3-4 6 = Rs 3-4-6 × 40 = Rs 131-4-0.
But 1 goat is worth Rs 5-7-6.

$$\therefore \text{ the number of goats he gets} = \frac{\text{Rs } 131\text{-}4\text{-}0}{\text{Rs } 5\text{-}7\text{-}6} = \frac{25,208 \text{ p}}{1,050 \text{ p}}$$
$$= 24. \quad \textit{Ans.}$$

Exercise 116

1. How much muslin at Re 1-5-8 per yard is equal in value to 143 yds. of cambric at Rs. 3-13-8 yard ?

2. A man gave away 24 lb of coffee worth 3*s*. 4*d* a lb. and received in exchange 15 lb of tea What is the value of the tea per lb. ?

3. A gives away 50 exercise-books worth 4 as 7 pies each and Rs 7-8-10 in cash and receives in exchange 100 ink-stands What is the value of each ink-stand ?

4 A man exchanged 20 sheep worth Rs. 12-8-6 each and 8 cows worth Rs 50-4-9 each for bulls worth Rs [illegible]-13-4 each. How many bulls did he get ?

Model 6.—A stationer buys $8\frac{1}{2}$ reams of paper for Rs. 30-8-0 and pays Rs 5-2-0 for carriage. Find, *correct to a pie*, the average cost per quire. [1 ream = 20 quires.]

Solution

The total cost of $8^1/_2$ reams or 170 quires of paper = Rs. 30-8-0. + Rs 5-2-0 = Rs. 35-10-0.

$$\therefore \text{ the average cost per quire} = \frac{\text{Rs } 35\text{-}10\text{-}0}{170}$$

$$= 3 \text{ as. } 4^{40}/_{170} \text{ pies.}$$

Now since 40/170 pie is less than ½ a pie, the answer *correct* to a pie is 3 as. 4 pies omitting the fraction 40/170

Exercise 117

1. If a clerk's salary is Rs 25 a month, how much is it (*correct* to a pie) for a day in the month of August?

2. If 235 articles cost £1,000, find (*correct* to a penny) the cost of each.

3 If 25 boxes of sugar together weigh 1 ton 10 cwt, find to the *nearest* pound the average weight of the sugar in each box given that each empty box weighs 22 lb.

4. A man buys 20 eggs at 2 for 9 pies and 12 more at 3 pies each. What is the *average* cost of each of the 32 eggs? (The answer to be *correct* to a pie)

Model 7.—The *sum* of two quantities is Rs. 405-7-6, and their *difference* is Rs. 180-0 6. What are the two quantities?

[For the method of solution see *Model* 21 in Chap XVI

Exercise 118.

(A) 1. The sum of two quantities is £800-5 4 and their *difference* is £200-8-6 Find the two quantities

2. Solve the above sum substituting the following for £800-5-4 and £200-8 6—

(*a*) £700-0-0 . £200-18-7.
(*b*) 120 miles 6 fur 100 yds. . 20 miles 4 fur
(*c*) 100 mds : 25 mds. 5 vis 4 seers.

(B) 1. The sum of two quantities is £87-4-5, and one of them is £17-6-5 less than the other. Find the two quantities.

2. A man walks 48 miles 4 fur in 2 days. If on the second day he walks 3 miles 2 fur less than on the first day, what distance does he travel on each day?

3. A house and a garden together cost Rs 10,000, if the house costs Rs. 400-8-6 less than the garden, what is the cost of each?

4. The sum of two quantities is Rs 665½, if one of them is greater than the other by Rs 60, find the two quantities.

5. Divide Rs. 200-8-0 into two parts, such that one part may be Rs. 8-7-6 more than the other.

Model 8 —I divide a certain quantity by 12 and from the quotient subtract 1 fur, 10 yds and get 1 fur. 30 yds. Find the quantity.

[For the method of solution see *Model* 28 in Chap. XV.]

Exercise 119.

1. I take a certain quantity, multiply it by 20, subtract £18-10-4 from the product, and get £285.19-8. Find the quantity taken

2. If we add Rs 40-9-3 to 10 times a certain quantity, we get Rs 462-12-3. Find the quantity.

3 If to 1/9 of a quantity we add 4 mds 3 vis 2 seers, we get 5 mds. 6 vis. What is the quantity?

4. I take a certain quantity, divide it by 11 subtract £1-7-9 from the quotient and get £1-0-1 What is the quantity taken?

Exercise 120.

(Miscellaneous.)

1. A merchant buys 1 md. 4 vis of sugar at 2 as. 6 pies a seer and 2 mds. 5 vis. of coffee at Rs. 1-12-6 a viss. Find the average cost per seer

2. I distribute Rs. 253-14-6 equally among 750 persons, how much will each of them get, and how much will 10 of them get together?

3. A buys tea for £ 4-7-6 at 3*s* 6*d* a lb. and reserves 4 lb. for his own use. For how much can he sell the remainder at 4*s* 3*d* a lb.?

4. A man whose annual income is Rs. 3,000 spends Rs 20-8-9 a week: how much does he save in a year? (A year = 52 weeks) Also find *correct* to a pie his average daily expenditure.

5 A gentleman's monthly income is Rs. 250, if he spends 1/4 of his income, how much will he save (*a*) in a month, (*b*) in a year? And what will be his daily savings (*correct* to a pie) in a leap year?

6. If a clock ticks 60 times in a minute, how many days, hours and minutes will it take to tick two lakhs and forty-thousand times?

7. How many guineas are there in £53,025?

8. How many packages of 11 oz. each can be made from 34 tons 3 qrs. 20 lb. 8 oz? And how many ounces will remain over?

9. A man bought tea for £15 at 3*s* a lb. and after reserving a certain quantity for himself sold the remainder for £18 at 4*s*. a lb. How much tea did he reserve for himself?

10. If Henry has £16-7-6 in six-penny pieces and Joseph has £ 50-7-6 in half-crewns, how many more coins has Henry than Joseph?

11. If telegraph posts are placed 40 yards apart, how many will there be in 5 miles and 7 furlongs?

12. What shall I have to pay for sending through book-post 143 packets, 50 of which weigh 7 tolas each, 50 more 15 tolas each, and the remaining packets 24 tolas each, if the postage is half an annn for every 5 tolas, and parts of 5 tolas are counted as 5 tolas, the postage being payable on each packet separately?

13. What is the greatest number of articles worth Rs. 2-8-6 each that can be bought for Rs. 100, and what sum will remain over?

14. If a man's salary is Rs. 35 a month, how much was it (*correct* to a pie) for 5 days in the month of Feb. 1924.

15. A man takes £ 200 to the market and buys the largest number of cows that can be bought for the money at £11-0-7 each, and for the remainder buys a dozen fowls. Find the cost *correct* to a penny of each fowl.

16. A gun is fired at a distance of 2 miles 4 fur. How long after it is fired will its report be heard, supposing that sound travels at the rate of 1 120 feet per second? [Answer to be correct to a second.]

CHAPTER XXVII.

PRACTICE AND BAZAAR TRANSACTIONS.

148. Suppose a tradesman wants to calculate the price of 125 things at Rs. 2-6-8. He does not multiply Rs. 2-6-8 by 125, but finds the price of 125 things at Rs. 2, at 4 as. (or $\frac{1}{4}$ of 1 Re.), at 2 as (or $\frac{1}{2}$ of 4 as), and at 8 pies (or $\frac{1}{3}$ of 2 as.), and adds these prices together, thus :—

		Rs.	as.	p.	
Cost of 125 things @ Rs 1	=	125	0	0	
Cost of 125 things @ Rs 2	=	250	0	0	
Cost of 125 things @ 4 as. or $\frac{1}{4}$ of 1 Re	=	31	4	0	
Cost of 125 things @ 2 as. or $\frac{1}{2}$ of 4 as.	=	15	10	0	
Cost of 125 things @ 8 pies or $\frac{1}{3}$ of 2 as.	=	5	3	4	
Cost of 125 things @ Rs 2-6-8	=	302	1	4	*Ans.*

Note—'@ Re. 1' means 'at Re. 1 each.'

149. This method is called **Practice**, and 4 as., 2 as., 8 pies are called **aliquot parts** respectively of Rs. 1, 4 as., 2 as.

Note.—A concrete quantity equal to a fraction like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc. of another concrete quantity (of the same kind) is called an *aliquot part* of that quantity.

Exercise 121—(*Oral*).

1. What parts of Rs 1 are 8 as., 4 as., 2 as. 2 as. 8 p., 5 as. 4 p.?
2. What parts of 8 as. are 4 as., 2 as., 1 a., 2 as. 8 p.?
3. What parts of £ 1 are 10 *s*., 5 *s*., 4 *s*., 2 *s*., 6 *s*., 8 *d*.?
4. What parts of 10 *s*. are 5 *s*., 1 *s*., 2 *s*. 6 *d*., 3 *s*. 4 *d*.?
5. What parts of 4 as are 2 as., 1 a., 1 a. 3 pies?
6. What part of 5 *s*. are 1 *s*., 2 *s*. 6 *d*., 1 *s*. 8 *d*., 10 *d*.?

150. Division into aliquot parts.—To find by *Practice* the cost (say) of 345 things @ £0-13-8, we must *first* divide 13*s*. 8*d*. into convenient *aliquot* parts.

Thus:	10*s*.		= 1/2 of £ 1.
	2*s*.		= 1/5 of 10*s*.
	1*s*.		= 1/2 of 2*s*.
		8*d*	= 1/3 or 2*s*.
	13*s*.	8*d*	= total of the aliquot parts.

Exercise 122.

Find by practice, the values of the following numbers of things. Also verify the answers by Compound Multiplication.

A. 1. 345 at Re. 1-12-6 each. 2. 245 at Re. 1-13-4 each.
3. 268 at £1 3-6 each 4. 3895 at £1-7-4 each.
5. 298 at Re 1-14-8 each. 6 240 at Re. 1-5-9 each.

B 1 2521 at 12*s* 9*d*. each. 2 249 at 17 *s*. 4*d*. each.
3. 252 at 10 as 8 p each. 4. 300 at 6 as. 8 p. each.
5. 361 at 2 ys $5\frac{1}{4}$ p each. 6. 868 at 6*s*. 4*d*. each.

C. 1. 345 at Rs 2-13-4 each. 2. 240 at Rs. 3-10-8 each.
3. 720 at £5-17-6 each. 4. 4200 at Rs 8.14-8 each.
5. 527 at £4-11-3 each. 6. 8307 at £3-15-$6\frac{1}{2}$ each.

---

151. To find the cost of $141\frac{1}{2}$ things at Rs. 2-5-6 each, we must take the price of $141\frac{1}{2}$ things @ Re. 1 as Rs. 141-8-0; to find the cost of $75\frac{1}{3}$ things at £3-6-9, we must take the cost of $75\frac{1}{3}$ things at £1 each as £75-6 8; and so on.

Exercise 123.

Find the cost of the following numbers of things:—

1 $81\frac{1}{2}$ @ Rs 3-6-6 2. $273\frac{1}{2}$ @ 5 as. 3 pies
3. 124¹/[illegible] @ 13*s*. 6*d*. 4. $25\frac{1}{6}$ @ £1-7-0.
5. $47\frac{1}{4}$ @ Re. 1-10-8. 6 $125\frac{1}{3}$ @ 3*s*. $7\frac{1}{2}$*d*.

---

152. *To find the cost of 775 mangoes at Rs. 4-7-6 a hundred.*

Solution

	Cost of 100 mangoes	= Rs. 4-7-6
∴	Cost of 700 mangoes or 100 × 7 mangoes	= Rs. 31-4-6
	Cost of 50 mangoes or 100×1/2 mangoes	= Rs. 2-3-9
	Cost of 25 mangoes or 100×1/4 mangoes	= Rs 1-1-10½
∴	Cost of 775 mangoes	= Rs. 34-10-1½
	or Rs 34-10-2 correct to a pie	*Ans.*

NOTE — We may otherwise find the required cost by subtracting the cost of 25 mangoes from the cost of 800 mangoes.

Exercise 124

Find by practice the cost of—

1. 550 cocoanuts at Rs. 6-8-3 a *hundred*.
2. 8750 bricks at Rs 12-4-6 a *thousand*.
3. 81 exercise-books at Rs 4 5-3 a *dozen*.
4. 95 articles at £1 4-6 a *score*.
5. 15 palams of coffee at Rs. 2-3-6 a *viss*.
6. 23 bullocks at Rs. 295-8-0 a *pair*

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153. A Bill of Parcels or an **Invoice** is a statement of goods sold, showing particulars of their quantity and price. The following is an example :—

Madras, *Jan.* 9. 1897.

K. Ramaswami Aiyar, Esq.,

Bought of *Addison & Co.*—

		Rs.	a.	p.
8 Note-books at Re 1-4-6 each ...	...	10	4	0
4½ quires of paper at 3 as. 4 p. per quire	...	0	15	0
4 dozen pencils at 8 as. a dozen ...	...	2	0	0
Total ...	Rs	13	3	0

Exercise 125.

(*a*) Make out bills for the following articles :—

1. Three pieces of muslin at Rs. 5-4 3 a piece. 1- yds. of longcloth at 3 as 8 pies per yard, 4 caps at 5 as. 8 pies each, and packing charges 1 a. 3 pies.

2 One cwt of sugar at 4 as 9 pies per lb 5 seers of coffee at 14 as 3 pies per seer, and 4 dozen mangoes at Rs. 2 4 as a dozen.

3. Twenty-four pairs of spectacles at £1 3*s*. 5*d*, a pair 8 dozen slates at 2*s* 6*d*. each, 20 note-books at $8^1/_2d$ each, and packing charges 1*s* 4*d*

4. Fifteen horses at £150-17-6 each 24 bulls at £75 4*s* a pair, 36 sheep at £26 8*s*. a dozen, and 3 mules at £15-10-6 each.

5. Two gross of steel nibs at 9 pies a dozen, 100 pencils at 8 pies each, 6 buttons at 8 as 6 pies per dozen, and 1 automatic pen at 10 as 9 pies.

6. Sixteen oranges at Re. 1-5-0 a dozen, thirty apples at Rs 2-8-0 a dozen, packing 4 as, 9 pies, and cooly 2 as, 9 pies.

7. Three pieces of cloth each forty yards long at 4 as, 3 pies a yard, 11 yards of calico at 6 annas a yard, and one hundred reels of cotton at Rs 2 a dozen

8. Five seers of sugar at Rs, 4-10-0 a maund, eight palams of jaggery at 6 as 3 pies a viss and three palams of pepper at 3 as 4 pies a seer.

(*b*) Make out a bill for the following articles, working out each item by practice:—148 mangoes at 2 as 3 pies each, 28 viss of coffee at Re 1-4-$7^1/_2$ a viss, and $18^1/_4$ seers of sugar at 1 anna 8 pies a seer

Exercise 126.

1. Bought 3 palams of ghee at 8 as. 6 pies a seer, $4^1/_2$ palams of asafœtida at 2 as 3 pies a palam, and 2 marakals of black gram at Rs 10-6-0 a kalam If a five-rupee note be given to the bazaarman, what change should he give?

2. Bought 63 plantains at 14 for an anna, 120 arecanuts at 3 as. 4 pies a hundred, $3^1/_4$ bundles of leaves at 2 as. 7 pies a bundle. If Rs. 3 be given to the bazaarman what change should he give?

3. 33 limes at 2 for 3 pies, 51 guavas at $8^1/_2$ for an anna and 30 wood-apples at Rs 3-2-0 a hundred were bought and Re 1-4-6 was paid to the bazaarman. How much is still due to him?

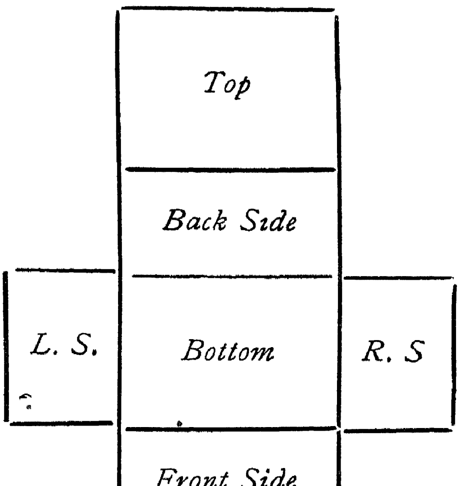
4. A man went to the fair with Rs 1000 and made the following purchases:—15 bulls at Rs 84-4-0 a pair, 100 fowls at Rs 10-6-0 a dozen, 25 geese at a pagoda per pair How much money had he remaining?

CHAPTER XXVIII.

CARD-BOARD MODELLING.

154. To make a card-board model of a cuboid:—

Draw on card-board the following figure which is the uet* of a cuboid 1 inch long, 3/4 inch broad, and 1/2 inch thich, and cut out the figure with a pair of scissors Then make a cut with a knife half way through the four boundaiy lines of the *bottom* and through the lower line of the *top*. Then bend these flaps through a right angle and fix the edges with gummed paper of tough quality The resulting solid will be a *cuboid*.



Net of a cuboid

Fig. 17.

NOTE.—Instead of drawing the above figure on card-board it may be drawn on squared paper or on plain paper and cut out and pasted on card-board.

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* If a box made of some thin material be cut through the side edges and laid out so that the sides and ends lie in a plane with the bottom, this plane figure is called the net of the box.

Exercise 127—(*Practical.*)

1. Draw on (1) squared paper, (2) on plain paper the net of—

(*a*) a one inch cube, (*b*) a 1 1/2" cube, (*c*) a cuboid 2" by 1 1/2" by 1", (*d*) a cuboid 10 cm. by 4 cm. by 6 cm.

2. Make card-board *cubical* boxes of the following lengths:—

(*a*) 1/2" : (*b*) 1 1/4" : (*c*) 2" (*d*) 8 cm , (*e*) 12 cm.

3 Make card-board *cuboidal* boxes of the following length, breadth and height,—(*a*) 2", 1 1/2", 1" : (*b*) 1", 1", 3/4" (*c*) 10 cm , 8 cm , 8 cm.

4. Make card-board boxes *without lid* of the following dimensions :—(*a*) 3" by 2" by 1 1/2", (*b*) 1" by 1" by 1" : (*c*) 12 cm. by 8 cm by 6 cm.

CHAPTER XXIX.

ODD, EVEN AND CONSECUTIVE NUMBERS.

155. Odd and Even numbers.—A whole number which is exactly divisible by 2 is called an *even* number: and a whole number which is *not* exactly divisible by 2 is called an *odd* number.

Thus 2, 4, 8, 12 are even numbers, and 1, 3, 7, 13 are *odd* numbers

156. Consecutive numbers.—Whole numbers which follow one another in their natural order are called *consecutive* numbers.

Thus 1, 2, 3 are three *consecutive* numbers in *ascending* order. 8, 7, 6, 5 are four *consecutive* numbers in *descending* order.

1, 3, 5 are three consecutive *odd* numbers in *ascending* order, and 10, 8, 6, 4 are four *even* consecutive numbers in *descending* order.

Exercise 128—(1—7) (*Oral*).

1. What is an *odd* number? What is an *even* number? What are *consecutive* numbers?

2 Which of the following numbers are *odd* and which *even*? 8, 7, 12 20, 100, 75, 68, 99

3 Name 4 *consecutive* numbers in *ascending* order, and give the same numbers in *descending* order,

4, Name the first five *odd* numbers and the first five *even* numbers.

5. Find the sum of the 5 consecutive numbers in *aseending* order commencing from 11

6. From the sum of all the *odd* numbers lying between 6 and 14, subtract the sum of all the *even* numbers lying between 1 and 11.

7. How many *odd* numbers are there (i) between 40 and 50 (ii) between 90 and 70? And how many *even* numbers are there (iii) between 45 and 65, (iv) between 99 and 77?

8 Add together by the simplest method the five consecutivs numbers of which the middle one is 3451

9, From the sum of the five consecutive *odd* numbers of which 323 is the middle one, subtract the sum of the five consecutive *even* numbers of which the middle one is 288.

10, Write down two sets of five conseeutive numbers commencing from 1C00, and find the difference between the sums of the two sets,

11. Subtract the product of the first 8 consecutive numbers from half-a-lakh.

12. Take any four consecutive numbers greater than 650 and shew that the sum of the 1st and 4th is equal to the sum of the 2nd and 3rd

13. Take any five consecutive numbers greater than 75 and show that the sum of the 1st, 2nd, 4th and 5th is 3 times the 3rd.

14, Take six consecutive numbers greater than 124 and show that the sum of the 1st, 2nd, 5th and 6th is twice the sum of the remaining two numbers.

15. Take seven consecutive numbers of one digit and find the relation between the middle number and the sum of the other six numbers. Also show that the same relation holds in the case of any 7 consecutive numbers of *two* or *more* digits.

CHAPTER XXX.

COMMON FACTORS.

157. Common factors.—Any number (except 1) which divides each of two or more numbers is called a *common factor* of those numbers.

For example, since 3 divides both 9 and 15 3 is a *common factor* of 9 and 15 Similarly 5 is a *common factor* of 20 and 25, 7 is a *common factor* of 14 and 21 : 4 is a *common factor* of 12, 20 and 28 : and so on.

NOTE —The number 1 is not considered as a factor of any number

Exercise 129—(*Oral*).

Name the *common factors* of—

1.	20 and 15	2.	9 and 21.	3.	15 and 18.
4	7 and 14	5	21 and 35.	6.	33 and 55.
7	105 and 80	8.	143 and 121.	9	65 and 169.

158. It often happens that there are more factors than one which are *common* to two or more numbers.

For example, the *common factors* of 18 and 30 are 2, 3, and 6 (since each of these three numbers divides 18 and 30 exactly): similarly the *common factors* of 12, 16 and 24 are 2 and 4

Exercise 130—(*Oral*).

Name *all* the common factors of—

1.	12 and 18.	2.	18 and 20	3	20 and 30.
4.	36 and 42.	5.	18 and 24.	6	45 and 30.
7.	12 and 30	8.	100 and 125.	9	48 and 80.

CHAPTER XXXI.

DIVISION BY CANCELLATION.

159. Division by cancellation of common factors.—The process of division can be sometimes shortened by rejecting or *cancelling* common factors from both dividend and divisor, since this does not change the value of the quotient, as will be seen from the following examples. This process is briefly called *division by cancellation*.

Examples. Find the value of (1) $\frac{15}{6}$; (2) $\frac{50}{15}$.

Solution.

By Cancellation

(1) $\frac{15}{6}=\frac{3\times5}{3\times2}=\frac{5}{2}$. (by removing the common factor 3) $=2^1/_2$. *Ans.*

(2) $\frac{50}{15}=\frac{10\times5}{5\times3}=\frac{10}{3}$ (cancelling the common factor 5) $=3^1/_3$. *Ans.*

By the Ordinary method.

(1) $\frac{15}{6}$: $6\times2=12$; remainder 3: $6\times\frac{1}{2}=3$. The quotient is $3\frac{1}{2}$. *Ans.*

(2) $\frac{50}{15}$: 15)50
3—remainder 5.

The quotient is $3\frac{5}{15}$ or $3\frac{1}{5}$ $\left(\text{since } \frac{5}{15}=\frac{5\times1}{5\times3}=\frac{1}{3}\right)$.

Exercise 131—(*Oral*).

Find the values of the following by *cancellation*:—

1. $\frac{25}{10}$. 2. $\frac{21}{6}$. 3. $\frac{36}{14}$. 4. $\frac{56}{21}$. 5. $\frac{63}{35}$. 6. $\frac{105}{49}$.

160. Another Example.—Find the value of $\frac{120}{75}$.

Solution by Cancellation.

$$\frac{\overset{8}{\cancel{\overset{24}{\cancel{120}}}}}{\underset{5}{\cancel{\underset{15}{\cancel{75}}}}}=\frac{8}{5}=1\frac{3}{5}.\quad Ans$$

NOTE.—We first remove the common factor 5 and then the common factor 3. We might also first remove 3 and then 5

Exercise 132.

(*a*) Find the values of the following expressions by cancelling the common factors of dividend and divisor *one after another* —

1. $\frac{189}{42}$. 2. $\frac{595}{105}$ 3. $\frac{315}{120}$. 4. $\frac{762}{96}$. 5 $\frac{1813}{784}$.

(*b*) Show that—

1. $\frac{120}{70}=\frac{12}{7}$. 2 $\frac{1500}{800}=\frac{15}{8}$. 3. $\frac{21000}{6000}=\frac{21}{6}=\frac{7}{2}$

(*c*) Find the values of—

1. $\frac{4200}{3200}$. 2. $\frac{2500}{1000}$. 3. $\frac{14400}{12000}$. 4 $\frac{21000}{2800}$

161. To find the value of $\frac{44 \times 15}{8 \times 9}$.

Solution.

$$\frac{\overset{11}{\cancel{44}} \times \overset{5}{\cancel{15}}}{\underset{2}{\cancel{8}} \times \underset{3}{\cancel{9}}} = \frac{11 \times 5}{2 \times 3} = \frac{55}{6} = 9\frac{1}{6}. \quad \textit{Ans.}$$

Explanation.—We divide 44 and 8 by their common factor 4 and set down the quotients 11 and 2 near them after crossing out 44 and 8, similarly we cross out 15 and 9 and set down 5 and 3 by their side.

Exercise 133

Find the values of the following by *cancellation*.—

1. $\frac{63 \times 15}{36 \times 20}$. 2. $\frac{49 \times 77}{24 \times 33}$. 3. $\frac{65 \times 21}{50 \times 12}$. 4. $\frac{121 \times 64}{33 \times 24}$.

5. $\frac{48 \times 40}{28 \times 15}$. 6. $\frac{65 \times 63}{55 \times 49}$. 7. $\frac{84 \times 48}{60 \times 36}$. 8. $\frac{169 \times 56}{143 \times 35}$.

9. $\frac{40 \times 70}{30 \times 50}$. 10. $\frac{110 \times 81}{33 \times 90}$. 11. $\frac{140 \times 25}{21 \times 35}$. 12. $\frac{144 \times 70}{84 \times 100}$.

CHAPTER XXXII.

USE OF LETTERS FOR NUMBERS.

162. In addition to the ten *numerals* 0, 1, 2, etc., the *letters* of the alphabet are sometimes used to denote numbers. But, while each of the *figures* 0, 1, 2, etc., has a *fixed* value, we may assign to *letters* used for numbers any value we please. For example—

(1) $a + b$ which denotes the *sum* of any two numbers will be equal to $5 + 8$ or 13, when $a = 5$ and $b = 8$, or to $7 + 3\frac{1}{2}$ or $10\frac{1}{2}$, when $a = 7$ and $b = 3\frac{1}{2}$: or to $8 + 2{\cdot}4$ or $10{\cdot}4$ if $a = 8$ and $b = 2{\cdot}4$, and so on.

(2) $x - y$ which denotes the *difference* of any two numbers will be equal to $8 - 2$ or 6, if $x = 8$ and $y = 2$, or to $3/4 - 1/2$ or $1/4$, if $x = 3/4$ and $y = 1/2$, or to $5{\cdot}4 - 3{\cdot}5$ or $1{\cdot}9$, if $x = 5{\cdot}4$ and $y = 3{\cdot}5$, and so on.

(3) $2a$ which denotes *twice* any number will be equal to 2×5 or 10, if $a = 5$, or to $2 \times 7\frac{1}{2}$ or 15, if $a = 7\frac{1}{2}$, or to 2×4.3 or 8.6, when $x = 4.3$, and so on.

(4) $x \times y$ or xy which denotes the *product* of two numbers will be equal to 3×4 or 12, if $x = 3$ and $y = 4$, or to $1/2 \times 30$ or 15 if $x = \frac{1}{2}$ and $y = 30$, or to 4×2.5 or ~~100~~, if $x = 4$ and $y = 2.5$ and so on 10.0

(5) $p \div q$ or $\frac{p}{q}$ which denotes the *quotient* of one number divided by any another number will be equal to $5 \div 2$ or $2\frac{1}{2}$, if $p = 5$ and $q = 2$, or to $3/4$ if $p = 3$ and $q = 4$, or to $4.8 \div 6$ or .8, if $x = 4.8$ and $y = 6$, and so on.

(6) a^2 which denotes the *square* of any number will be equal to 5^2 or 25, if $a = 5$, or to $(2\frac{1}{2})^2$ or $6\frac{1}{4}$, if $a = 2\frac{1}{2}$, and so on.

(7) x^3 which denotes the *cube* of any number will be equal to 4^3 or 64, if $x = 4$, or to $(1/2)^3$ or $1/8$, if $x = 1/2$, and so on.

(8) $(m + n)^2$ which denotes the *square of the sum of any two numbers* will be equal to $(4 + 3)^2$ or 49, if $m = 4$ and $n = 3$, or to $(10\frac{1}{4} + 3/4)^2$ or 121, if $m = 10\frac{1}{4}$ and $n = 3/4$; and so on.

(9) $(c - d)^2$ which denotes the *square of the difference of any two numbers* will be equal to $(8 - 2)^2$ or 36, if $c = 8$ and $d = 2$, or to $(1\frac{1}{2} - 3/4)^2$ or $9/16$, if $c = 1\frac{1}{2}$ and $d = 3/4$, and so on.

(10) $(x + y)(a - b)$ which denotes the *product* of the *sum* of any two numbers multiplied by the *difference* of any two numbers will be equal to $(3 + 5)(6 - 4)$ or 16, if $x = 3$, $y = 5$, $a = 6$, $b = 4$, or to $(1/2 + 4)(4 - 1\frac{1}{2})$ or $4\frac{1}{2} \times 2\frac{1}{2}$, if $x = 1/2$, $y = 4$, $a = 4$ and $b = 1\frac{1}{2}$, and so on.

Exercise 134 —(A) & (B) (*Oral*).

(A) What is denoted by each of the following expressions?

(*a*) $x + y$ (*b*) $m - n$ (*c*) $a + b + c$. (*d*) $2x$.

(*e*) mn. (*f*) $2l + 3m$. (*g*) $2x^2$. (*h*) x/y.

(*i*) $(x + y)^3$. (*j*) $(l - m)^2$. (*k*) $(a - b)(c + d)$.

(*l*) $a(b + c)$. (*m*) $2(x - y)^2$. (*n*) $4xyz$.

(B) Find the value of—

1. $a+b$, if $a=5$, $b=7$: $a=7\frac{1}{2}$, $b=3\frac{3}{4}$ · $a=4$, $b=1\cdot5$.

2 $x-y$, if $x=10$ $y=3$, $x=7$, $y=3\frac{1}{2}$, $x=2\cdot8$ $y=1\cdot9$.

3 mn, if $m=4$, $n=3$, $m=8$ $n=1\frac{1}{2}$, $m=4\cdot5$, $n=8$.

4. $2l+m$, if $l=8$, $m=2$, $l=4\frac{1}{2}$, $m=3$, $l=100$. $m=2\cdot5$.

5 x^2, if $x=5$, $x=7$, $x=12$, $x=5\frac{1}{2}$; $x=1/2$.

6 c^3, if $c=5$, $c=4$, $c=12$, $c=1/2$.

7. $(a+b)^2$, if $a=3$ $b=4$, $a=1\frac{1}{2}$, $b=3\frac{1}{2}$, $a=3$, $b=3\frac{1}{4}$.

8. $(m-n)^2$, if $m=15$, $n=5$, $m=10$, $n=2\frac{1}{2}$; $m=3$, $n=2\frac{3}{8}$.

9. $2ab$ if $a=3$ $b=4$, $a=1/2$, $b=5$, $a=3\cdot5$, $b=7$

10. xyz, if $x=3$ $y=4$, $z=2$ $x=1$, $y=3$, $z=7$

11. $(a+b)(c+d)$ if $a=1$ $b=0$, $c=7$, $d=8$; $a=2$ $b=15$. $c=12$, $d=5$, $a=1/2$, $b=1$, $c=0$, $d=5$, $a=2\cdot5$, $b=4\cdot5$, $c=8$, $d=20$.

12 $4(a+b)^2$, if $a=4$ $b=3$, $a=3/4$, $b=1\frac{1}{4}$, $a=4\cdot5$, $b=6\cdot5$.

13. $x-y$, if $x=2$, $y=1\cdot2$, $x=4$, $y=3\cdot4$. $x=4$, $y=7$, $x=2\cdot5$, $y=5\cdot2$.

14. n, if $n^2=144$, 49, 81, 400, $1/4$, or $30\frac{1}{4}$.

15. a, if $a^3=125$, 216, 27 : 1000, or 1728.

(C) Find the value of the following expressions, if $l=5\frac{1}{2}$, $m=3\frac{1}{4}$, $n=2\frac{3}{4}$, $a=4\cdot3$, $b=3\cdot4$, $c=5\cdot2$ ·—

1. $l+m+n$ 2. $a+b+c$. 3 $2l-3m+n$,

4. $4a-3b$. 5. $5b+6c$. 6. $8n-2m-l$.

7. $2(l+m)$. 8. $20(m-n)$. 9 $12(a+b)$.

10 $\dfrac{a+b+c}{m-1/4}$ 11. $\dfrac{m+n}{a+b+3}$. 12. $\dfrac{l-2n}{5b}$.

13. $(m+n)(2l-m-n)$ 14. $(a+b-c)(l-1\frac{1}{2})(a+2)$.

163. To find the value of $\frac{xy}{z}$, when $x=44$, $y=30$, and $z = 18$.

Solution

$$\frac{xy}{z} = \frac{44 \times \overset{5}{\cancel{30}}}{\underset{3}{\cancel{18}}} = \frac{44 \times 5}{3} = \frac{220}{3} = 73\tfrac{1}{3} \quad Ans$$

Exercise 135.

Find the values of—

1. $\frac{xy}{12z}$ when $x = 38$, $y = 16$, and $z = 19$.
2. $\frac{ab}{6c}$ when $a = 144$, $b = 11$, and $c = 21$.
3. $\frac{lm}{xy}$ when $l = 154$, $m = 23$, $x = 11$, and $y = 16$
4. $\frac{pq}{r}$ when $p = 26$, $q = 91$, and $r = 169$.
5. $\frac{51a}{bc}$ when $a = 72$, $b = 17$, and $c = 48$.

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164. If x be taken to denote any number, then it will be readily seen that—

(1) $x + 3$ will denote the number *increased* by 3,

(2) $x - 3$ will denote the number diminished by 3,

(3) $\frac{x}{3}$ will denote the number divided by 3,

(4) $8x - 5x$ will denote 8 times the number diminished by 5 times the number,

(5) $1/2x + 1/3x$ will denote the sum of *half* the number and *one-third* of the number,

(6) $2x + 7$ will denote *twice* the number increased by 7

(7) $3x - 4$ will denote *three times* the number diminished by 4,

(8) $2(x + 3)$ will denote the number increased by 3 and the result multiplied by 2,

(9) $\frac{x - 8}{8}$ will denote the number diminished by 8 and the result divided by 8,

(10) $\frac{x}{4} - 10$ will denote the number divided by 4 and the quotient diminished by 10.

Exercise 136.

If x stands for a certain number, what expression involving x will represent each of the following?

(1) The number increased by 5, (2) twice the number, (3) 5 times the number *minus* $3^1/_2$ times the number, (4) four times the number increased by 12 (5) the number increased by 2 and the sum multiplied by 3. (6) the number diminished by 8 and the result divided by 5, (7) the *difference* between the *square* of the number and *half* the number, (8) the number divided by 5 and the quotient increased by 4, (9) half the number diminished by 4.5, (10) 8 times the square of the number increased by 14·8, (11) the cube of the number divided by 7, (12) 4 times the cube of the number diminished by 6.

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165. We shall now illustrate the method of solving problems by using the letter x to denote numbers.

Example 1.—If I add 15 to a certain number, I obtain 38. What is that number?

Solution.

Let x denote the number

Then by the question, $x + 15 = 38$.

That is, if x be *increased* by 15, it becomes 38,

$$\therefore\ x = 38 - 15 = 23$$

Hence the number required is 23. *Ans*

[Verification: $23 + 15 = 38$]

Example 2.—By subtracting 8 from a certain number I get 28. Find that number.

Solution.

Let x denote the number.

Then, by the question, $x - 8 = 28$.

That is, if x be *diminished* by 8 it becomes 28.

$$\therefore\ x = 28 + 8 = 36.$$

Thus the number required is 36. *Ans.*

[Verification : $36 - 8 = 28$.]

Example 3.—A certain number, when multiplied by 4, produces 38. What is the number?

Solution

Let x be the number required.

Then, by the question, $x \times 4$ or $4x = 38$.

$$\therefore x = 38/4 = 9\tfrac{1}{2}.$$

That is, the number required is $9\tfrac{1}{2}$. *Ans.*

[Verification : $9\tfrac{1}{2} \times 4 = 36 + 2 = 38$.]

Example 4.—I take a number, divide it by 5, and get 32 for the quotient. What is the number taken?

Solution.

Let x denote the number taken.

Then, by the question $x \div 5 = 32$.

$$\therefore x = 32 \times 5 = 160.$$

That is, the number taken $= 160$ *Ans.*

[Verification : $160 \div 5 = 32$.]

Exercise 137.

(*The answers are to be verified.*)

1 A certain number *increased* by 7 becomes 39 What is the number?

2. What number when *decreased* by 7 will become 28?

3. A certain number when multiplied by 12 produces 96. Find the number.

4. I take a number and by dividing it by 8 get $4\tfrac{1}{2}$. What is the number taken?

5. If 4·8 be added to a certain number, it becomes 30. Find the number.

6. A merchant bought a certain number of bags of rice He sold away 325 bags and then had 728 bags remaining. How many bags did he buy?

7. A person travelled a certain distance by rail and 40 5 miles by boat. If the total distance travelled be 100 miles, find the distance travelled by rail

8 If 125 times a certain number is a lakh, find the number.

9 If one-twelfth of a certain number is 15 4, what is the number?

10. If one-sixteenth of a certain number is 7,716, find the number.

Example 5.—I take a certain number, multiply it by 7, add 24 to the product, and get 80. What is the number taken?

Solution.

Let x denote the required number.
Then, by the question, $7x + 24 = 80$.,
$\therefore 7x = 80 - 24 = 56$
$\therefore x = 56 \div 7 = 8.$

Hence the number taken is 8. *Ans*

[Verification, $8 \times 7 + 24 = 56 + 24 = 80$.]

Example 6.—I take a certain number, subtract 7 from it, multiply the remainder by 4, and get 52. What is the number taken?

Solution.

Let x denote the number taken.
Then $(x - 7)$ multiplied by $4 = 52$
That is $4(x - 7) = 52$.
$\therefore x - 7 = 52 \div 4 = 13$
$\therefore x = 13 + 7 = 20.$

Hence the number taken is 20 *Ans.*

[Verification; $20 - 7 = 13$, $13 \times 4 = 52$.]

Exercise 138

(The answers are to be verified.)

1. I take a number, multiply it by 8, add 20 to the product and get 108. Find the number taken.

2. I take a number, divide it by 12, subtract 3 from the quotient and get $2\frac{1}{2}$. What is the number taken?

3. If I take a certain number, add 12 to it, and multiply the sum by 8, I get 216. What is number taken?

4. I take a number, divide it by 6, add 12 to the quotient, and get 21. What is the number taken?

5. I take a number subtract 10 from it, divide the remainder by 6 and get 25. What is the number taken?

6. I take *twice* a number, and add to it *thrice* the same number and get 70. What is the number?

7 To 5 times a certain number I add 6, divide the sum by 3 and get 17. What is the number ?

8 From 12 times a certain number I subtract 8,000, multiply the remainder by 5, and get half a lakh. What is that number ?

9, I multiply a certain number by 5, divide the product by 6, and add 7 to the quotient. If the final result is 22, find the number.

Example 7 —The sum of two numbers is 48. If one of them is 13, find the other.

Solution.

Let x denote the number.
Then $x + 13 = 48$.
$\therefore \quad x = 48 - 13 = 35$ *Ans.*

[Verification : $x + 13 = 35 + 13 = 48$.]

Example 8.—The difference of two numbers is 35. If the smaller number be 25, find the larger number.

Solution.

Let x be the larger number.
Then $x - 25 = 35$.
$\therefore \quad x = 35 + 25 = 60$ *Ans.*

[Verification $60 - 25 = 35$.]

Exercise 139.

[*The answers are to be verified.*]

1. The sum of two numbers is 750 , if one of them is 328, find the other

2. The difference of two numbers is 48 If the smaller number be 24, find the larger.

3 I buy a cow and a buffalo for Rs 120, If the cost of the buffalo is Rs 48, find the cost of the cow.

4. The cost of a house and a garden is Rs. 4,500 , if the cost of the garden is Rs 750, find the cost of the house.

5 The difference of two numbers is 75 4. If the smaller number is 38 8, find the larger number.

6. The sum of *three* numbers is one-fourth of a lakh ; if the 2nd number is 10,240 and the 3rd is 728½, find the *first* number.

Example 9.—The sum of two numbers is 75. If the 2nd number is greater than the first by 3, find the two numbers.

Solution.

Let x denote the first number

Then $x + 3$ will denote the second number.

Then $x + (x + 3) = 75$.

That is $x + x + 3$ or $2x + 3 = 75$.

$\therefore \quad 2x = 75 - 3 = 72$.

$\therefore \quad x = \frac{72}{2} = 36$.

And $\quad x + 3 = 36 + 3 = 39$.

That is, the required numbers are 36 and 39. *Ans.*

[Verification : $36 + 39 = 75$, and $39 = 36 + 3$]

Exercise 140.

(*The answers are to be verified.*)

1. The sum of two numbers is 100; if the second number is greater than the first by 18, find the two numbers.

2. The sum of two numbers is 44, if the 2nd number is greater than the first by 3, find the two numbers.

3 The sum of two numbers is 15, if the smaller number is 3 4 less than the larger, find the two numbers.

4. A cow and a horse together cost Rs. 500, and the cost of the horse is Rs 320 more than that of the cow. Find the cost of each.

5 A garden costs Rs. 720 less than a house. If the cost of both together be Rs. 4,050, find the cost of each.

Example 10,—If I add 14 to the square of a certain number, I get 63. Find the number.

Solution.

Let x denote the number required.

Then, by the question $x^2 + 14 = 63$

$x^2 = 63 - 14 = 49$.

$\therefore \quad x = 7$.

$\therefore$ the required number is 7. *Ans.*

[Verification : $7^2 + 14 = 49 + 14 = 63$.]

Exercise 141.

(Answers to be verified.)

1. I multiply a certain number by itself add 30 to the product and obtain 111. Find the number.

2 By subtracting 18 from the square of a number I get 151. Find the number.

3. I add 16 to the cube of a certain number and get 141. What is the number?

4 I multiply the square of a number by 3 and subtract 8 from the product. If I thus get 139, find the number.

5. I divide' the square of a certain number by 4 and get 36 Find the number.

6 I divide the cube of a certain number by 8 and subtract 15 from the quotient. If the remainder is 1?, find the number.

7. If the square of a certain number increased by $7^3/_4$ is equal to 50, find the number

8. If from 3 times the square of a certain number I subtract $20^3/_4$ I get 70, what is the number?

Example 11 —The *sum* of two numbers is 240 and their *difference* is 120. Find the two numbers

Solution

Let x denote the *greater* of the two numbers and y the *smaller*,
Then $x + y = 240$ and $x - y = 120$
$\therefore \quad x + y + x - y = 240 + 120 = 360$, or $2x = 360$.
$\therefore x = 180$ and $y = 240 - 180 = 60$.
Hence the required numbers are 180 and 60.

[Verification . $180 + 60 = \mathbf{240}$; $180 - 60 = \mathbf{120}$]

Exercise 142. *(Answers to be verified)*

1. The *sum* of two numbers is 225 and their *difference* is 75. What are the two numbers?

2 Work out the above sum substituting the following numbers for 225 and 75:—
(*a*) 170 and 70. (*b*) 470 and 220. (*c*) 352 and 49 (*d*) 202·8 and 42 (*e*) 1111 and 197 6.

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* Another method of solution is to denote the *larger* by *x* and and the *smaller* no. by $x - 120$, or the *smaller* by x and the *larger* by $x + 120$.

CHAPTER XXXIII.

UNITARY METHOD.

166. The examples given below illustrate what is called the *unitary method* of solving problems, *i.e.*, a method in which we find what is wanted from what is given *by passing through a unit common to both.* If, for example, we are required to find the cost of 9 yards of cloth when 7 yards cost Rs 21, we first find the cost of *one* yard of cloth (the *common unit*) and then find the cost of 9 yards. Similarly, if the cost of 1 cwt. 8 lb. of tin is given, as in Example 3 below, and we are required to find the cost of 27 lb., we take the cost of 120 lb. (= 1 cwt. 8 lb) of tin as given, find therefrom the value of 1 lb. of tin (which may be taken as the *common unit*) and then proceed to find the cost of 27 lb.

Model 1.—If 16 cows cost Rs. 500, what will 22 cows cost ?

Solution.

Cost of 16 cows = Rs. 500.

$\therefore$ cost of 1 cow = Rs $\frac{500}{16}$.

$$\therefore \text{ cost of 22 cows} = \text{Rs } \frac{500}{16} \times 22 = \text{Rs. } \frac{\overset{125}{\cancel{500}} \times \overset{11}{\cancel{22}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{16}}}$$

$$= \text{Rs } \frac{125 \times 11}{2} = \text{Rs. } \frac{1375}{2} = \text{Rs, } 687\tfrac{1}{2}. \quad \textit{Ans.}$$

Note.—The answer may be given as Rs 688 correct to a Re.

Exercise 143

(*Answers to be given as whole numbers*)

1. If 24 sheep cost Rs. 202, find the cost of 15 sheep.
2. If 19 men earn Rs. 750, how much will 21 men earn ?
3. If a train runs 1000 miles in 36 hours, how far will it run in 21 hours ?

4. Find *correct to a rupee* the salary of a clerk for 15 days in the month of August, supposing his monthly salary is Rs. 70.

5. If x cows cost y rupees what will be the cost of z cows ? Find the value of your answer when $x = 15, y = 755, z = 21$.

Model 2.—If 8 men can build a house in 36 days, in how many days can 12 men build it ?

Solution.

8 men can build the house in 36 days.

∴ I man can build it in 8 times as many days, or 36×8 days.

∴ 12 men can build it in $\frac{\overset{3}{\cancel{36}} \times 8}{\cancel{12}}$ days or 24 days. *Ans.*

Exercise 144.

1. If 12 men can reap a field in 28 hours, in how many hours can 21 men reap it ?

2. If 3 horses can plough a field in 9 days, in how many days can 8 horses plough the same field ?

3. Fifteen men can build a wall in 18 days, in how many days can 6 men build it ?

4. If x men can dig a canal in y days, in how many days can z men dig the canal ? Find the value of your answer if $x = 120$, $y = 75$, $z = 90$.

Model 3.—If 1 cwt. 8 lb. of tin costs Rs. 168, what will be the cost of 27 lb. ?

Solution.

Cost of 1 cwt. lb. or 120 lb. of tin = Rs. 168.

∴ cost of 1 lb. of tin = Rs. $\frac{\overset{\overset{7}{\cancel{84}}}{\cancel{168}}}{\underset{\underset{5}{\cancel{30}}}{\cancel{120}}}$ = Rs. $\frac{7}{5}$.

∴ cost of 27 lb. of tin = Rs $\frac{7}{5} \times 27$ = Rs. $\frac{189}{5}$

= Rs. $37\frac{4}{5}$ or Rs. 38 correct to a rupee. *Ans.*

Exercise 145

1. If 1 cwt. of copper costs Rs 168, what will be the cost of 25 lb ?

2. If 4 vis, 2 srs of sugar sosts Rs 5, what will 1 md. 1 vis., 3 srs cost ?

3. If in 1 hr. 10 min a trsin runs 8¼ miles, how many miles will it run in 2 hrs. 5 minutes ?

4 If we can buy 320 mangoes for Rs 3 8 as, how many mangoes can we buy for Rs 1-4 as. ?

CHAPTER XXXIV.

USE OF THE COMPASSES.

1. Describing Circles.

167. Circle.—A surface like the bottom of the solid called a *cone* or the ends of the solid called a *cylinder*, is called a *circle*.

Exercise (*a*). Name some common objects which are *circular* in shape (*b*) Name some Indian coins that are round and some that are *not* round.

168. Centre of a Circle.—In the three circles given below the points M, N, and O which are exactly in the middle of the circles are called their *centres*.

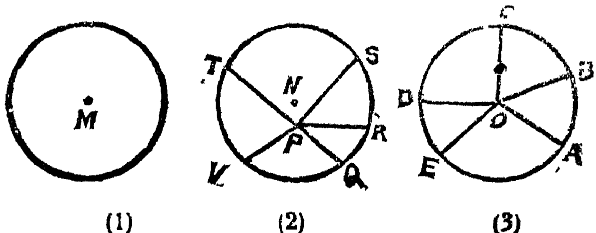

Fig. 18.

NOTE 1.—The lines OA, OB, OC, OD and OE, which are drawn from O the *centre* of the circle marked (3) to its *boundary line* are *equal to one another*, whereas the lines drawn in circle (2) from the point P which is *not* the centre to the *boundary line* are *not* equal to one another.

NOTE 2.—A circle has only *one* centre, which may be defined as a point within it such that all the straight lines drawn from it to the circumference (boundary line) are equal to one another.

169. Circumference, Radius, Diameter.—The boundary line of a circle is called its *circumference*; any line drawn from the *centre* of a circle to *circumference* is called its *radius*, and any line drawn *through* the centre of a circle and terminated by the circumference at both ends is called a *diameter* of the circle.

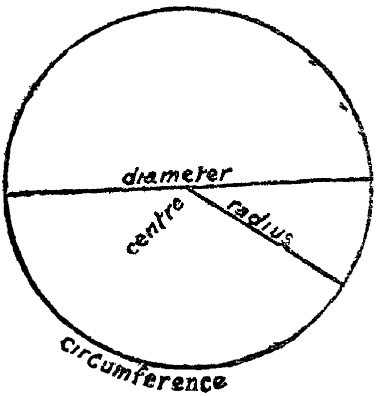

Fig 19.

NOTE 1.—*A* diameter of a circle is equal to *twice* its radius.

NOTE 2.—Learn that the plural of *radius* is *radii*.

NOTE 3.—Any number of *radii* can be drawn in a circle, and also any number of *diameters*..

170. Describing Circles with Compasses.

To describe a circle with the compasses, fix the pencil (sharpened to a fine point) in one leg, so that when the compasses are closed, the pencil point *just* projects beyond the steel point of the other leg. Next open the compasses so that the distance between the pin point and the pencil point is equal to the radius of the circle to be described, Now take hold of the head of the compasses, and fixing the pin point on the centre of the circle so that that leg may stand upright, rotate the other leg round the pin point with the pencil point in contact with the paper.

Exercise 146—(*Practical*).

[Questions marked with an asterisk are also to be done on the black-board with B. B compasses.]

1*. Practise describing with the help of your compasses circles of different sizes.

2*. Practise describing *free-hand* circles of various sizes.

* **To the Teacher.**—(1) It must be clearly pointed out to the pupil that the word *circle* denotes the *space* enclosed by its circumference and *not* the *circumference* merely which is only its boundary line.

(2) The pupil may well be made to practise describing circles *free-hand* on paper and on the black-board, as this affords valuable training both to the *eye* and the *hand*.

3. How many *radii* can a circle have ?

4. Describe a circle, draw a few radii, and show by means of your dividers that all these radii are *equal*.

5. Describe circles of the following radii .—2 inches, $1^1/_2$ inches, 10 cm., 5·6 cm.

6 Describe circles of the following diameters :— 3 inches, 5 inches, 10 cm , 13 cm.

7*. Draw some circles *free-hand* and mark their centres by judging by the eye. Draw 4 or 5 lines from this point to the circumference and measure them to see if this point is far removed from the exact centre.

8. Cut out from paper and thin card-board two or three paper circles and card-board circles.

9*. With any point O as centre draw 2 or 3 circles of different radii. Learn that circles having a common centre are called *concentric* circles. [See Fig 20 below]

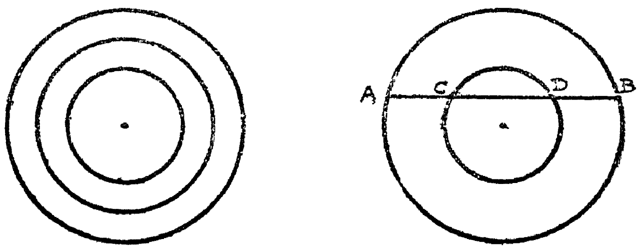

Fig. 20.

10.* In the right-hand figure above, prove by using the *dividers* that AC = BD.

11. Describe three concentric circles of the following radii :— (*a*) 3 in., 2 in., 1 in. , (*b*) 3 cm., 5 cm., 6 cm. , (*c*) 4 5 cm , 5 4 cm., 7 cm.

12*. On the circumference of a circle take any two points A and B. The portion of the circumference lying between A and B is called an arc of the circle If the points A and B are the ends of a diameter of the circle, each of the arcs between them is one-half of the circumference and is therefore called semi-circumference. [See Fig 21 on the next page.]

13*. Take any two points A and B on the circumference of a circle and join AB Learn that the line AB is called a chord of the circle and that the two parts of the circle into which a chord divides it are called segments of the circle. Note that, if the *chord* AB is a diameter of the circle, then the two segments are semi circles. [See Fig. 21 on the next page.]

14*. Take an *arc* AB on the circumference of a circle. Join A and B with the centre of the circle. The figure enclosed by the two radii and the arc between them is called a **sector** of the circle. It will be noticed that any two radii of a circle (not forming a diameter) divide it into two *sectors*, one of which is larger than the other. It will also be seen that the diameter (= two radii forming one straight line) divides the circle into two *equal* sectors. In this case each of the sectors becomes a semi-circle and the arc of each a semi-circumference. [See Fig. 21 below.]

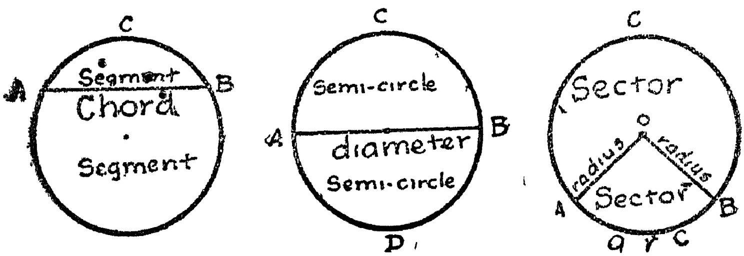

Fig. 21.

15. Learn to draw *free hand* an *Ellipse*. This will be useful later on in drawing sketches of *cones* and *cylinders*.

---

171. Quadrant.—If a circle be divided into four equal sectors by two diameters drawn at right angles to each other, each of these 4 equal parts is called a *quadrant* of the circle.

Exercise 147.

1. What are the three boundaries of a *quadrant* of a circle? What fraction of the *entire* circumference of a circle is the arc of a quadrant of it?

2. Cut out of card-board a quadrant of a circle.

---

2 Pattern Drawing.

Exercise 148—(*Practical*).

1. Copy the following designs on plain paper after first drawing rectangles or squares of suitable size and dividing them into equal squares:—

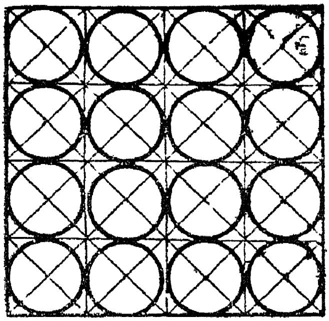

Fig. 22.

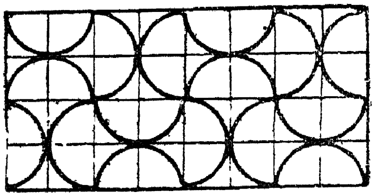

Fig 23.

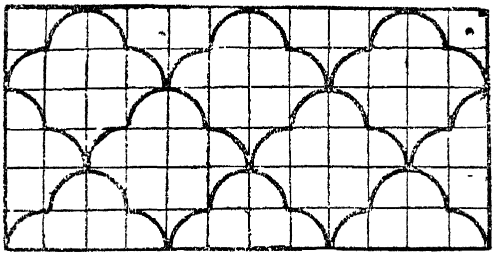

Fig. 24

2 Describe a circle of 2 inch radius, Draw two diameters *at right angles* to each other , and with each of these four radii as diameter describe a circle.

3. Describe a circle taking 5 cm for its radius Draw two diameters AB, CD *at right angles* to each other Join AC, CB, BD, DA, and on each of these four lines describe a *semi-circle* inside the circle ADBC

4 Draw a line AD 9 cm. long and describe a circle with AD as diameter. Divide AD into 3 *equal* parts at the points B, C. On AB and AC as diameters describe semi-circles *downwards*, and on BD and CD describe semi-circles *upwards*

5. With centre O and radius 4 cm , describe a circle , and draw in it two diameters AC, BD cutting each other *at right angles*. With centre O and radius 2 cm., describe a circle , again with centres A, B, C, D and radii 2 cm., describe arcs *inwards*, so that their ends may lie on the circumference of the large circle.

CHAPTER XXXV

INDEX NOTATION.

172. Powers.—The student has already learnt (Art. 73) that the product of a number multiplied by itself any number of times is called a *power* of that number.

Thus 5×5 is the second power or *square* of 5, and is denoted for the sake of brevity by 5^2, which is read '5 squared' or '5 raised to the power of 2', $5 \times 5 \times 5$ is the third power or *cube* of 5, and is denoted by 5^3, which is read '5 cubed' or '5 raised to the power of 3', $5 \times 5 \times 5 \times 5$ is the 4th power of 5, and is denoted by 5^4, which is read '5 raised to the power of 4', and so on for 5^5, 5^6, x^7, x^6, etc.

173. Index.—The small figure placed above and to the right of a number or letter is called the *index* (plural *indices*) because it *indicates* the number of equal factors that are to be multiplied together to form a *power*.

Thus in 5^2, 2 is the index, in 5^3, 3 is the index: in 10^5, 5 is the index, in a^6, 6 is the index.

NOTE.—Any number 5 is called the *first power* of 5 and its *index* is 1. Similarly a is the *1st power* of a, its *index* being 1.

174. In the following examples the powers of 10 are involved.

Example 1.—Find the value of (*a*) 10^3; (*b*) 10^5; (*c*) 3×10^4.

Solution.

(*a*) $10^3 = 10 \times 10 \times 10 = 100$. *Ans.*

(*b*) $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100000$. *Ans.*

(*c*) $3 \times 10^4 = 3 \times 10 \times 10 \times 10 \times 10 = 3 \times 10000 = 3000$. *Ans.*

Example 2.—What powers of 10 are (*a*) 100 (*b*) 100000?

Solutions.

(*a*) $100 = 10 \times 10 = 10^2$. *Ans.*

(*b*) $1000000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$. *Ans.*

NOTE.—It will be seen from the above examples that any number consisting of 1 followed by ciphers can be expressed as a power of 10, of which the index is the same as the number of ciphers in the given number.

Example 3.—Show that $60000 = 6 \times 10^4$.

Solution.—$60000 = 6 \times 10000 = 6 \times 10^4$. *Ans.*

Exercise 149.

(*a*) Find the value of 10^3, 10^4, 5^4, 2^5, 2×10^2, 4×10^3.

(*b*) What powers of 10 are 150, 1000, a lakh, a million, a crore?

(*c*) What power of 3 is 27, of 5 is 625, of 4 is 64?

(*d*) Express each of the numbers 4000, 60000, 1100000 as the product of a power of 10 and another number.

(*e*) Show that $32647 = 3\times10^4+2\times10^3+6\times10^2+4\times10+7$, and that $205143 = 2\times10^5+5\times10^3+10^2+4\times10+3$.

(*f*) If $x = 10$, find the value of—

1. $4x^5+3x^4+2x^2+x+1$. 2. x^4+7x^3+6x+5.

3 $2x^4+x^2+8x+6$ 4. $9x^4+9x^2+9x$.

---

175. Multiplication and Division of powers of the same number :—

Examples-

(1) $7^2\times7^3=\underline{7\times7}\times\underline{7\times7\times7}=7^5$ [$5=2+3$]

(2) $10\times10^3=\underline{10}\times\underline{10\times10\times10}=10^4$. [$4=1+3$.]

(3) $x^3\times x=\underline{x\times x\times x}\times\underline{x}=x^4$. [$4=3+1$.]

From the above examples we see that *to multiply together two powers of the same number, we have to add the indices.*

(4) $\dfrac{10^5}{10^2}=\dfrac{10\times10\times10\times10\times10}{10\times10}=10\times10\times10=10^3$.

[$3=5-2$]

(5) $\dfrac{x^4}{x}=\dfrac{x\times x\times x\times x}{x}=x\times x\times x=x^3$. [$3=4-1$.]

From examples (4) and (5) we see that *to divide one power of a number by another power, we have to subtract the index of the latter from that of the former.*

Exercise 150.—(*Oral.*)

Perform the following multiplications and divisions.—

(*a*) 1. $2^3\times2^4$. 2. $10^3\times10^2$. 3. $x^4\times x$. 4. $2\times10\times10^5$.

(*b*) 1. $5^4\div5^2$. 2. $10^4\div10$. 3. $x^5\div x^2$. 4. $10^5\div10^4$.

CHAPTER XXXVI.

DECIMAL FRACTIONS.

1. Introductory.

176. Names of Fractions,—We have in *whole numbers* the denominations *ten, hundred, thousand*, etc., of which *ten* is ten times the unit and the rest are each 10 times the preceding one. Similarly we have *in fractions* the denominations *tenth, hundredth, thousandth, ten-thousandth, hundred-thousandth, millionth*, etc., of which *tenth* is one-tenth of the unit and the rest are each one-tenth of the preceding one.

177. Decimal Fractions.—A number consisting of one or more of the fractions *tenths, hundredths*, etc., is called a *decimal fraction* or simply a *decimal*.

For example a number consisting of 4 *tenths*, 3 hundredths and 4 *ten-thousandths* is a decimal.

NOTE.—A number consisting of a whole number and a decimal is also called a decimal.

178 Local value:—In the *whole number* 5432, the figures 2, 3, 4, 5 denote in order 2 *units*, 3 *tens*, 4 *hundreds*, and 5 *thousands*; similarly in the *decimal* 2·345 the figures 2, 3, 4, 5 denote in order 2 *units*, 3 *tenths*, 4 *hundredths*, and 5 *thousandths*

NOTE.—In 2·345 the dot which separates the *integral* from the *fractional* parts of the number, is called *decimal point* or simply *point*.

Exercise 151—(*Oral*).

What is the *local value* of each significant digit in the following decimals?

1. 25·34.	2. 16·048.	3. 3 0045.
4. ·8009.	5. 666·0666.	6. 4·0005691.

---

179. Graphical Representation of Decimals—If the subjoined figure ABCD, which is a square cut up into 100 small equal squares, represent the *unit*, then every one of

the *ten* vertical and of the *ten* horizontal rows in it consisting of 10 small squares each, will represent *one-tenth* (·1) of the unit and every one of the hundred small squares like HBEG will denote *one-hundredth* (·01) of the unit. Again, if every one of these hundred squares be sub-divided like AKLP into 10 equal strips, each of these strips will represent *one-thousandth* (·001) of the unit, and so on.

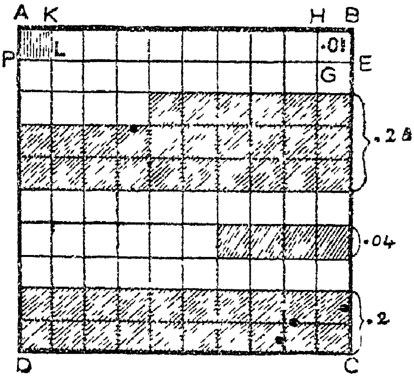

Fig. 25.

Thus the shaded portions of the figure ABCD taken in order from the bottom will respectively represent ·2, ·04, and ·26 of the unit.

Exercise 152 —(*Practical*).

(*a*) Represent *graphically* on squared paper the following decimals :—

1 Four-tenths. 2. Three-tenths and seven-hundredths
3. 7-tenths. 4 2-tenths and 7-hundredths
5 ·03 6. ·52. 7 ·19 8 ·66.

(*b*) If the square AKLP in Fig. 25 above be divided into 100 equal parts, what decimal of the whole square ABCD will each of these hundred parts represent ? And how many of these parts will represent ·0004 of the unit ?

(*c*)* Divide a sheet of note paper into 10 pieces, which may be taken as *equal* to one another. What decimal fraction of the whole sheet is each piece ? Give 9 of these pieces to a pupil A. and divide the 10th piece into 10 *equal* parts What decimal of the whole sheet is each of these 10 pieces ? Give 9 of those pieces to B. Proceed in this way till C gets 9 pieces which are each *one-thousandth* of the whole sheet, and the remaining piece is given to D

Now answer the following questions ;—(1) What decimal of the entire sheet will 4 pieces taken from A 3 from B and 1 from C make ? (2) How many pieces must be taken from A B and C to make ·369 ·204, ·042, ·001 of the entire sheet ?

---

*To the Teacher.—This exercise is intended to give the pupil a clear idea of decimal fractions and should *not* therefore be passed over.

2 Numeration and Notation of Decimals.

Examples.

(1) ·451 is read as four-tenths, five hundredths and one thousandth, or as decimal (or point) four, five, one.

(2) 2·0503 is read as two hundredths and five and 3 ten-thousandths, or as two, point, nought, five, nought, three.*

(3) Fourteen, and five tenths, seven thousandths, and eight millionths is written as 14·507008.

Caution —·576 should never be read "decimal five hundred and seventy-six"

Exercise 153.

(*a*) Express in figures—

1. Seven-tenths 2. Four and four-tenths.
3. Thirty, and seven-tenths and one-hundredth.
4. One, and one-tenth, two-hundredths and three-thousandths
5. Seventy-five, and four-thousandths.
6. Eight-tenths and eight-hundredths.
7. Six-hundredths. 8 Nine-thousandths.
9. One-millionth. 10. Two ten-thousandths.
11. One-tenth, one-thousandth and one-millionth.
12. One thousandth and one-millionth.
13. *Eight-hundred and forty-five* thousandths.
14 Five-tenths and fifty-five thousandths
15 Seven-tenths and seven-thousandths.
16. *Two hundred and thirty-eight* tenths.
17. Fifty, decimal, nought, nought, eight, one.
18. Four, point, three noughts, five, seven.

(*b*) Express in words—

1. ·7. 2. ·9. 3 ·1. 4. ·02.
5 ·13. ·45. 7. 2·3 8. 14·0405.
9 ·109005. 10. 120·00007.

---

* To the Teacher —It is of the utmost importance that beginners are taught to read decimals using the words *tenths, hundredths, &c* Unless this is done in the earlier stages of teaching, the pupil is apt to read a number ·456 as '*decimal four, five, six,*' without the slightest idea of the real meaning of the figures.

3. Comparison of Decimals.

180. In a row of figures denoting a *whole number*, the value of any *significant* figure is greater than that of all the figures that follow it.

For example, in 219805 the value of 1 (which denotes 10000) is greater than 9305.

Similarly, in a row of figures denoting a *decimal*, the value of any significant figure is greater than the value of all the figures that follow it.

For example, in 51 92034, 1 is greater than ·92034, the value of 2 (which denotes ·02) is greater than the sum of the values of 3 and 4, *i e.*, greater than 00034.

181. The following examples in the *comparison of decimals* can be easily understood.

(1) ·2 is greater than ·156, since 2 in the *tenths* place is greater than 1 in the same place

(2) ·234 is greater than 2194, since 3 in the *hundredths* place is greater than 1 in the same place,

(3) 1·0 is greater than ·999, since 1 0 is greater than 9

(4) ·1 is greater than ·0598, since ·1 is greater than ·0.

(5) of the decimals ·348 ·4, and, ·35 the greatest is ·4, the next is ·35, and the smallest is ·348.

Exercise 154—(*Oral*).

(*a*) In each of the following parts of decimals, find which is the greater:—

1. ·35, ·29 2 ·82 ·9, 3. ·643, ·634 4, ·1232, ·12319.
5. ·076, ·12, 6. 2, ·145 7. ·106, ·12. 8. ·046. ·0098.

(*b*) Arrange the decimals in each of the following groups (i) in *descending* order of magnitude, (ii) in *ascending* order of magnitude.

1. ·4, ·35. ·41. 2. ·05, ·12, ·043 3. ·405, ·50, ·51.
4. 1 43, 1·1, 1 089. 5. ·0011, ·01, ·003, 6. ·0345, ·14336.

(*c*) Write down the least decimal less than 1 containing 4 places of decimals and the greatest decimal less than 1 containing 5 places of decimals

---

* **To the Teacher.**—The beginner must be cautioned against supposing that the magnitude of a decimal is determined by the number of figures it contains though it is so in the case of *integers*.

4. Affixing and Prefixing Ciphers to Decimals.

182. **The value of a decimal is altered by affixing ciphers to it,** *i.e.*, by adding ciphers to the right of it.

For example 2300 is the same as ·23, since each of these decimals denotes 2 *tenths* and 3 *hundredths*.

(*b*) **The value of a decimal is altered and diminished by prefixing ciphers to it.**

For example ·023 and ·0023 are each less than ·23, since the digits 2 and 3 which denote 2 *tenths* and 3 *hundredths* in ·23, denote 2 *hundredths* and 3 *thousandths* in ·023 and 2 *thousandths* and 3 *ten-thousandths* in ·0023.

(*c*) It is evident that an integer like ·43 may be written in the forms 43·0, 43·00, 43·000, etc.

(*d*) It should also be noted that the insertion of ciphers in the middle of a decimal diminishes its value.

For example ·2304, ·23004, ·2034 are all less than ·234.

Exercise 155.—(*Oral*).

Show that—

1. ·3500 and 35000 are the same as 35.
2. ·035 and 0035 are each less than ·35.
3. 49·0, 49·00, 49 000 are the same as ·49.
4. 1023, ·10203, ·1203, ·12003 are all *less* than ·123.

---

5. Addition of Decimals.

183. We shall illustrate the *addition of decimals by three examples.*

Examples.

Add together (1) 4·566, 15·055, 9·644 and ·503
(2) 4·56, 5 0689, 3·0456 and 4·76.
(3) 15·745, 1 436, 0·603 and 2·816.

(1)	(2)	(3)
4·566	4·5600	15 745
15 055	5 0698	1 436
9·644	3 0456	0·603
·503	4·7600	2·816
29 768. *Ans.*	17·4354. *Ans.*	20·600 *Ans.*

In *Example* 2, we affix ciphers at the end of the first and fourth numbers for the sake of convenience, since the affixing of ciphers does not affect the value of the numbers.

In *Example* 3, we score out the two ciphers at the end of the answer, as they do not in any way affect its value.

. **Verification.**—The answers can be verified as in the addition of integers.

Exercise 156.

(*a*) Add together the following groups of decimals and verify your answers :—

1.	2.	3.	4.
45·708	123 005	1234 5	1 0546
53 087	83·694	769 8	8·8765
75·869	30·048	669 9	4 6954
20 405	6·804	2687·6	8·1735

(*b*) Find the sum of—

1. 46·79, 7 732: ·452, 6·006.
2. 17·0091: 7923, 20·4579; 34 0007.
3. 14 05, 140 5; 1·405, ·1405

4. Four-tenths and six-hundredths; three-tenths and two-hundredths, seven-tenths, five-tenths and two-hundredths

5. Three, and three-tenths, three-hundredths: three, and three-thousanths: one-thousandth.

(*c*) Find the sum of all the decimals of 3 places that can be formed with the digits 1, 2, 3 in every possible way, each figure being used only once in each decimal.

(*d*) Find the value of $x + y + z$, if (1) $x = 3{\cdot}25$, $y = 4{\cdot}8$ and $z = 1\,056$ (2) $x = 45\,6$, $y = 4{\cdot}56$, and $z = 456$

(*e*) Find the sum of the least and greatest decimals less than 1 containing 4 places of decimals

6 Subtraction of Decimals.

184. The following examples illustrate the *subtraction of decimals*

Examples.—Subtract (1) 3·485 from 4·506 : (2) 50·5468 from 213 12; (3) 1·6095 from 14.

(1)	(2)	(3)
4·506	213·1200	14 0000
3 485	50 5468	1·6095
1·021 *Ans*	162 5732 *Ans.*	12 3905. *Ans.*

Verification—The answers can be verified as in the subtraction of *whole numbers*.

Exercise 157.

(*a*) Perform the following subtractions and verify your answers.—

1.	3·456 2·148	2	14 156 3·044	3.	7·605 6·786	4	120 1 12 01

5. ·3004 from 3·004. 6. 4·549 from 16. 7 2 004 from 2 06

(*b*) Take away 14·5605 fzom 1456 05.

(*c*) Subtract 8 76 from 19, and from the result take away 4·005.

(*d*) Find the difference between 1 005 and 10 06.

(*e*) By how much does 1 469 exceed 1 43?

(*f*) What number must be added to ·0049 to make it equal to unity?

(*g*) The sum of two numbers is 2·0054 and one of them is 1 0008, find the other.

(*h*) The difference of two numbers is 4 04, and the greater of them is 5 06, find the smaller.

(*i*) By how much does the sum of 3 5, 35, and 35 exceed that of 5, 5 and ·05?

(*k*) Find the value of $x - y$ when $x = 3·5$ and $y = ·005$.

(*l*) Find the value of $x - y$ when (i) $x = 3·85$ and $y = 4\ 386$ (ii) $x = ·057$ and $y = 1·08$, (iii) $x = 2\ 07$ and $y = 4\ 046$

(*m*) Find the value of x when (1) $x + 3·456 = 10·5$, (2) $x - 10·05 = ·65$

---

185. We will now solve two examples involving both addition and subtraction of decimals.

Example 1.—Simplify 3·4—1 76 + 5·4.

Example 2.—Find the value of 4·5 + 6·005—8·5—1·66.

(1)	3·4	(2) 8 4	4·5
plus	5·4	1·66	*plus* 6 005
	8·80	10 06	10·505
minus	1·76		*minus* 10 06
	7·04. *Ans.*		·445 *Ans.*

Exercise 158.

(*a*) Find the value of the following expressions :—

1. 45·4—4·54 + ·454 2. 45·4—(4·54 + ·454).
3. 1·2 + 1·3 + 1·4—2·4—·24.
4. 1004—100·4 + 10·04—1·004.
5. 2·35 + 4·56 — 1·23 — 2·34.

(*b*) Subtract the sum of 1·404, 14·04 and 140·4 from 1404.

(*c*) From the sum of 1·03, 10·3 and 103 subtract ·103.

(*d*) Find the value of (i) $x - y + z$, and (ii) of $x + y - z$, when $x = 10·2$, $y = 8·05$, and $z = 1·876$.

(*e*) Find the value of $a - b - c$ when (i) $a = 4·5$, $b = 3·05$, $c = 1·208$, (ii) $a = 100·5$, $b = 1·005$, $c = 10·05$.

---

7. Multiplication of Decimals by Integers

186. Multiplication of Decimals by powers of 10:—

In the number 1234·567, suppose the decimal point is moved one place to the right so that it becomes 12345·67. Now comparing these two numbers which are composed of the same figures in the same order, we find that every figure in the latter has *ten* times the value which it has in the former, and therefore the latter number is *ten* times the former. Hence *to multiply a decimal by 10, we have only to shift the decimal point one place to the right.* Similarly, *to multiply a decimal by 100, 1000 &c., we have to shift the decimal point two places, three, &c. places, respectively, to the right, i.e., as many places to the right as there are ciphers in the multiplier—affixing ciphers, if necessary.*

Examples.

3·4561 × 1000 = 3456·1	3·504 × 1000 = 3504· = 3504
1·0504 × 100 = 105·04	3·504 × 10000 = 35040· = 35040
·054 × 10 = 0·54 or ·54.	·04 × 10000 = 0400· = 400

NOTE.—In the fifth example, we add a cipher at the end, as there are *four* ciphers in the multiplier and only *three* decimal places in the multiplicand. Similarly in the sixth example we add *two* ciphers at the end, since the multiplier contains *four* ciphers and there are only *two* decimal places in the multiplicand.

Exercise 159.—[(*a*) and (*b*) Oral.]

(*a*) Multiply each of the following numbers separately by 10, 100, 1000, 10000, &c.,—

1. 1·23456. 2. 12·3405. 3. 690·0456. 4. 0432.
5. 00432. 6. ·0000432. 7. ·02. 8. ·004.

(*b*) 1. How many *tenths* of a minute are there in 3·76 minutes?

2. How many *hundredths* of an inch are there in 1·085 inches?

3. How many *thousandths* of a mile are there in ·45 mile?

(*c*) If $x = 4·08$, $y = ·0062$, $z = 1·54$, find the value of—
(i) $10x + 100y + 1000z$, (ii) $100x - 1000y$.

---

187. Multiplication of Decimals by any Integer.—We shall explain this by some examples.

Example (1) Multiply 1·234 by 6.

Solution

```
 1·234
     6
 -----
 7·404   Ans.
```

Explanation.

4 *thousnndths*×6=24 *thousandths*=2 *hundredths* and 4 *thousandths*. We set down 4 under 4 (—*i.e.*, in the place of *thousandths*), and carry 2 to the *hundredths'* place. Again, 6 × 3 = 18, plus 2 is 20, set down 0 and carry 2, and so on.

Example (2) 13·425 × 14.

Solution

```
 13·425
 -------
 197·950.   Ans
```

Explanation.

5 × 14, 70, set down 0 under 5 and carry 7.

2 × 14, 28; plus 7, 35, set down 5 under 2 and carry 3, and so on.

Note 1.—The right-hand figure of the *multiplier* is placed under the right-hand figure of the *multiplicand*.

Note 2.—The decimal point in the *product* is exactly below the decimal point in the *multiplicand*.

Verification.—The product can be verified *by casting out nines*.

Exercise 160.

(*a*) Find the following products and verify them :—

1. 3 708 × 9.	2. 6 065 × 8.	3. 2 345 × 16.
4 4 182 × 11	5 6 054 × 15	6. 8·625 × 16.
7 2 303 × 13	8. 5·055 × 14	9. 121 35 × 6.

(*b*) Find the value of : (i) $8x$, (ii) $12x$, when $x = 180\ 05$.

(*c*) Find the value of $9x - 11y$ when $x = 18\ 43$ and $y = 15{\cdot}05$

(*d*) Find the value of $8a + 7b$ when $a = 3{\cdot}125$ and $b = 49\ 006$.

Example (3). Multiply 4 321 by 432.*

Solution

Method (A)		Method (B)	
4 321		4·321	
432		432	
8 642	... (i)	1728·4	... (iii)
129 63	... (ii)	129 63	... (ii)
1728·4	... (iii)	8 642	... (i)
1866 672. *Ans* .		1866 672. *Ans*	

Explanation.

(*a*) 1 *thousandth* × 400 = 001 × 400 = ·001 × 100 × 4 = 1 × 4 = 4 Hence in (A) and B) the 4 obtained from 1 × 4 is placed in the *tenths*' place. [See partial products numbered (iii)].

(*b*) Similarly 001 × 30 = 001 × 10 × 3 = ·01 × 3 = 03. Hence 3 got from 1 × 3 is placed in the *hundredths*' place of the partial products marked (ii).

(*c*) Again, since ·001 × 2 = ·002, the 2 is placed in the *thousandths*' place of the partial products marked (i).

Exercise 161.

(*To be done by two methods.*)

(*a*) Perform the following multiplications and verify your products by *casting out nines* :—

1. 1·234 × 432	2. 3·048 × 122.	3. 4 403 × 333.
4. 8 045 × 234.	5. 1·324 × 125.	6. 7·076 × 208.

---

* To the Teacher —It should be elicited from, or pointed out to, the pupil that the multiplier should be written under the multiplicand so that the *units*' figure of the former may stand exactly below the *right-hand* figure of the latter.

7	6·104 × 26	8.	·045 × 37.	9.	42·34 × 35.
10.	12·7 × 555.	11.	30·6 × 306.	12.	18 8 × 245.
13.	·0004 × 46.	14.	·0004 × 326.	15.	·0012 × 104.

(*b*) Find the value of—(i) $8x$ (ii) $15x$, (iii) $203x$ when $x =$ 2·304.

(*c*) Find the value of (i) $15x + 101y$, (ii) $25x - 12y$ when $x =$ ·008 and $y =$ 0042.

(*d*) Find the value of ab when (i) $a = 21·308$ and $b = 125$, (ii) $a = 1\ 245$ and $b = 48$.

(*e*) Find the value of $xy - ab$ when $x = 24$, $y = 10\ 08$, $a =$ 3·045, $b = 36$.

(*f*) Find the value of x when (1) $\frac{x}{8} = 2\ 225$, (2) $\frac{x}{19} = ·045$.

Exercise 162

Perform the following multiplications using the *factors* of the multiplier and verify the products by multiplying by the multiplier itself and by casting out nines :—

1.	3·923 × 24.	2.	18 04 × 36.	3	44·44 × 44.
4.	·0235 × 49.	5.	1 005 × 64.	6.	2·801 × 121.

188. To multiply 23·048 by 3400 we may first multiply 23·048 by 34 and then the product by 100, or by multiplying first by 100 and the product by 44.

Exercise 163.

Perform the following multiplications by the method of the above article and verify your products by casting out nines :—

1.	8 304 × 400.	2	15·084 × 80.	3.	465 27 × 700.
4.	12 084 × 1500.	5	·0084 × 700	6.	15 084 × 9000
7.	·0008 × 12000	8.	00456 × 4300	9.	1·10001 × 13100.

8. Division of Decimals by Integers.

189. Division of Decimals by powers of 10—

Taking the number 1234·567, and moving the decimal point one place to the left, we get 123·4567 In the latter number, each figure has *one-tenth* of the value it has in the former, and the latter number is therefore *one-tenth* of the former Hence *to divide a decimal by 10, we have only to move the decimal point one place to the left.* Similarly *to divide a decimal by 100, 1000, &c., we have to move the decimal point two, three, &c., places respectively to the left, i.e., as many places to the left as there are ciphers in the divisor—prefixing ciphers, if necessary.*

Examples.

(1) $3454 \div 10 = \cdot 3454.$	(4) $304 \cdot 34 \div 100 = 3 \cdot 0434.$
(2) $134 \cdot 5 \div 1000 = \cdot 1345$	(5) $1412 \cdot 06 \div 100 = 14 \cdot 1206.$
(3) $4 \cdot 5 \div 100 = \cdot 045$	(6) $4 \cdot 5 \div 1000 = \cdot 0045.$

Exercise 164.—(*Oral*)

(*a*) Divide each of the following numbers separately by 10, 100, 1000, etc.

1. 26405·12.	2. 12345·6.	3. 10·045.	4. 9376·54.
5. 94·5043.	6. ·0045.	7. 124000	8. 205640.
9. 104.	10. 1.	11. 12	12. ·13.

(*b*) 1. How many *tens* of miles are there in 13·24 miles ?

2. How many *hundreds* of inches are there in 203·5 inches ?

3. How many *thousands* of centimetres are there in 327·5 centimetres ?

(*c*) If $x = 10 \cdot 45$ and $y = 123 \cdot 4$, find the value of (i) $\frac{x}{10} + \frac{y}{100}$.

(ii) $\frac{x}{100} - \frac{y}{1000}$.

190. Division of Decimals by any Integer*.—The method of dividing a decimal by any integer is similar to the ordinary method of simple division, but has to be carried beyond the units' place to the right.

191. Short Division.—Let us find the quotient of 46·45, divided by 5.

Tens.	Units.	*Tenths.*	*Hundredths.*
5)4	6	4	5
	9	2	9

There is no difficulty as to the place of the decimal point in the quotient, since the first figure in the quotient agrees in position with that of the last figure 6 of the first partial dividend 46. Hence we have the following :—

* To the Teacher.—Beginners experience very great difficulty in fixing the position of the decimal point in the quotient. It is hoped that this difficulty will completely disappear if, in working out examples in division, the student is made to set down the figures of the quotients in their proper places as shown in the worked-out examples.

RULE.—Divide as in whole numbers, *taking care to place each figure of the quotient under the last figure of the corresponding partial dividend and the decimal point under the decimal point*

If the division does not terminate with the last figure of the dividend, affix ciphers (or conceive ciphers to be affixed) to the dividend, and continue the operation as far as may be necessary as in *Examples* 9, 10, 11 and 12 below.

Example 1. 234·528 ÷ 8.

8)234·528
 29·316 *Ans.*

Example 2. 809·112÷12

12)809·112
 67·426. *Ans.*

In *Example* 1, we place 2 under 3, 9 under 4, then the point under the point, and so on In *Example* 2, we place 6 under 0, 7 under 9, and so on

[VERIFICATION.—The quotient may, in every case where the division terminates, be verified by multiplying it by the divisor].

Exercise 165

(*a*) Perform the following divisions and verify each answer.—

1.	105·903 ÷ 7	2	248·045 ÷ 5.	3	75·054 ÷ 6
4.	905·67 ÷ 9.	5	905·7653 ÷ 11.	6	146·56 ÷ 8.
7.	16·18012 ÷ 11.	8	30·5481 ÷ 3	9.	100·044 ÷ 6

(*b*) Find the value of (i) $\frac{x}{3}$ (ii) $\frac{x}{8}$ when $x = 92·16$

Example 3. 25·152 ÷ 12.

12)25·152
 2·096. *Ans.*

Example 4. 90·315 ÷ 15.

15)90·315
 6·021 *Ans*

In *Example* 3, we place 2 under 5 and have a remainder 1, then we place the point after 2 and take the next figure 1, which gives 11, now, since 11 is not contained in 12, we put a cipher in the quotient, and proceed In *Example* 4, 15 time 6 is 90, no remainder, we place the point after 6 and take 3, now since 3 is not contained in 15, we put a cipher in the quotient, and proceed.

Exercise 166.

Find the value of the following and verify each answer :—

1.	4·0144 ÷ 4.	2.	300·468 ÷ 12.	3.	40·0548 ÷ 4.
4	48·108 ÷ 6.	5.	305·145 ÷ 5.	6.	14·1414 ÷ 7.
7.	3·10824 ÷ 12.	8.	12·0144 ÷ 3.	9.	160·3752 ÷ 8.

Example 5. 1·425 ÷ 5.

5)1 425

0·285 *Ans.*

Example 6. 3·69456 ÷ 16.

16)3 69456

0·23091. *Ans.*

In *Example* 5, since 5 is not contained in 1, we place a cipher in the quotient below 1, then we put the decimal point after 0 and take 14 for the next partial dividend, and continue the division.

In *Example* 6, after getting 3 in the quotient, we have a partial dividend 14, which being less than 16 gives a cipher after 3.

N B.—In *Examples* 5 and 6 the cipher in the quotient, before the decimal point may be, and is usually, omitted.

Exercise 167

Divide (verifying the quotient in each case)—

1.	1 484 by 7.	2.	1·5654 by 6.	3	1·25648 by 8.
4.	2·8809 by 9.	5.	1·45624 by 4.	6.	4·50048 by 8.
7.	3·5126014 by 7.	8.	3·54024 by 12.	9	6 677 by 11.

Example 7 ·3456 ÷ 4

4) 3456

·0864. *Ans*

Example 8 ·01287 ÷ 13.

13) 01287

00099. *Ans*

In *Examples* 7 and 8, we begin the quotient with a decimal point.

NOTE.—It may be pointed out that the figures in the dividend must be taken and disposed of *one by one*, so that there may be a figure in the quotient corresponding to *every* figure in the dividend.

Exercise 168

Perform the following divisions and verify the answer in each case.—

1.	·5454 ÷ 6	2	·2331 ÷ 7.	3.	·456016 ÷ 8.
4.	·12348 ÷ 9.	5.	0548 ÷ 4.	6.	·04545 ÷ 5.
7.	001236 ÷ 6.	8	0048048 ÷ 8.	9	·001463 ÷ 7.

Example 9. 12 5 ÷ 8

8)12·5000

1·5625. *Ans.*

Example 10. ·0717 ÷ 12

12) 0717

·005975. *Ans.*

In *Examples* 9 and 10 the division does not terminate with the last figure of the dividend, and we therefore affix ciphers (or conceive ciphers to be affixed) to the dividend, since ciphers affixed to a decimal do not affect its value.

Exercise 169.

Divide (verifying the answer in each case) :—

1.	4·85 by 4.	2.	12·55 by 2.	3.	37·65 by 6.
4.	15·54 by 4.	5.	1·25 by 8.	6.	·123 by 8.
7.	·0239 by 4.	8.	·123 by 6.	9.	·00498 by 6.

Example 11. $7 \div 4$

4)7·0
1·75 *Ans.*

Example 12. $1 \div 16$.

16)1·0000
·0625 *Ans.*

In *Examples* 11 and 12, we place the decimal point after the dividend and affix ciphers.

Exercise 170.

Perform the following divisions, verifying each answer:—

1.	$9 \div 8$.	2.	$30 \div 8$.	3.	$3 \div 4$.
4.	$7 \div 2$.	5.	$1 \div 4$.	6.	$3 \div 16$.
7.	$1 \div 8$.	8.	$5 \div 16$.	9.	$45 \div 12$.

Exercise 171.—(*Promiscuous.*)

(*A*) Find the value of—

1.	552·5 ÷ 13.	2.	43·26112 ÷ 14.	3.	61·050135 ÷ 15.
4.	·0962 ÷ 13.	5.	·014378 ÷ 14.	6.	·0546 ÷ 15.
7.	4·94554 ÷ 16.	8.	1 ÷ 16.	9.	·81 ÷ 15.

(*B*) Find the value of x when—

(1) $12x = 12·3$. (2) $13x = 135·5$.

(3) $8x + 1·23 = 41·55$. (4) $15x - 2·05 = 1·01$.

(C) If $9x = 11·25$, what is the value of $12x$?

192. Long Division.—In finding the quotient by employing *long division*, the process is the same as that for short division. It will often be found convenient, however, to set down the figures of the quotient *above* the dividend instead of *to the right* of it. If this plan be adopted, the position of the decimal point will easily get fixed as in short division where the quotient is placed *below* the dividend.

The rule may be given as follows :—

Rule —Divide as in whole numbers, *taking care to place the decimal point in the quotient just before bringing down the first decimal figure of the dividend.*

If the division does not terminate with the last figure of the dividend, affix ciphers (or conceive ciphers to be affixed) to the dividend and continue the operation as far as may be necessary. [See *Examples* 3, 4, 5 & 6 below.]

Example (1)—Divide 388·1296 by 23.

Example (2)—Divide ·16564 by 41.

```
           16·875   Ans.
(1)  23) 388·1296(
         23
         158        172
         138        161
          201        119
          184        115
           17         46
                      46
```

```
          ·00404.  Ans.
(2)  41)  16564(
          164
           164
           164
```

In *Example* (1) the first figure 1 of the quotient holds the same place (the tens' place) as the 8 in 38, the first partial dividend, does in the dividend

In *Example* (2) we begin the quotient with a decimal point; then since 41 does not contain 1 we put a nought in the quotient; again since 41 does not contain 16, we place another nought in the quotient, and so on.

Exercise 172.

Perform the following divisions and verify your answers :—

1. 52·992 ÷ 23	2. 92·15 ÷ 19.	3. 190·162 ÷ 47.
4. ·78384 ÷ 71.	5. 15·1782 ÷ 123	6. 58·145 ÷ 29.
7. 1·395 ÷ 31	8. 7828 ÷ 19	9. ·8845 ÷ 61.

Example 3. 147·43 ÷ 46.

```
       3·205   Ans
46) 147·430(
    138
      94
      92
       230
       230
```

Example 4. 171 ÷ 76.

```
       2·25.  Ans.
76) 171·00(
    152
     190
     152
     380
     380
```

In *Example* 3, since the division does not terminate with the last figure of the dividend, we affix ciphers to it.

In *Example* 4, since the dividend is an integer, we place the decimal point to the right of it and then affix ciphers.

Exercise 173

Perform the following divisions and verify your answers :—

1. 361·1 ÷ 115.	2. 502 2 ÷ 124.	3. 2492 8 ÷ 205.
4. 1394 ÷ 328.	5 40 ÷ ~~424~~. 256	6 126 9 ÷ 188.
7. 949 ÷ 584.	8. 163 4 ÷ 152.	9. 775 ÷ 248,

193. Division by Factors.—When the divisor can be split into factors which are less than 16 or are powers of ten, we may employ *short division*.

Ex. 1. 400·4 ÷ 385

385 = 5 × 7 × 11.

5)400·4
7)80·08
11)11·44
1·04. *Ans*.

Ex. 2. 404 ÷ 3200

3200 = 100 × 8 × 4

100)404.
8)4·04
4)·505
·12625. *Ans*.

Exercise 174

Work out the following examples by *short division* and verify your answers by *short* or *long multiplication* —

(A)	1 113 5 ÷ 25.	2. 261 ÷ 18.	3. 17 ÷ 40.
	4 51 45 ÷ 49	5. ·970 ÷ 320	6. 4680 ÷ 45000
(B)	1 1264·2 ÷ 105.	2 8289 6 ÷ 264.	3. ·8442 ÷ 252
	4 406·56 ÷ 385	5. 736 008 ÷ 364.	6. 210210 ÷ 420

194. When the division does not terminate, or when the quotient will contain a large number of decimal places, we are often required to find the quotient *as far as* a given number of decimal places.

Example 1.—Divide 57 by 28 *as far as* 5 places of decimals
Ans. 2 03571 .....

Example 2.—Divide 13·6 by 45 *as far as* 3 places of decimals.
Ans 302 ....

Example 3.—Divide 12 7 by 64 *as far as* 4 places of decimals.
Ans ·1984 ....

Exercise 175.

Divide (*as far as* 3 places of decimals).—

1. 58 6 by 45 2 120·4 by 23. 3. 71·3 by 150.
4 290 by 140. 5. 68 by 44 6 ·483 by 16

Perform the following divisions *as far as* 4 places of decimals :—

7. 23·5 ÷ 11. 8 15 ÷ 140. 9. 40 ÷ 27.
10 5·2 ÷ 70. 11 24·5768 — 12 12. 258 12 by 32.

Perform the following divisions *as far as* 5 places of decimals —

13. 20 1 ÷ 96 14. 327 1 ÷ 9 15. 3 40002 ÷ 72.

CHAPTER XXXVII.

STRAIGHT LINES AND ANGLES.

Exercise 176 —(*Practical*).

(*The straight edge to be used for drawing the lines*)

1. From any point A draw two straight lines AB and AC. The *corner* A at which the lines AB and AC meet is called an *angle*.

NOTE 1 —AB and AC which form the angle at A are called the arms of the angle the point A is called the vertex of the angle : and the arms are said to *contain* the angle between them

NOTE 2 —An angle is named by three letters of which the middle one is the letter at the vertex and the others are letters on the arms. For example, the angle drawn in Qn. 1 is called the angle BAC or CAB When there is only one angle at any point A, it may be briefly called "*angle at* A" or "angle A"

2 (*a*) Draw a horizontal line OA , from O draw an *oblique* line OB *slanting to the right*, a *vertical* line OC, and another *oblique* line OD slanting to the left Now name the angles formed respectively by OB, OC, and OD with OA

NOTE —It will be seen that the angles just formed at the point O differ from each other in size, the smallest angle being the angle BOA, and the largest DOA.

An angle like COA is called a right angle one like BOA which is less than a *right angle* is called an acute angle and one like DOA which is *greater* than a *right* angle is called an **obtuse** angle.

(*b*) Name three more angles in the figure of 2 (*a*)

3. Take your pair of dividers and widen the arms so as to form (i) a *right* angle. (ii) an *acute* angle, and (iii) an *obtuse* angle.

NOTE —It will be noticed that as the arms are opened wider and wider, the angle between them goes on increasing in magnitude.

4. Draw an angle BOC. Produce the arms OB, OC Is there any change in the size of the angle? No Hence we see that the *size* or *magnitude* of an angle is not altered if the length of the arms is increased or decreased.

5 Is the angle between the hands of a clock *right, obtuse,* or *acute* at 2 o'clock, at 5 o'clock, at 9 o'clock?

6. Take a piece of thick paper (regular or irregular in shape) with a straight edge Fold the paper at any point on the straight edge, so that it is bent back on itself (Fig 27) and press well along the fold with your fingers Then open out the fold, flatten down the paper, and mark the crease of the fold with your pencil (Fig. 26). Note that each of the two angles formed by the pencil line (or line of crease) with the straight edge of the paper is a *right angle*

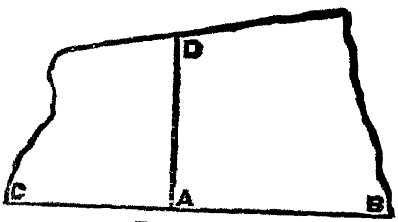

Fig 26.

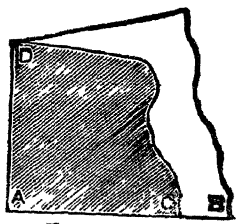

Fig 27.

NOTE.—If the paper be folded so that one part of the edge does *not* fall along the other part, one of the angles formed at the point of the fold is *obtuse* and the other *acute*. Show this.

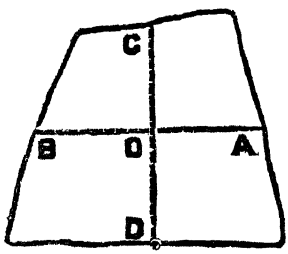

Fig 28.

7. Fold a piece of thick paper and get a straight edge by pressing along the fold with your fingers Fold it again so that this straight edge is bent on itself as in Exercise 6 above Unfold the paper completely, flatten it down, and mark the two creases with your pencil. Note that each of the 4 angles formed where the two creases cut each other is a *right angle*.

NOTE —If the straight edge of the paper be folded so that one part of it does *not* fall along the other, two of the four angles formed at the point of the 2nd fold will be *obtuse* and two *acute*. Show this.

8. Draw a straight line AB Take a stout pin with a piece of thread tied to it near the point Fix the pin at the point A. Then holding the thread tight, move the other end of it round from the point B At first the angle between the pencil line and the thread is small. As the thread turns round, the angle increases. Notice that the size of the angle depends on the *amount of turning* and *not* on the length of AB or of the thread.

9. Suppose a stick is placed in the direction OP and revolved round O so as to face the direction OS. From OP to OA it must have turned through one right angle and from OA to OS through another right angle The angle POS is therefore equal to *two* right angles and is called a *straight angle* as its two arms OP and OS form a straight line.

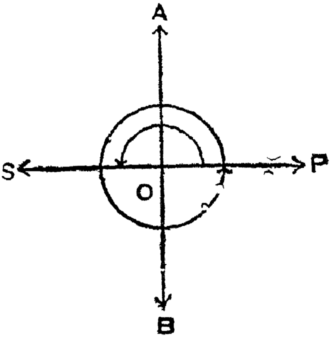

Fig 29.

Again suppose that the stick is revolved from OS so as to return to the position OP. It will be seen that in one complete revolution the stick has turned through *four* right angles

10 (i) Through how many right angles have you to turn in drill for a *right-turn*, a *left-turn* and an *about-turn* ?

(ii) At what hours will the angle between the two hands of a clock be a *right angle*, a *straight angle* ?

11. To campare the angles AOB and A'O'B'.

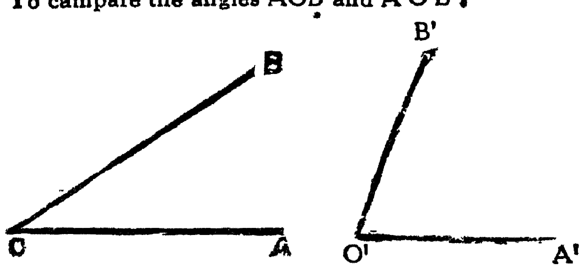

Fig. 30.

1st Method. Make a trace of $\angle AOB$ on tracing paper and put the trace on $\angle A'O'B'$ so that OA on the tracing paper may fall on O'A'. Then if OB on the tracing paper falls exactly on

O'B', the two angles are equal: if not they are unequal, ∠AOB being less than or greater than ∠A'O'B' according as OB falls between O'A' and O'B' or outside ∠A'O'B' This method of comparing angles is called the *method of superposition* *

2nd Method Cut out the two angles along their arms and apply one of the cuttings on the other, and find out whether the angles are equal or unequal*,

12 Show by the above methods that all right angles are equal to one another

Exercise 177.—(*Practical and Freehand*).

1. Mark any two points and join them by a straight line and by curved lines Note that there is only one straight line between any two points and it is the shortest distance between them

2. Draw two straight lines from any point, forming a *very small* angle with each other, and note that they become wider and wider apart as they are produced and can therefore never meet again however far they may be produced Hence learn that two straight lines cannot enclose a space.

CHAPTER XXXVIII

METRIC MEASURES OF LENGTH.

195. The Metric Measures of Length are so called from the *Metre* which is the fundamental unit of length in that system of lineal measures, from which all the other units of length are derived

NOTE 1.—The *Metric* Measures of Length are also called the *French* Measures of Length, since they were first used in France where they were invented.

NOTE 2 —The metre is supposed to be the *ten-millionth* part of the distance from the equator to the pole, measured along a meridian of the earth, and is equal to 39·37079 inches.

196. The Metre and its Multiples and Sub-multiples :—

From the table of *Metric Measures of Length* given at page 10 it will be seen that the denominations *Kilometre*, (Km.), *Hectometre*, (Hm.) *Decametre* (Dm), *Metre* (m.), *decimetre* (dm), *centimetre* (cm.), and *millimetre* (mm) are each one-tenth (1) of the preceding one and 10 times the succeeding one, so that—

* Note that angles may also be compared by measuring them with the *protractor* (angle-measurer).

(*a*) 1 Km. = 1000 m , 1 Hm = 100 m. ; 1 Dm. = 10 m. ;
(*b*) 1 m. = 1 Dm = ·01 Hm. = ·001 Km. ;
(*c*) 1 m. = 10 dm = 100 cm. = 1000 mm. ;
(*d*) 1 dm. = 1 m. , 1 cm -- ·01 m. , 1 mm. = ·001 m.

Exercise 178.

1 Why is the *metric system* of lengths so called ? By what other name is it known and why ?

2 Fill up the blanks in the following statements :—

(*a*) 1 Km = m. (*b*) 1 m. = ... Km. (*c*) 1 m = cm.
(*d*) 1 cm. = . m (*e*) 1 cm. = . mm. (*f*) 1 mm. = ..cm.

3 Given that the length of a metre is 39·37079 inches, find the length of a *Decametre*, a *Hectometre*, and a *Kilometre*

4. Taking the length of the *metre* as 39 37 inches, what is the length of 1 dm , 1 cm , and 1 mm ? And of 1 Dm. , 1 Hm., and 1 Km.

5. Reduce—

(*a*) 5 cm. to mms , 5 cm. 4 mm to mms , 2 dm 5 cm. to cms.
(*b*) 4 mm to cm , 3 cm 4 mm to cms., 3 dm. 2 cm. to dms.
(*c*) 7 mm to ms , 1 m 3 mm. to ms. , 2 m. 6 cm. to ms.

6 Add together (*a*) 5 cm 2 mm and 4 cm , 3 mm. ; (*b*) 1 cm 5 mm , 5 cm 8 mm , and 2 cm 1 mm.

7 Subtract (*a*) 6 cm 4 mm from 9 cm 8 mm ; (*b*) 7 cm., 4 mm from 10 cm.

8 Find the value of (*a*) 4 8 cm. + 3·4 cm., (*b*) 4·7 cm.— 2 9 cm. (*c*) 8 cm. 4 mm. — 3 cm 5 mm. + 2 cm 3 mm.

9. Multiply (*a*) 2 cm. 5 mm by 3 , (*b*) 1 cm. 4 mm. by 5 ; (*c*) 6 cm. 5 mm. by 8.

10 Divide (*a*) 6 cm 4 mm. by 4 , (*b*) 12 cm. 6 mm. by 7 ; (*c*) 20 cm. 3 mm by 7.

11 How many pieces 4 cm. 3 mm long can be cut off from a tape 35 cm. 6 mm long ? And what length will remain over ?

12 Find the value of x when—

(*a*) $\frac{x}{8}$ = 5 cm. 4 mm. (*b*) $\frac{x}{10}$ = 4 cm. 2 mm.

(*c*) $7x$ = 10 cm. 5 mm. (*d*) $10x$ = 18 cm. 4 mm.

To the Teacher —The pupil may be taught that Deka, Hecto, and Kilo which are Greek words for Ten, Hundred, and Thousand, respectively, are used for denoting the *higher multiples* of the metre, while *deci, centi*, and *milli* which are Latin words for 10, 100, and 1000 respectively, are used for denoting the *sub-multiples* of the metre

13. Multiply 8 cm 4 mm by 8 and divide the product by 10.

14. If the length of 7 equal pieces of tape be 30 cm. 1 mm, what will be the length of 9 such pieces ?

15. How many pieces each 1 dm 3 cm long can be cut off from a string 1 metre long, and what length will remain over ?

16. From a length of 4 m 5 cm., eight lengths of 3 dm. 4 cm. 5 mm. each are cut off, into how many lengths of 2 dm 1 cm. 5 mm. can the remaining length be cut off

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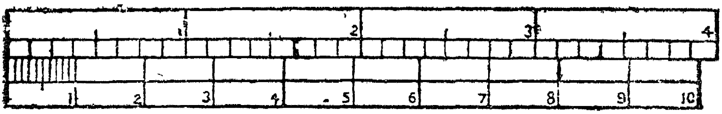
Exercise 179.

1. From the following diagram showing the relation between *inches* and *centimetres* show that (*a*) 10 cms are nearly equal

inches



centimetres

Fig 31

to 4 inches, (*b*) 2 inches are a little more than 5 cms, (*c*) 1 cm. is a little more than 4 of an inch, (*d*) 7 cms are equal to $2\frac{3}{4}$ inches

2 Taking the metre as 40 inches show that (i) dm. = 4 inches, (ii) 1 cm = 4 inch, (iii) 1 inch = 2 5 cm or $2\frac{1}{2}$ cm. Also find in inches the equivalents of (*a*) 3 cm., (*b*) 4 cm., etc.

3. If 1 m. = 39 37 inches, how many inches will a Km be equal to ? If 1 m = 40 inches, find the difference in inches between these two results.

4 Taking the metre as 40 inches, find the difference in yards between a kilometre and 5 furlongs.

5 What will be the difference in inches between 8 Km. and 5 miles, if a metre is taken (*a*) as 39 37 inches. (*b*) as 40 inches ?

6. If 1 Km, be taken as 5 furlongs, how many inches will a metre be equal to ?

7 Taking the metre as 39'37 inches, show that 915 metres are roughly equal to 1000 yards

8. For what length of the metre will 1 mile be equal to *a*) 1609 metres. (*b*) 1600 metres ?

9 The *metre* is the ten-millionth part of the distance from the earth's equator to the North pole. Taking this distance as 6,200 miles, find the length of the metre in inches to 3 places of decimals.

CHAPTER XXXIX.

USE OF THE PROTRACTOR.

1. The Right Angle and its Sub-divisions.

197. Units of Angle Measurement.—The chief units employed for measuring angles are (1) the *right angle*, and (2) the *degree* (which is the 90th part of a right angle).

The symbol for *degree* is °, so that 1 rt. angle = 90°, 2 rt. angles = 180°, 4 rt angles = 360°, $1\frac{1}{2}$ rt. angles = 135°, 1 rt. angle and 60° = 150°, and so on.

Exercise 180—(*Oral*).

(A) 1. Express the following angles in degrees —
1/2 a rt. angle ; ⅓ of a rt. angle . 1 rt, angle 20°

2. What fraction of a rt angle is each of the following ?
30°, 45°; $22\frac{1}{2}$°, 18°, 15°, 10°, 12½°

3. How many rt angles are equal to 120°, 112½°, 225°, 300°?

(B) 1. How many degrees are there in a *straight angle*? And in 4 rt angles ?

2. Through how many degrees of angle does the *minute-hand* of a clock revolve in an *hour* ? In a *minute* ? In 5 *min.* ?

3. Through how many degrees of angle does the *hour*-hand of a clock move in 1 *hour* ? In $2\frac{1}{2}$ *hours*?

4. Through how many degrees of angle does the *second*-hand of a watch move in a *second* ?

2. Graduation of the Protractor.*

198. The Circular Protractor —

(1) The curved edge of the protractor is divided into 90 equal parts by thin and short lines which if produced downward will pass through the mid point of the base (or straight edge), which is also the centre of the semi-circle, so that these lines divide the straight angle at the centre (which is equal to two right angles into 180 degrees,

(2) The lines dividing the curved edge into 180 parts are numbered 10, 20, 30. up to 180 at intervals of 10° each, once commencing from the right end of the base and proceeding towards the left and again commencing from the left end of the base and proceeding towards the right.

---

* The *protractor* is an instrument for measuring and constructing angles.

(3) These two sets of numbers enable us to find at once the magnitudes of the *two* angles which any straight line makes with another straight line on which it stands.

For example, if one angle be 70° the other angle will be 110° (= 180° — 70°), if one angle be 125°, the other angle will be 55° (= 180° — 125°), and so on.

3. Constructing Angles with the Protractor.

199. To Construct an Angle of 60° at a point —

Draw a line PQ and place your protractor so that the mid point of the base falls exactly on P and the base along PQ Mark a point R on your paper as close as possible to the 60th division on the curved edge of the protractor counting from the end of the part of the straight edge which falls on PQ. Now remove the protractor and join PR. Then the angle RPQ will be 60°.

Exercise 181 —(*Practical.*)

[*Questions 1, 3, 4 5, 7 and 8 may also be done on the B. B. with the B. B. Protractor and Ruler*]

1 At any point draw the following angles by using the protractor :— 45°, 30°, 90°, 82°, 95°, 120°, 150°, 175°.

2 Can you draw with the protractor an angle greater than 180° ?

3. Draw a straight line AB and from a point C in it draw CD and CE, so that each of the angles made by these lines with AB may be 50°. Similarly draw angles of 110 , 165 , and so on

4 From a point O in a straight line PQ draw a line OR to make an angle of 60° with OQ What must the magnitude of the angle POR ?

5. Describe angles of 22½°, 30½°. etc , as accurately as you can.

6 Describe by using the protractor *rectangles* and *squares* of the following dimensions :—

1. 2 in. by 1·5 in.	2. 7 cm by 5 cm.
3. 10 cm. by 7·5 cm.	4. 3″ square.
5. 2½″ square.	6. 8 cm. square.

7 Construct an angle of 120° and divide it into (*a*) 2 equal parts (*b*) 3 equal parts.

8 Construct a right angle ABC and from B draw a line BD so that the angle ABD may be 35°.

4. Measuring Angles with the Protractor.

200. To measure a given angle—

To measure any angle PQR, place the protractor so that the mid point of its base may be exactly on the angular point Q and the base along one of the lines QP and QR Read the number of the division on the curved edge below which QR or QP passes, this will be the number of degrees in PQR

NOTE 1—If an arm of the angle is not long enough to pass beyond the curved edge, we may produce it or guess the division below which it will pass if produced

NOTE 2—If an arm of the angle measured passes *between two* divisions the measurement is to be given *correct* to 1° as judged by the eye.

Exercise 182—(*Practical*).

[*To be done on the Black Board first by the teacher and then by the pupils, using the* B B *Protractor and Ruler.*

1. Draw an *acute* angle AOB with *long* arms and measure it correct to 1°, by placing the base of the protractor (*a*) along OA. (*b*) along OB

2 Similarly draw an *obtuse* angle AOB with *long* arms and measure it likewise in two ways.

3. Repeat Exer. 1 and 2 above several times, drawing the arms of the angles in various directions

4. Repeat Exer. 1 and 2 above several times, taking angles with *short* arms.

Exercise 183.—(*Practical*)

1 Repeat Questions 1 to 4 of Exercise 180 above on *paper*, using your *protractor* and *ruler*.

2 Measure with your *protractor* the angles of your *set squares*.

3. Measure correct to 1° the four angles given below.

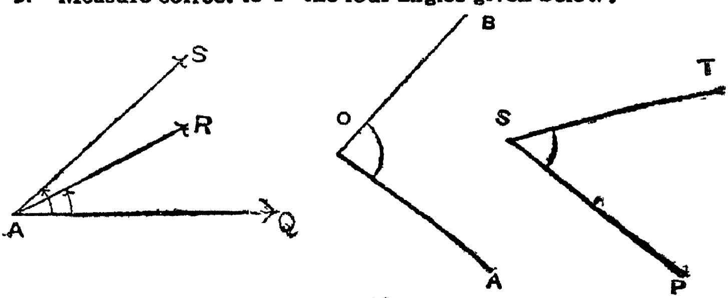


Fig. 32.

4 Measure the angles FQR and YQR in Fig. 33 and find the sum of the two angles.

5. Measure the three angles of the following triangle (Fig. 34) and find the sum of the three angles.

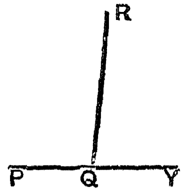


Fig. 33.

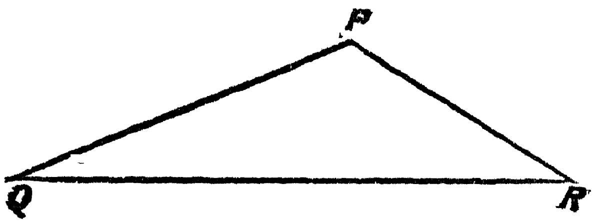


Fig. 34.

6. Measure the angles in the adjoining figure and fill up the following tables:—

(*a*) $\angle$ABD =
plus $\angle$DBC =
$\therefore$ $\angle$ABC =

Check your result by measuring $\angle$ABC.

(*b*) $\angle$ABK =
minus $\angle$ABD =
$\therefore$ $\angle$DBK =

Check your result by measuring $\angle$DEK.

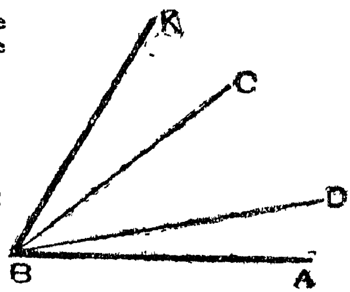


Fig. 35.

CHAPTER XL.

RESOLUTION INTO PRIME FACTORS.

201. Prime Factors.—When a composite number is resolved into factors, each of which is a *prime* number, such factors are called the *prime factors* of the number.

Thus since $60 = 2 \times 2 \times 3 \times 5$, the *prime factors* of 60 are 2, 2, 3, 5. [See Art. 210.]

202. To resolve any composite number into its *prime factors*, we divide it by the smallest prime number which will divide it exactly; then divide the quotient by the smallest prime number which it contains; and proceed in this way till the quotient is a prime number. The successive divisors and the last quotient are the prime factors of the number.

Example.—Resolve 13860 into *prime factors.*

$$\begin{array}{r|l} 2 & 13860 \\ 2 & 6930 \\ 3 & 3465 \\ 3 & 1155 \\ 5 & 385 \\ 7 & 77 \\ & 11 \end{array}$$

Hence $13860 = 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 11$ or $2^2 \times 3^2 \times 5 \times 7 \times 11$, using the *index notation.* *Ans.*

203. The following tests of the divisibility of numbers will be found very useful in resolving large numbers into their factors.

(A) A number is exactly divisible—

(i) by 2, when its last digit is either 0 or divisible by 2.

Example.—30, 376.

(ii) by 3, when the sum of its digits is divisible by 3.

Example.—4458 is divisible by 3, since $4 + 4 + 5 + 8$ or 21 is divisible by 3.

(iii) by 4, when the number expressed by its last two digits is divisible by 4.

Example.—1732 is divisible by 4, since 32 is divisible by 4.

(iv) by 5, when its last digit is 0 or 5.

Example.—140, 3025.

(v) by 8, when the number expressed by its last 3 digits is divisible by 8.

Example.—30360 is divisible by 8, since 360 is divisible by 8.

(vi) by 9, when the sum of its digits is divisible by 9.

Example —154206 is divisible by 9, since 1+5+4+2+0+6 or 18 is divisible by 9.

(vii) by 11, when the difference between the sum of the digits in the odd places and that of the digits in the even places is either 0 or divisible by 11.

Examples.—In 776985, 7 + 6 + 8 = 21, and 7 + 9 + 5 = 21, and 21 − 21 = 0, In 80317, 8 + 8+7=23 0 + 1 = 1, and 23−1 = 22, which is divisible by 11. Hence both the numbers are divisible by 11.

(B) A number is divisible by 6, 15 and 12 when it is respectively divisible by 2 and 3, 3 and 5, and 3 and 4.

Examples —5382 is divisible both by 2 and by 3 and therefore by 6 also 9705 is divisible both by 3 and by 3, and therefore by 15 also ; 7728 is divisible both by 3 and by 4, and therefore by 12 also.

Exercise 184—(*Oral*).

(*a*) By which of the numbers 2, 3, 4, 5 8, 9, 11 is each of the following numbers divisible ?

1. 3648.	2. 340128.	3. 707850.	4. 702750.
5. 6875.	6. 142857.	7. 85008.	8. 7290919.

(*b*) 1. What is the *least* digit that must be substituted for 0 in 32704 so that the new number may be divisible by 3 ? And what is the *largest* digit ?

2. By what digit must 8 in 170548 be replaced so that the new number may be divisible by 9 ?

3. By what digit must 6 in 296074 be replaced so that the resulting number may be divisible by 11 ?

4. Supply the missing figure in 460*896 which is divisible by 11.

(*c*) Show that (1) 9546 is divisible by 6 ; (2) 26028 is divisible by 12 ; (3) 22605 is divisible by 15, (4) 54120 is divisible by 165.

Exercise 185.

Resolve the following numbers into *prime* factors and write them down in the *shortest* form :—

1. 270.	2. 228.	3. 363.	4. 748.
5. 539.	6. 3960.	7. 1750.	8. 12705.
9. 2970.	10. 6435.	11. 450450.	12. 592900.

204. Example—What is the least number by which 1008 must be multiplied so that the product may be a *perfect square*?

2)3150 3)1575 3)525 5)175 5)35 7	Resolving 3150 into prime factors, we find that it is equal to $2 \times 3^2 \times 5^2 \times 7$. Hence it must be multiplied by 2×7 or 14 to become a perfect square.

Exercise 186.

(A) Find the least number by which each of the following numbers must be multiplied, so that the product may be a *perfect square*:—

1. 1728.	2. 1875.	3 972	4. 726.
5 2205.	6. 775.	7. 13013.	8. 1250.

(B) Find the least number by which each of the following numbers must be divided, so that the quotient may be a *perfect square*:—

1. 2205.	2. 24200.	3. 2646.	4. 2535.

CHAPTER XLI.

MEASURES AND MULTIPLES.

205. Measure.—A number which *measures* another (*i.e.* divides it exactly) is called a **measure** of it.

Thus 5 is a *measure* of 15; Rs 4 is a *measure* of Rs. 12; and so on.

NOTE 1—Any number is a *measure* of itself. Thus 5 is a *measure* of 5. Unity is not considered as a measure of any number.

NOTE 2—The term *measure* has the same meaning as *factor* so that 5 may be called a *measure* or *factor* of 15.

NOTE 3—The quotient of a *concrete quantity* divided by a measure of it is an *abstract number*. For example 20 ft. ÷ 5 ft. = 4.

Exercise 187—(*Oral*).

1. Name all the *measures* (including itself) of each of the following numbers:—15, 18, 21, 24, 30, 48, 50, 64.

2. Name all the measures (of the same denomination) of each of the following quantities.—Rs. 15, 30 feet, 44 tolas; 72 metres

3 Name the *smallest measure* of 8, 12, 20, 18, 35, 72 £30, 18 yards, 35 cm

4. Name the *largest measure* (but itself) of 18, 24, 36, 48, 60, 100; 20 lb., 14 miles, 28 Km.

Exercise 188.—(*Practical*).

1. Draw two straight lines AB and CD, 15 cm and 5 cm. in length and by stepping off with the dividers on AB lengths equal to CD, show that CD is a *measure* of AB.

2. Similarly show that both 2 cm. and 3 cm. are *measures* of 12 cm?

3 Cut a narrow strip of paper 24'' long, and cut three others 3'', 4'' and 6'' long And by actual measurement, find how many times each of the latter slips is contained in the former.

4 Draw a long line AB and a short line XY. Examine whether XY is a measure of AB If it is not, produce AB to the nearest point C, so that XY may be a *measure* of AC.

---

206. Multiple—When one number contains another number an *exact* number of times, the former is called a *multiple* of the latter and the latter is called a *measure* of the former.

Thus 15 is a *multiple* of 5, and 5 is a measure of 15; 18 is a *multiple* of 9, and 9 is a measure of 18, and so on.

NOTE—The term *measure* is the correlative of *multiple*. Whenever one number is a measure of another the latter is a multiple of the former, and conversely whenever one number is a multiple of another, the latter is a measure of the former.

Thus, since 5 is a *measure* of 20, 20 is a *multiple* of 5.*

207. A multiple of any number is so called, because it can be obtained from the number by *multiplying* it by any *integer*.

For example, taking the number 7 and multiplying it *separately* by 2, 3 and 4, we obtain the numbers 14, 21 and 28, which are all *multiples* of 7.

* To the Teacher —The teacher should explain that the multiplication table contains a large number of measures and multiples

NOTE —Any number can be called a *multiple* of itself, since it can be obtained by multiplying it by 1. Thus since $8 \times 1 = 8$, 8 is a multiple of itself.

Exercise 189 —(*Oral*).

1. Name 3 or 4 multiples of 5, of 6, of 8, and of 10.

2. Name three or four multiples of Rs. 3, of 7 pies, of 4 yards, of 9 shillings.

3. Name some numbers of which (*a*) 24 is a *multiple*, (*b*) Rs 42 is a *multiple*, (*c*) 64 cm. is a *multiple*

4. Name all the multiples of 5, 7, 9, 10, 12 not exceeding 50.

5. Name all the multiples of 9, 10, and 11 lying between 32 and 62

6. Name the four multiples of 24 lying between 100 and 200.

2 Find the 4 multiples of Rs 35 which lie between Rs. 120 and Rs. 250

8. Find the *multiples* of 25 inches which lie between 60 inches and 160 inches What is the quotient obtained by dividing each of these multiples by 25 inches ?

9. Show by means of examples that the number of *measures* of any number is *limited* while the number of its *multiples* is *unlimited*.

10 Taking x as a whole number, note that $2x$ is a multiple of it and name 3 more multiples of it.

CHAPTER XLII.

SOME RELATED ANGLES.

1, Adjacent Angles and Opposite Angles.

Exercise 190 —(*Practical*.)

1, Draw a straight line AB and from any point C in it draw another straight line CD at right angles to it or otherwise Learn that the two angles ACD and BCD are called *adjacent angles*.

2. Draw two straight lines AB and CD to cut each other at O, Learn that the angles AOC and BOD are called *opposite angles* (or *vertically opposite angles*), and so also the angles AOD and BOC.

3. Name the four pairs of *adjacent angles* in each of figures 36 and 37 below. Also name the two pairs of *opposite angles* in figure 37.

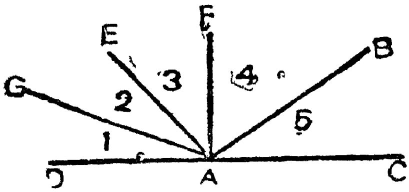


Fig. 36.

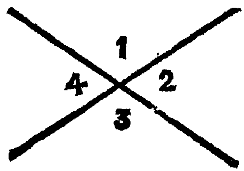


Fig. 37.

4. From any point C in a line AB, draw a straight line CD in any direction. Measure the angles DCA and DCB and find the sum of these two angles Repeat this exercise half-a-dozen times by taking different pairs of lines What do you infer from this regarding the sum of the two *adjacent* angles formed by any straight line with another straight line ?

5 Draw two straight lines to cut each other at any inclination: and measure the opposite angles Repeat this half-a-dozen times. What do you infer from this regarding the relation between the vertically opposite angles formed by two straight lines cutting each other ?

6. If one of two adjacent angles is 50°, what is the magnitude of the other ?

7. If two lines PR and QS intersect (cut) each other at O and the angle POQ is 60°, what must be the magnitude of each of the other three angles ?

8. Show, by measuring each of the angles 1, 2, 3, 4 and 5 in Fig. 36 in Question 3 above, that the sum of all the angles is 180°.

2. Angles at a point.

Exercise 191.

1. Take a point O and from it draw 5 or 6 straight lines in different directions Measure each angle at O and find the sum of all the angles at the point. Repeat this exercise half-a-dozen times, and state your inference regarding the sum of all the angles formed by a number of straight lines meeting at a point.

2. If 5 out of the 6 angles round a point are respectively 120°, 25°, 75°, 40° and 30°, what must be the magnitude of the sixth ?

3 From a point O are drawn 8 lines making 8 equal angles. What is the magnitude of each of these angles ?

3. Supplementary and Complementary Angles.

Exercise 192.

A. 1. Learn that two angles whose sum is 2 rt. angles or 180° are called *supplementary angles*, each of the two being the *supplement* of the other.

2. If one of two *supplementary angles* be 105°, what is the magnitude of the other? If 100°? If 75°? If 45°?

3. What is the *supplement* of each of the following angles? 30°, 60°, 75°, 120°, 150°, 170°.

4. Draw an angle of 65° and produce one of its arms. What must be the magnitude of the new angle formed thereby? Verify your answer by measurement.

5. If one of two *supplementary* angles be 3 times the other, what is the magnitude of each?

6. If one of two *supplementary* angles be 18° *more* than the other, what is the magnitude of each?

B. 1. Learn that two angles whose sum is 1 rt. angle or 90° are called *complementary angles*, each of the two being the *complement* of the other.

2. If one of two *complementary angles* is 40°, what is the other? If 70°? If 35°? If 22½°?

3. What is the *complement* of each of the following angles? 60°, 45°, 48°, 37°, 36°, 72°, 23½°.

4. If one of two *complementary angles* be 4 times the other, what is the magnitude of each?

5. If an angle is 20° less than its *complement*, what is the magnitude of each?

CHAPTER XLIII.

GREATEST COMMON MEASURE.

208. Common Measure.—A number which divides *each* of two or more numbers *exactly* is called a *common measure* of those numbers.

For example, 5 is a *common measure* of 15 and 20, 6 is a *common measure* of 12, 24 and 30, 2 is a *common measure* of 2^3 and 2^4.

209. Greatest Common Measure—The *greatest* of the measures *common* to two or more numbers is called their *Greatest Common Measure.*

For example of the *three* common measure of 18 and 24, *viz.*, 2, 3 and 6, 6 is the *greatest*, and therefore the *greatest common measure* of 18 and 24 is 6. of the *three* common measures 3, 5 and 15 of 60, 105 and 120, the *greatest* is 15, and the *Greatest Common Measure* of these three numbers is therefore 15. 2, 2^3 are the common measures of 2^3, 2^4 and 2^5, and 2^3 is therefore their *Greatest Common Measure*.

NOTE 1.—The contraction for Greatest Common Measure is G C M

NOTE 2.—When there is only *one* common measure to two or more numbers, that measure is still called their G. C. M. For example, 5 is the G C M of 15 and 20.

Exercise 193.

Write down all the *common measures* of the numbers in each of the following groups and hence find their G.C.M.

1. 18 30. 2. 35, 28 3 60, 42, 4. 75, 105.
5. 16, 24 36 6. 18, 30 42. 7 12, 21. 45. 8. 7/. 49, 21.
9 $3 \times 5 \times 7$, $2 \times 3 \times 5$ $3 \times 5 \times 11$.
10. 2^2, 2^5, 11. 3^4, 3^5. 3^3. 12. 7, 7^3, 7^2.

Exercise 194—(*Graphical*).

1. Show *graphically* on squared paper that (*a*) 2 is a common measure of 10 and 8, (*b*) 4 is a common measure of 8, 12 and 16.

2 Draw two straight lines 12 cm. and 8 cm long. Find the length of lines which measure each of these two lines. Which of them is the *greatest*?

3 On squared paper take two lines containing 24 and 18 small divisions of the paper, and find all the common measures of 24 and 18. Which is the *greatest* of those common measures?

210. Numbers prime to one another.—When two or more numbers have no *common measure* but unity, they are said to be *prime to one another*.

For example, 8 and 11 are *prime to each other*: 5, 7 and 12 are *prime to one another*: 8 and 9 are *prime to each other*.

211. Quotients of numbers divided by their G C.M —

If we divide 18 and 24 by their G C M 6 we get the quotients 3 and 4 which are *prime to each other*: if we divide 16, 24 and 32 by their G. C M 8 we get the quotients 2, 3, 4, which are also *prime to one another*.

Hence we infer that *when two or more numbers are divided by their G. C. M., the quotients are prime to one another.*

Exercise 195.

Find the G. C M of each of the following sets of numbers and see if the quotients obtained by dividing each set by their G.C M. are *prime to each other*.

1. 50, 70, 2. 45 99. 3 105, 60. 4. 70, 42.
5. 63, 14, 49. 6. 30, 42, 84. 7. 66, 55, 88.
8. 3×5, $3\times2\times5$. 9. $5\times7\times11$, $2\times3\times11$.

---

212. To find the G. C M of :—(1) $2\times3^2\times5^3$, (2) $3\times5^2\times7$, (3) $2^2\times3^3\times5^2$.

Solution.

(A) The only *prime factors* common to (1), (2) and (3) are 3 and 5 ;

(B). 1. The G. C M of 3^2, 3 and 3^3 is 3 :
 2. The G C. M of 5^3, 5^2 and 5^2 is 5^2 :

(C). Hence the G C. M required is 3×5^2. *Ans.*.

It may be noticed that the factors 2 and 7 are *not* taken for the G. C. M since they are *not* found in *all* the three quantities (1), (2) and (3).

Verification.

$$\frac{2\times3^2\times5^3}{3\times5^2}=2\times3\times5:\ \frac{3\times5^2\times7}{3\times5^2}=7:\frac{2^2\times3^3\times5^2}{3\times5^2}=2\times3^2.$$

And $2\times3\times5$, 7 and 2×3^2 are *prime to one another*, (*i e.* have no factors common to *all*).

Exercise 196.

Find the G C M of the following groups of quantities and verify your answers :—

1. $2\times3^3\times5$, $2^2\times3^2\times7$. 2 $2^3\times3^4$, $2^2\times3^2\times5$.
3. $5\times7^2\times11$ $7\times11^2\times13$. 4 $2\times5^3\times7^2$, $2^3\times5\times7^2$.
5. $2^2\times3\times5^2$ $2\times3^2\times5^2$, 3×5^3. 6. $3^2\times5$, 3×5^2, $3\times5\times7$.
7. $2\times3\times5\times7$, $3\times5\times7\times11$. 8. $3\times5^2\times7\times11$, $5\times7^2\times11\times13$.

213. From the foregoing Examples and Exercises we derive the following

Rule.—The *Greatest Common Measure* of two or more numbers is the product of *all different prime factors* of the numbers which are *common* to *all* the numbers, each such prime factor being taken in the *least power* in which it occurs in *any one* of the numbers.

Exercise 197.

Find the G. C. M. of the following groups of numbers by resolving them into *prime factors*. Then divide each group of numbers by their G. C. M. and see if the resulting quotients are *prime to each other.*

(*a*) 1. 40 and 100. 2. 90 and 525. 3. 100 and 150.
4. 350 and 60. 5. 495 and 450. 6. 1425 and 380.

(*b*) 1. 120, 450 and 750. 2. 450, 150 and 675.
3. 735, 3675 and 1155. 4. 825, 495 and 1210.
5. 504, 1512 and 3920. 6. 3325, 570 and 3800.

214. To find *graphically* the G. C. M. of two small numbers.

Take on squared paper two lines AB, CD of lengths 24 and 9 divisions respectively. (Fig. 38.)

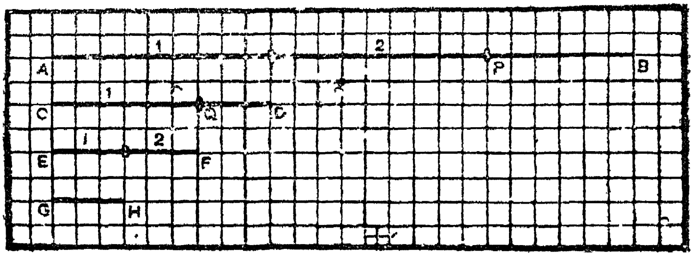


Fig. 38.

With the dividers, measure lengths equal to CD along AB until you come to a length PB which is less than CD.

Take a line EF equal to PB.

Measure lengths equal to EF along CD until you come to a length QD which is less than EF.

Take a line GH equal to QD.

Again measure lengths equal to GH along EF. You find that GH exactly divides EF. GH is the greatest measure common to AB and CD. And as it contains 3 divisions, 3 is the G.C.M. of 24 and 9.

Exercise 198—(*Graphical*).

Find *graphically* on squared paper the G. C. M. of—

1. 8 and 12	2. 14 and 21,	3. 9 and 39.
4. 16 and 24.	5. 27 and 39.	6. 20 and 36.

---

215. From the *graphical* method of Art. 214 for finding the G. C. M. of two small numbers, we may deduce the following rule for finding the G. C. M. of two numbers without resolving them into factors:—

Divide the greater of the two numbers by the less, the less by the first remainder, the first remainder by the second remainder, and repeat the process until there is no remainder. The last divisor is the G. C. M. required.

Example.—Find the G. C. M. of 154 and 350.

Solution

```
154)350(2
    308
     42)154(3
        126
         28)42(1
            28
            14)28(2
               28
```

The G. C. M. required is 14. *Ans.*

Verification *

$$\frac{154}{14}=11.\ \frac{350}{14}=25.$$

Here 11 and 25 are *prime to each other*, as they should be.

---

* To find out whether two given numbers are *prime to each other*, we may find their G. C. M. by the method of division. If the G. C. M. so found be 1, the two numbers are *prime to each other*. For example, 323 and 480 will be found to be *prime to each other*.

Exercise 199.

(*A*) Find the G.C M of the following pairs of numbers by the method of Art 215 (*i e* without resolving them into factors). Verify your answers as shown in the above example.

1. 169 and 377.	2. 570 and 225	3 312 and 696
4. 1178 and 465.	5. 234 and 522.	6. 2318 and 915.
7. 1518 and 627	8 1050 and 4032	9. 4223 and 1927.

(*B*) Find by the Division Method the G C M. of the pairs of numbers given in Exercise 197 (*a*).

---

216 *To find the G. C. M of three numbers without resolving them into factors.*

RULE.—Find the G. C. M. of any two of the numbers, then find the G. C. M of this G. C M and the remaining number. The G. C. M. *last* found is the G.C.M. required.

Example.—Find the G.C.M of 208, 176, and 280.

Solution.

```
176)208(1                         Again.      16)280(17
    176                                          16
     32)176(5                                    120
        160                                      112
         16)32(2                                   8)16(2
            32                                       16
```

Thus the G C.M. required is 8. *Ans.*

Verification.

$$\frac{208}{8} = 26\,;\ \frac{176}{8} = 22\,.\ \frac{280}{8} = 35.$$

And 26, 22 and 35 are *prime to one another* as they should be.

Exercise 200.

(A) Find by the method of Art 216 the G C M. of the following groups of numbers and verify your answers —

1. 837, 1134 and 1347.	2. 108, 1116 and 6144.
3. 484 5256 and 128.	4. 378, 651 and 525.
5. 28770, 98574 and 40782.	6. 215441, 81719 and 24871.

(B) Find by the *Division Method* the G.C.M. of the groups of numbers given in Exercise 197 (*b*).

217. G.C.M. of Compound Quantities.

—To find the G.C.M. of *compound quantities*, we must first reduce them to the *lowest* denomination contained in any one of them.

Example—Find the G.C.M. of Re. 1-2-1 and Re. 1-15-0.

Solution.

Re 1-2-1 = 217 pies; and Re 1-15-0 = 372 pies.

The G.C.M of 217 ps. and 372 ps. will be found to be 31 pies.

Hence the G.C.M. required is 31 pies or 2 as 7 pies. *Ans.*

Verification — $\frac{\text{Re } 1\text{-}2\text{-}1}{2 \text{ as } 7 \text{ p}} = 7$, $\frac{\text{Re } 1\text{-}15\text{-}0}{2 \text{ as } 7 \text{ p}} = 12$. And 7 and 12 are *prime to each other*, as they should be.

Exercise 201

Find the G.C.M. of the following quantities and verify your answers:—

1. Rs 7-8-0 and Rs. 17-8-0.
2. Rs 3-12-8 and Rs. 2-0-8.
3. Re. 0-12-9 and Re. 0-14-3.
4. Rs. 60-2-6 and Rs 16-6-6.
5. £1-13-11 and £2-3-1.
6. £4-10-0 and £6-15-0.
7. £1-2-0, £1-7-6 and £2-9-6.
8. 8 hrs 45 min and 20 hrs.
9. 4 mds 7 vis. 2 seers 6 pal. and 7 mds 6 vis. 6 pal.
10. Rs 17-8-0. Rs 27-8-0 and Rs 15-0-0.
11. £0-1-4, £0-2-0, £0-2-8 and £0-3-4.
12. 3 cwt 2 qrs. 4 lb. and 3 cwt 3 qrs. 12 lb.
13. (*a*) 7 cm. 5 mm. and 6 cm; (*b*) 4·2 m. 3·5 m. and 2·1 m.
    (*c*) 21·6 cm., 24·0 cm., and 163·8 cm.
    (*d*) 5·46 m., 6·30 m., 16·38 m.

---

218. Problems on G.C.M.

Model 1—Find the greatest number that will divide 1325, 1649, and 1265, leaving a remainder 5 in each case.

Solution.

In other words the question is, "Find the greatest number that will divide 1325—5, 1649—5 and 1265—5 exactly." Hence the required number is the G.C.M. of 1320, 1644 and 1260, which is found to be 12. *Ans.*

Model 2.—What is the length of the *longest* pole with which you can *exactly* measure 126 feet, 144 feet, and 156 feet?

Solution.

The required length must be the G.C.M. of 126 ft., 144 ft. and 156 ft. which will be found to be 6 ft. Hence the required length is 6 feet. *Ans.*

NOTE.—The student can verify the answer for himself.

Exercise 202.

1. Find the greatest number that will divide 1317, 819 and 8463 without a remainder.

2. Find the *greatest* number that will divide 383 and 311, leaving a remainder 5 in each case.

3. Find the *greatest* number that will divide 315, 427 and 567, leaving a remainder 7 in each case.

4. Find the *greatest* number that will divide 700 and 356, leaving remainders 7 and 3 respectively.

5. What is the *greatest* number that will divide 500, 631 and 910, leaving remainders 5, 6 and 10 respectively?

6. Find the length of the *longest* pole with which you can *exactly* measure the sides of a rectangular room 18 ft. 9 in. long and 12 ft. 6 in. broad.

7. What must be the *greatest* length of a rope with which you can *exactly* measure a kilometre (= 3280 ft.) and a mile (= 5280 ft.)?

8. There are two strings 120 inches and 88 inches long. They are to be cut up into an *exact* number of equal pieces of the *greatest* possible length. What must be the length of each piece? And what is the total number of pieces into which the two strings can be so cut up?

9. What must be the *highest* price of a cow so that you can buy an *exact* number for Rs. 241, or Rs. 301-4 as., or Rs. 361-8 as.

10. What must be the *greatest* length of a pole so that you can *exactly* measure with it the two sides and diagonal of a rectangular plot of ground, which are 42 ft. 55 ft., and 70 ft. respectively?

11. A farmer has 225 marakals of paddy and 165 marakals of rice. He wishes to put them separately in bags of equal size containing the greatest possible number of marakals. How many marakals should each bag hold? And how many such bags will be required for the paddy and rice together?

12. Three pieces of cloth 14, 72 and 28 cubits long are to be cut up into an exact number of smaller pieces of one and the same length. Find the *maximum* length of each of the smaller pieces. Among how many men can all the pieces so obtained be distributed at two to each?

CHAPTER XLIV

LEAST COMMON MULTIPLE.

219. A Common Multiple of two or more numbers is a number which contains each of them an *exact* number of times. For example.

(1) 24 is a *common multiple* of 3, 4, 6, etc.

(2) 80 is a *common multiple* of 4, 5 8, 10. etc.

Exercise 203—(*Graphical*).

Show *graphically*, on squared paper, by stepping with the dividers, that (i) 12 is a *common multiple* of 3 and 4, (ii) 30 is a common multiple of 3, 5, 6.

220. The number of multiples common to two or more numbers is *unlimited*. For example,

(1) The common multiples of 4 and 3 are 12, 24, 36, etc.

(2) The common multiples of 2, 3 and 5 are 30, 60, 90, etc.

221. Least Common Multiple.—The *smallest* of the common multiples of two or more numbers is called their *Least Common Multiple*. For example,

12 is the *L. C. M.* (Least Common Multiple) of 4 and 3, 60 is the *L. C. M.* of 3, 4 and 5.

222 From the examples in Art. 221 above it may be easily inferred that *every common multiple of two or more numbers is a multiple of their L C M.*

This can be shown *graphically* in the case of the three numbers 5, 3 and 2 from the following table of their common multiples:—

(1) Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60	etc.
(2) Com. Mult. of 5 and 3	×	×	15	×	×	30	×	×	45	×	×	60	etc.
(3) Com Mult. of 5, 3 and 2			×			30			×			60	etc.

Exercise 204—(*Oral*).

1. Prove by an example that every common multiple of any two or more numbers is a multiple of their L. C. M.

2. The L C M of 6 and 9 is known to be 18. Name two other common multiples of the same two numbers.

3. The L C M of 3, 4, 5 is given to be 60. Find all the common multiples of the same numbers lying between 150 and 350 and examine whether each of them is divisible by 3, 4, 5.

---

223. To find the **L.C.M.** of 4 and 3 graphically.—

Draw on squared paper two horizontal lines AB and PQ, one below the other, so that A and P are on the same vertical line and on the same side of it Let AB contain 4 small divisions of the paper and PQ, 3. Produce AB and mark points C, D, E, etc. in the produced part so that BC, CD, etc. may each be equal to AB. Similarly produce PQ and mark points R, S, T, etc, on the produced part so that QR, RS etc., may each be equal to PQ You will then find that the L.C M of 4 and 3 is 12.

NOTE —By continuing the above process sufficiently far, you can find the *higher* common multiples of 4 and 3, *viz*, 24, 36, etc.

Exercise 205 —(*Graphical*).

(A) Find *graphically* the L, C M. of—

1. 3 and 5. 2 5 and 4. 3. 4 and 6. 4. 6 and 9.

(B) Find *graphically* the L. C. M. of 2 and 3 and the next three *higher* common multiples,

---

224. *To find the L.C.M. of two or more* prime *numbers or of composite numbers which are given as the product of* **powers of prime** numbers :—

We shall work out a few examples with the remark that the L C M. of several numbers must contain *each* of the *prime* factors occurring in *all* the given numbers.

Examples.

(1) The L.C.M. of 3, 5 and 7 is $\underline{3 \times 5 \times 7}$ or 105, which contains *each* of the given *prime numbers,*

(2) The L.C.M of 2×3 and 3×5 is $\overline{2 \times 3 \times 5}$ or 30, which contains *each* of the given *prime factors* $\overline{2, 3, 5}$.

(3) The L.C.M. of 3^2 and 3^5 is 3^5 which is the *least* of the powers 3^5, 3^6, etc of 3 which contain both 3^2 and 3^5

(4) The L.C.M of 2×3^2, 3×5 and 5^3 is $\underline{2 \times 3^2 \times 5^3}$, which contains *all* the prime factors occurring in the given numbers, *each factor being taken in its highest power*.

Exercise 206.

Find the L.C.M. of—

1. $2, 3, 7.$ 2. $2\times5, 3\times5$ 3. $2^4, 2^3.$
4 $5^2, 5, 5^3.$ 5. $3\times5^2, 2\times5. 2\times3^2.$
6. $2\times7, 3\times7^2, 2\times3^2.$ 7 $3^2\times5, 2^3\times3\times5.$

---

225. From the examples and exercises given above we deduce the following rule for finding the L.C.M of two or more numbers :—

Rule.—Resolve the given numbers into their *prime* factors and for their L. C. M. take the product of *all* the *different* prime factors occurring in them, each prime factor being taken in the *highest* power in which it occurs in *any one* of them.

Example.—Find the L.C M. of 120, 168 and 180.

Solution.

$120 = 2^3\times3\times5$	The L. C. M required
$168 = 2^3\times3\times7$	$= 2^3\times3^2\times5\times7 = 8\times9\times5\times7$
$180 = 2^2\times3^2\times5.$	$= 40\times9\times7 = 360\times7 = 2520.$

Verification:—The quotients obtained by dividing the L C M. of two or more numbers by each of them must be *prime to each other*. Thus in the above example,

$\frac{2520}{120} = 21;\quad \frac{2520}{168} = 15.\quad \frac{2520}{180} = 14;$ and 21, 15, and 14 are *prime to one another*.

Exercise 207.

Find the L. C. M of the following *composite* numbers by resolving them into their *prime factors*, and verify your answers :—

1. 30 and 24. 2. 45 and 54. 3 32 and 36.
4 48 and 72. 5. 35 and 105. 6. 400 and 60.
7. 15, 18 and 24 8. 108, 96 and 84.
9 63, 105 and 189 10. 100, 375 and 90.
11. 12, 15, 18 and 21. 12. 36, 54, 108 and 72.
13. 24, 18, 12 and 8. 14 36, 24, 18 and 9.
15. 245, 70 and 147. 16. 216, 180 and 250.

---

226. *Relation between any two numbers and their G.C.M. and L.C.M.*—The product of any *two* (*not* more) numbers is equal to the product of their G.C.M. and L.C.M.

Illustration.

The G.C.M. of 48 and 104 is 8 and their L.C.M. is 624.

$48 \times 104 = 4992$,

and $8 \times 624 = 4992$.

Exercise 208.

1. Prove the truth of the statement in the above article by taking the following pairs of numbers :—

(1) 24 and 36. 2. 35 and 42. 3. 64 and 80 4 65 and 78

(2) Given that the G.C.M. of 832 and 650 is 26, show that their L.C.M. is 20800.

(3) If the G.C.M. of 781 and 923 is 71, what is their L.C.M.?

(4) If 812 be the L.C.M. of 203 and 116, show that their G.C.M. is 29.

(5) Given that the L.C.M. of 75 and 135 is 675, what is their G.C.M.?

---

227. To find the L.C.M. of two numbers which cannot be readily resolved into factors, divide one of the numbers by their G.C.M. and multiply the other number by this quotient.

Example.—To find the L.C.M. of 1274 and 532.

The G.C.M. of 1274 and 532 is 14.

And $532 \div 14 = 38$.

$\therefore$ the L.C.M. required $= 1274 \times 38 = 48412$. *Ans.*

Verification.—$48412 \div 1274 = 38$, $48412 \div 532 = 91$; and 38 and 91 are *prime to each other*

Exercise 209.

Find the L.C.M. of the following pairs of numbers and verify a few of your answers :—

1. 391, 4633.	2. 1271, 984	3. 351, 1833.
4. 1764, 819.	5. 7200, 2232.	6. 2880, 1725.
7. 3267, 3960.	8. 1296, 900	9. 1428, 9240.
10. 2368, 3200.	11. 3905, 3025.	12. 5625, 4575.

228. The L.C.M. of *several small numbers* is usually found by the following rule:—

Set down the numbers in a line; divide as many of them as possible by any *prime* number, such as 2, 3, 5, 7, 11, &c., which is a common measure of any two or more of them, and set down the quotients and the undivided numbers in a second line; proceed in this way till you get a row of numbers in which no two have a common factor. The continued product of all the divisors and the numbers in the lowest line will be the L.C.M. required.

Example.—To find the L.C.M. of (*a*) 6, 9, 14, 20, 24; (*b*) 20, 24, 28, 42, 18, 21.

(*a*)
2) ~~6~~, 9, 14, 20, 24
2) 9, 7, 10, 12
3) 9, 7, 5, 6
3, 7, 5, 2

∴ the L.C.M.
$= 2\times2\times3\times3\times7\times5\times2$
$= 36\times70 = 2520$. *Ans.*

(*b*)
2) 20, 24, 28, 42, 18, ~~21~~
2) 10, 12, 14, 21, 9
3) 5, 6, ~~7~~, 21, 9
5, 2, 7, 3

∴ the L.C.M.
$= 2\times2\times3\times5\times2\times7\times3$
$= 12\times210 = 2520$. *Ans.*

NOTE 1.—The learner should be careful to use only *prime* numbers as divisors, and to use each prime number as *often* as may be necessary.

NOTE 2.—In any line, every one of the numbers which is exactly contained in another may be struck out. Thus in Example (*a*) above, 6 in the first line is struck out since it divides 24 exactly; and in Example (*b*), 21 in the first line and 7 in the third line are struck out for a similar reason.

Exercise 210.

(*a*) Find the Lowest Common Multiple of—

1. 3, 6, 8, 14, 28 and 32.
2. 15, 18, 20, 32, 12 and 100.
3. 27, 33, 54, 69 and 132.
4. 15, 14, 16 and 18.
5. 4, 6, 9, 15, 18, 20 and 21.
6. 21, 30, 54 and 27.
7. 10, 24, 25, 32 and 45.
8. 20, 12, 15, 18, and 4.
9. 12, 18, 30, 48 and 60.
10. 209, 133, 95 and 57.
11. 16, 40, 44, 48 and 66.
12. 30, 27, 24, 21, 18 and 9.

(*b*) Find the *least* number that is *exactly* divisible by the first seven numbers.

(*c*) Find the *least* number that is *exactly* divisible (*i*) by the first five *even* numbers, (*b*) by the first five *odd* numbers.

229 L.C.M. of Compound Quantities.

Example.—Find the L.C.M. of Re. 1-2-1 and Re. 1-15-0.

Solution

Re. 1-2-1 = 217 pies, Re. 1-15-0 = 372 pies. And the G.C.M. of 217 pies and 372 pies is 31 pies.

$$\text{Hence their L C M} = \frac{\overset{7}{\cancel{217}} \times 372}{\cancel{31}} \text{ pies}$$

$= 7 \times 372$ pies $= 2604$ pies = Rs 13-9-0. *Ans.*

Verification.

$$\frac{\text{Rs. 13-9-0}}{\text{Re. 1-2-1}} = \frac{2604 \text{ pies}}{217 \text{ pies}} = 12 \qquad \frac{\text{Rs. 13-9-0}}{\text{Rs. 1-15-0}} = \frac{2604 \text{ pies}}{372 \text{ pies}} = 7.$$

And 12 and 7 are *prime to each other*

Exercise 211.

Find the L.C M of the compound quantities given in Exercise 201 and verify a few of your answers

230. Problems on L. C. M.

Model 1—Find the least number which, when divided separately by 4, 5, 6 and 11, leaves in each case a remainder 3. What is the next such higher number ?

Solution.

Now the least number that is exactly divisible by 4 5, 6, and 11 is their L.C.M, which is 660.

∴ the required number is 660 + 3 or 663 *Ans.*

The next higher number required = $660 \times 2 + 3$ = 1323. *Ans.*

Model 2.—What is the smallest sum of money for which I can buy an exact number of sheep at Rs. 4-8-0, Rs. 6 or Rs. 7-8-0 ? And how many sheep of each kind can be bought for each such sum ?

Solution.

Here we are required to find the least sum of money which is exactly divisible by each of the quantities Rs 4-8-0, Rs. 6, and Rs. 7-8-0.

Hence the required sum of money is the L.C.M. of Rs. 4-8-0, Rs 6, and Rs 7-8-0 which is Rs. 90.* *Ans.*

---

* To the Teacher.—The student must be made to verify the answer to each of the above and similar examples since by doing so he will get a clear and intelligent grasp of the principles underlying the questions.

Again, the number of sheep of each kind required is 20, 15 and 12 respectively. *Ans.*

Exercise 212.

1. Find four common multiples of 4, 6, 9, and 21.

2 Find the least number of fruits that can be equally divided among 4, 5, 6, or 7 boys

3. Find the least number, which, when divided separately by 3, 4, 5 and 6, leaves in each case a remainder 2.

4 Find three numbers which, when divided separately by 3, 4, 5 and 6, will remainder 2 in each case What is the next higher number of the same kind ?

5 What is the smallest sum of money for which I can buy an exact number of books at Rs 2, Rs. 5, or Rs. 6 each ?

6. What is the smallest sum of money for which I can buy an exact number of pencils at 1 a. 3 p , 1 a., or 10 p each ?

7. What must be the shortest length of a rope, so that I may cut it exactly into pieces either 5 ft. 3 in , 4 ft. 1 in., or 6 ft. long?

8. Find the *least* number which when divided separately by each of the numbers 3 4, 5, 6 and 7, leaves in each case a remainder 2 Also find the next two higher numbers of the same kind

9 Find two numbers lying between 600 and 900 which, when divided separately by each of the numbers 8, 12 and 15 will leave in each case a remainder 6

10 What is the *smallest* number of 4 digits which is exactly divisible by 5, 16 and 24? And what is the *largest* such number?

11 Find the *smallest* number which when increased by 15 will be a common multiple of 18, 30 and 45.

12. Given that an acre = 4840 square yards, and a cawni = 6400 square yards, find the *least* number of square yards that can be divided into an *exact* number of acres or of cawnis.

13 The fore-wheel and hind-wheel of a carriage are respectively 10 feet and 13 feet 4 inches round In what distance will they have made a complete number of revolutions for the first time ? And how many revolutions will each have made in this distance ?

14. Three men A, B and C start together from the same place and run round a circular race-course in the same direction. If A can run the course in 10 minutes, B in 15 minutes, and C in 20 minutes in how many hours aftar starting will they be together again at the starting place ? And how many times will each have gone round the course?

15 Show that the least number of 3 digits that is exactly divisible by 3, 4 and 5 is 120.

CHAPTER XLV.

SOLIDS : SURFACES : LINES : POINTS.

231. The student must revise Arts. 126 and 127, and Exercise 99, before going through the following Exercise:—

Exercise 213 —(*Practical*).

(A) 1. Write in a tabular form the number of (1) *plane surfaces*, (2) *curved surfaces*, (3) *straight edges*, (4) *curved edges*, (5) *plane angles*, and (6) *solid angles* (or *corners*) in each of the following solids :—

(*a*) a cube, (*b*) a cuboid (*c*) a cylinder, (*d*) a cone, (*e*) a sphere, and (*f*) a hemi-sphere.

2. Draw rough free hand sketches of a *cube* and a *cuboid* similar to those given at page 87, and of a *cylinder* and a *cone* similar to those given below.

Cylinder.

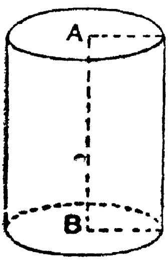


Fig 39.

Cone.

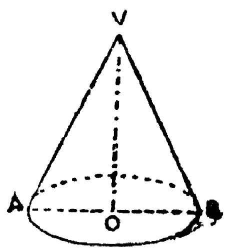


Fig. 40.

3. (*a*) *Test of a plane surface.*—To test whether a surface is plane apply a straight edge to it If it fits the surface in *every direction*, then the surface is plane.

(*b*) *Test of a curved surface.*—Test in the same way the curved surface of a *cylinder* and a *cone*. You will find that the straight edge fits only in some positions but not in all.

In the case of the surface of a sphere, the straight edge does not fit in any position.

4. Note that—

(i) A *solid* has *three* dimensions, namely, *length*, *breadth*, and *thickness* or *height* (ii) a *surface* has *two* dimensions, namely, *length* and *breadth*, (iii) a *line* has only *one* dimension, namely *length* and (iv) a *point* has no dimensions at all, (*i.e.*, has neither length, breadth, nor thickness) but merely denotes *position*

5. Note that—

(*a*) Two *plane* surface meet in a *straight* line.

(*b*) Two surfaces, one of which is *curved* and the other *plane* meet in a *curved* line

6. Note that—

(*a*) A *line* is traced by a *moving point*, a *straight* line being formed if the point moves in one and the same direction throughout and a *curved* line being formed if the point continually changes its direction

(*b*) A *surface* is formed by the motion of a *line* as in Fig. 41 below.

(*c*) A *solid* is formed by the motion of a surface as in Fig. 42 below.

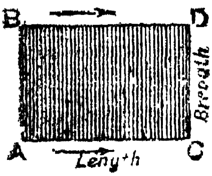


Fig. 41.

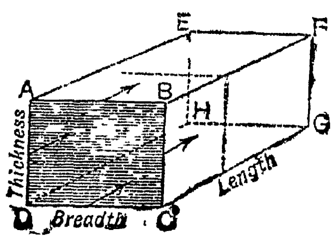


Fig. 42.

7. Note also that—

(*a*) A *cylinder* is generated by the revolution of a *rectangle* round one of its edges (*b*) a *cone* is generated by the revolution of a *right-angled triangle** round one of the two sides containing the right angle, (*c*) a *sphere* † is generated by the revolution of a *semi-circle*† round its *diameter*. [See Figs. 43, 44 and 45 on the next page]

* A *right-angled triangle* is a figure of 3 sides, two of which contain a *right angle*.

† The generation of a *sphere* can be well explained by means of a peeled orange.

Cylinder. *Cone.* *Sphere.*

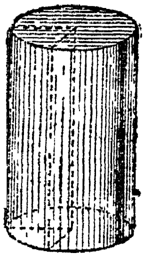


Fig. 43

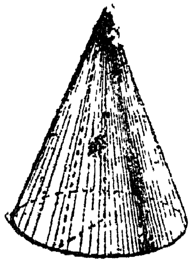


Fig. 44.

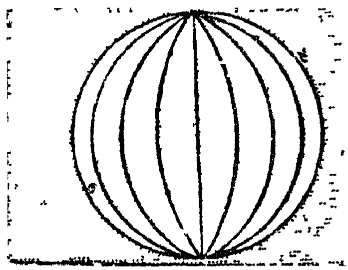


Fig 45.

(B) 1. Make clay models of different sizes of (i) *cubes*, (ii) *cuboids*, (iii) *cylinders*, and (iv) *spheres*.

2. Take 4 lumps of wet clay of the *same* weight and form them into a cube, a cuboid, a cylinder and a sphere respectively.* Note that these four solids are of the *same size* but of *different shapes*.

3. Take two or more lumps of wet clay of different weights, form them all into *spheres* (or all into *cubes*) and note that these solids are of the *same shape* but of *different sizes*.

4. Examine models of a *cuboid* and a *cube*, and note as you have done already, [See Art 127] (i) that both have 6 faces, (ii) that the opposite faces of a cuboid are equal to each other, and (iii) that all the 6 faces of a cube are *equal* squares.

5. Make a clay model of a *cone*, and point out its *height* (vertical height, *i.e.*, distance from the *top* or *apex* to the centre of the base) and its *slant height* (distance from the apex to the circumference of the base as measured by a straight edge applied to the curved surface).

6. Take a model of a *cylinder* of wet clay. Cut it into 3 equal parts Then each of these 3 parts can be formed into a *cone* with the same *base* and *height* as the cylinder. Hence learn that the volume of a *cone* is one-third of the volume of a *cylinder* of the same base and height.

---

* Or a single lump of clay may be taken and formed first into a *cube*, then into a *cuboid*, then into a *cylinder*, and lastly into a *sphere*.

Exercise 214—(*Practical*).

(A) 1. Suppose you take a large number of rectangular sheets of paper all of the same size and pile them one above another so that their sides may all coincide. What kind of solid will be formed thereby?

2. If you pile one above another a number of filter papers of the same size, so that their circumferences may coincide with one another, what kind of solid will be formed?

(B) 1. If you place 20 quires of paper over one another so that tha edges may all coincide, what will be the thickness of the *cuboid* so formed, given that the thickness of a quire is 1/2 inch?

2. A book consists of 600 pages (300 sheets) af paper and two covers Supposing the thickness of each sheet is ·001 inch and of each cover ·05 inch, what is the thickness of the book?

3. Bricks are made in the shape of a *quadrant* of a circle, and arranged in the form of a *cylindrical pillar* How many bricks will be required for a pillar 8 feet high, if each brick is 1 inch thick? And what will be the height of a pillar consisting of 480 such bricks?

CHAPTER XLVI.

VULGAR FRACTIONS.

1. Introductory.

232. Vulgar Fractions.—We have already seen that numbers like ¾ 7/10, 4, 83 are called *fractions*, and that fractions like ¾ and 7/10, which are expressed by two numbers written one above the other are called *Vulgar Fractions* [*vulgar* = common.]

233. Numerator and Denominator.—In a vulgar fraction ¾, the *lower* number 4 which denotes the number of *equal parts* into which the unit is divided is called the *Denominator*, and the *upper* number 3 which denotes the number of such *equal parts* taken is called the *Numerator.*

NOTE.—A *vulgar fraction* is often called a *fraction*, just as a *decimal fraction* is called a *decimal.*

234. Proper and Improper Fractions.—

If we divide the unit into four equal parts and take *one*, *two* or *three* of these parts, we get the fractions 1/4, 2/4 and 3/4, each

of which is *less* than unity: but if we take all the *four* parts, we get the fraction 4/4, which is equal to unity. Again if to these *four* parts, we add *one two* or *three* parts of another unit (of the same kind as the first) which is also divided into *four* equal parts, we get the fractions 5/4, 6/4, 7/4 each of which is *greater* than unity.

A fraction whose numerator is less than its denominator is called a **proper fraction.** A fraction whose numerator is greater than its denominator or equal to its denominator is called an **improper fraction.** Thus ⅔ and 12/27 are *proper fractions*; 7/5, 8/8 are *improper fractions.*

235. Mixed numbers—A number consisting of the sum of an integer and a fraction is called, as the student already knows, a *mixed number.* Thus 5 + 1/7 is a *mixed number* and is generally written $5^1/_7$ or $5\frac{1}{7}$ and read 'five and one seventh.'

Exercise 215.

(A) Express the following fractions in figures.—

1. Three-*eighths*.* 2. Six *twenty-firsts*,
3. One-hundredth, 4. Thirty-one twentieths.
5. Nine-twelfths.* 6 Two and a half.

(B) Express the following fractions in words :—

1. 2/3 2. 7/11. 3. 13/8. 4. 101/1001. 5 $7^{14}/_{38}$.

---

236. To prove that $\frac{3}{4} = 3 \div 4$.

If you divide one rupee into 4 equal parts and take 3 of these parts, you get ¾ of a rupee. Or, if you divide 3 rupees into 4 equal parts, you get for the quotient ¾ rupee. Hence we see that $\frac{3}{4} = 3 \div 4$. The same can be shown graphically thus :

Draw on squared paper a length AB of 4 small divisions and take in it a length AC of 3 divisions. Then AC will denote 3/4 of a unit, AB being taken as the unit.

---

* To the Teacher —The pupil should be warned against saying *three-fourth* instead of *three-fourths*, *two-third* instead of *two-thirds*, and so on.

Now produce AB to D. so that AD may be 3 times AB or equal to 3 units. Then, by stepping along AB lengths equal to AC, it will be easily seen that AC is contained 4 times in AB Hence we see that 3/4 has the same meaning as $3 \div 4$ that is, that a fraction represents the *quotient* of the numerator *divided* by the denominator.

Hence any fraction 5/12 is also read '5 by 12'

Exercise. Prove *graphically* that (1) $3/5 = 3 \div 5$,
(2) $5/8 = 5 \div 8$.

2. Transformation of Vulgar Fractions.

237. Suppose we divide an orange into 4 equal parts, and take 3 of these parts; this is the same thing as dividing the orange into 8 equal parts and taking 6 of these parts. Hence we see that the fractions ¾ and 6/8 are equal to each other.

238. *To show graphically that ½ = 3/6 = 12/24.**

Draw on squared paper a line AB containing 24 small divisions.

Fig. 46.

Let this line represent a unit. Divide it into 2 equal parts at C. Then AC = 1/2 a unit.

Again divide each of these two equal parts into 3 parts The whole line AB is then divided into 6 equal parts and AC = 3/6 of the unint.

Next divide each of these 6 parts into 4 equal parts. Then AC will be found to be equal to 12/24 of the unit.

Hence we see that 1/2 = 3/6 = 12/24.*

Exercise 216—(*Graphical*).

Prove graphically that—

(*a*) 1/2 = 4/8 = 8/16. (*b*) 2/3 = 4/6 = 12/18.

(*c*) 3/7 = 9/21 = 18/42. (*d*) 1/3 = 9/27 = 18/54.

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* To the Teacher — The student may be made to prove this by *paper-folding* and by rectangles divided into a suitable number of equal parts.

239. From Art. 237 above, $\frac{1}{2}=\frac{3}{6}$, but $\frac{3}{6}=\frac{1\times 3}{2\times 3}$;

therefore $\frac{1}{2}=\frac{1\times 3}{2\times 3}$.

Again $\frac{3}{6}=\frac{1}{2}$; but $\frac{1}{2}=\frac{3\div 3}{6\div 3}$; therefore $\frac{3}{6}=\frac{3\div 3}{6\div 3}$

Hence it follows that *the value of a fraction is not altered if the numerator and denominator are both multiplied or both divided by the same number.*

Example 1.—Reduce 8 to a fraction with denominator 5.

$8=\frac{8}{1}=\frac{8\times 5}{1\times 5}=\frac{40}{5}$. *Ans.*

Example 2.—Reduce ¾ to a fraction with denominator 24.

$4)\underline{24}$; $\therefore \frac{3}{4}=\frac{3\times 6}{4\times 6}=\frac{18}{24}$. *Ans.*
$\quad 6$

Example 3.—Reduce 3/4 to a fraction with numerator 24.

$3)\underline{24}$. $\therefore \frac{3}{4}=\frac{3\times 8}{4\times 8}=\frac{24}{32}$. *Ans.*
$\quad 8$

Example 4.—Show that $\frac{24}{40}=\frac{3}{5}$.

$\frac{24}{40}=\frac{3\times 8}{5\times 8}=\frac{3}{5}$ *Ans.*

Exercise 217—(*Oral*).

(a) Reduce the following to equivalent fractions with denominator 24 :—

1. 3/4. 2. 5/12 3. 7/8. 4. 2. 5. 2/3. 6. 5/4.

(b) Reduce the following to equivalent fractions with numerator 48 :—

1. 3/4 2. 6/11. 3. 2. 4 8/15. 5. 16/21 6. 16/15.

(c) Show that :—

1. $\frac{30}{36}=\frac{5}{6}$. 2. $\frac{75}{60}=\frac{5}{4}$. 3. $\frac{16}{100}=\frac{4}{25}$.

(d) Name the missing numbers in the following :—

1. $\frac{4}{5} = \frac{80}{-} = \frac{}{35}$. 2. $\frac{81}{90} = \frac{27}{-}$. 3. $\frac{7}{6} = \frac{}{72} = \frac{91}{-}$.

4. $\frac{11}{15} = \frac{}{225} = \frac{121}{-}$. 5. $\frac{30}{42} = \frac{}{7}$. 6. $\frac{16}{25} = \frac{256}{-} = \frac{}{625}$.

(e) Express 15 in *sevenths* and 23 in *ninths*.

(f) How many *elevenths* are there in 14 ?

---

240. Fractions in their lowest terms.—A fraction whose numerator and denominator are *prime to each other*, *i.e.*, have no common factors but 1, is said to be in its *lowest terms*. Thus the fractions 3/4, 5/13 are in their *lowest terms*; but 6/8 is not in its lowest terms, since 6 and 8 have a common factor 2.

241. Reduction of fractions to their lowest terms.—It follows from Art. 237 that a fraction may be reduced to its *lowest terms* by dividing its numerator and denominator by all their common factors in succession, or by their G.C.M. at once. This process is called *cancelling common factors*.

Example 1.—Reduce 600/750 to *lowest terms*.

Dividing numerator and denominator in succession by 10, 5, 3, we have $\frac{600}{750} = \frac{60}{75} = \frac{12}{15} = \frac{4}{5}$. *Ans.*

Example 2.—Reduce 506/690 to its simplest form.

The G.C.M. of 506 and 690 is 46.

Hence dividing the numerator and denominator by 46, we have

$$\frac{506}{690} = \frac{11}{15} \quad Ans.$$

Exercise 218.—(*Oral*)

Reduce the following fractions to their *lowest terms* :—

1. $\frac{4}{6}$. 2. $\frac{3}{9}$. 3. $\frac{4}{10}$. 4. $\frac{6}{9}$. 5. $\frac{12}{15}$.

6. $\frac{9}{21}$. 7. $\frac{45}{36}$. 8. $\frac{4}{16}$. 9. $\frac{18}{24}$. 10. $\frac{20}{30}$.

6. $\frac{11450}{225}$. 7. $\frac{373428}{3894}$. 8. $\frac{49255}{70}$. 9. $\frac{156990}{285}$. 10 $\frac{423284}{5723}$.

---

244. To reduce two or more fractions to equivalent ones with a common denominator.

Fractions with different denominators can be expressed as fractions with a common denominator by applying Art. 236.

Example 1.—Express $\frac{3}{4}$, $\frac{7}{12}$, $\frac{11}{18}$, with denominator 360.

$360 \div 4 = 90$, $\therefore$ $3/4 = \frac{3 \times 90}{4 \times 90} = \frac{270}{360}$.

$360 \div 12 = 30$; $\therefore$ $7/12 = \frac{7 \times 30}{12 \times 30} = \frac{210}{360}$;

$360 \div 18 = 20$; $\therefore$ $11/18 = \frac{11 \times 20}{18 \times 20} = \frac{220}{360}$.

$\therefore$ the equivalent fractions required are $\frac{270}{360}$, $\frac{210}{360}$, $\frac{220}{360}$. *Ans.*

Example 2.—Reduce 3/8, 7/12, 11/18 to equivalent ones with the *least common denominator.*

The L.C M. of 8, 12 and 18 being 72,

$72 \div 8 = 9$; $\therefore$ $\frac{3}{8} = \frac{3 \times 9}{8 \times 9} = \frac{27}{72}$;

$72 \div 12 = 6$, $\therefore$ $\frac{7}{12} = \frac{7 \times 6}{12 \times 6} = \frac{42}{72}$;

$72 \div 18 = 4$; $\therefore$ $\frac{11}{18} = \frac{11 \times 4}{18 \times 4} = \frac{44}{72}$;

Hence the equivalent fractions are $\frac{27}{72}$, $\frac{42}{72}$, $\frac{44}{72}$. *Ans.*

NOTE 1.—The common denominator may be any common multiple of the original denominator, but we generally take their L C M. in order to avoid large numbers.

NOTE 2.—If the given fractions be not in their lowest terms, we should first bring them to their lowest terms and then reduce them to equivalent ones with the least common denominator, as in the following example :—

Example—Reduce 5/12, 10/15, 39/104 to equivalent fractions with the lowest common denominator.

The given fractions in their lowest terms are 5/12, 2/3, 3/8.

Now the L. C.M. of 12, 3, 8 is 24

Hence the equivalent fractions are 10/24, 16/24, 9/24. *Ans.*

Exercise 223.

(*a*) Find fractions equal to 3/8, 5/12, 11/16 whose denominator shall be 960.

(*b*) Reduce 3/8, 3/42, 10/21 to equivalent fractions whose numerator shall be 120.

(*c*) Express 3/4, 7/8, 5/6 as fractions with denominator 216.

(*d*) Reduce the fractions in each of the following sets to equivalent ones having the *least* common denominator:—

1. $\frac{2}{3}, \frac{3}{5}, \frac{7}{10}$. 2. $\frac{8}{11}, \frac{13}{22}, \frac{3}{10}$. 3. $\frac{3}{4}, \frac{5}{8}, \frac{7}{12}$.

4. $\frac{8}{15}, \frac{7}{10}, \frac{11}{20}$. 5. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$. 6. $\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}$.

7. $\frac{2}{4}, \frac{4}{6}, \frac{24}{32}$. 8. $\frac{49}{56}, \frac{3}{14}, \frac{45}{105}, \frac{51}{112}$. 9. $\frac{9}{16}, \frac{17}{33}, \frac{11}{24}, \frac{5}{12}$.

---

245. Two fractions may be compared by multiplying the numerator of each by the denominator of the other and comparing the products, because these products are the numerators of fractions having a common denominator and equivalent to the given fractions.

Example—(1) Compare $\frac{4}{6}$ and $\frac{7}{9}$. (2) $\frac{5}{8}$ and $\frac{7}{12}$.

(1) $4 \times 9 = 36$; $6 \times 7 = 42$, ∴ 7/9 is the greater.

(2) $8 \times 7 = 56$; $12 \times 5 = 60$; ∴ 5/8 is the greater.

Exercise 224.—(*Oral*).

Find the greater in each of the following pairs of fractions:—

1. 3/4 2/3. 2 7/8, 5/6. 3. 2/5, 6/8. 4. 2/6, 4/10.
5. 1/3, 1/4. 6. 7/12, 2/3. 7. 3/8, 6/12. 8. 2/5, 5/21.

---

3. Comparison of Vulgar Fractions.

246. Of two fractions having a **common denominator** the greater is that which has the greater **numerator.**

Hence we can easily find which is the larger and which the smaller of two fractions, when the denominators of the fractions are the same. Thus 5/7 is clearly greater than 3/7.

Where the denominators are not the same, as for example, in comparing 3/7 and 4/5, we have first to express the fractions as fractions with the same number (preferably their L. C. M.) for denominators, and then compare them. Thus, the L. C. M. of 7 and 5 being 35, we have

$$\frac{3}{7}=\frac{3\times 5}{7\times 5}=\frac{15}{35} \text{ and } \frac{4}{5}=\frac{4\times 7}{5\times 7}=\frac{28}{35}.$$

And as $\frac{28}{35}$ is greater than $\frac{15}{35}$ so $\frac{4}{5}$ is greater than $\frac{3}{7}$.

Similarly we can compare three or more quantities as in the following

Example.—Arrange the fractions 3/4, 5/6, 2/3 in descending order of magnitude.

The L. C. M. of 4, 6, 3 being 12,

$12 \div 4 = 3$; $\therefore \frac{3}{4}=\frac{3\times 3}{4\times 3}=\frac{9}{12}$;

$12 \div 6 = 2$; $\therefore \frac{5}{6}=\frac{5\times 2}{6\times 2}=\frac{10}{12}$.

$12 \div 3 = 4$; $\therefore \frac{2}{3}=\frac{2\times 4}{3\times 4}=\frac{8}{12}$.

Hence the descending order of magnitude is

$\frac{10}{12}, \frac{9}{12}, \frac{8}{12}$;

i.e., $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}$ *Ans.*

NOTE 1.—The comparison of fractions may be graphically shown by means of diagrams like that under Art. 237 where, for example, it is clear that 3/4 is less than 5/6, 2/3 is greater than 1/2, &c.

NOTE 2.—Fractions to be compared may, for the sake of convenience, be brought to their *lowest terms* before being reduced to a common denominator as in the following

Example.—Compare the values of 9/15, 8/9, 49/70.

The fractions in their lowest terms are 3/5, 8/9, 7/10.

Now the L. C. M. of 5, 9, 10 is 90.

$\therefore \frac{3}{5}=\frac{3\times 18}{5\times 18}=\frac{54}{90}$;

$\frac{8}{9}=\frac{8\times 10}{9\times 10}=\frac{80}{90}$;

$\frac{7}{10}=\frac{7\times 9}{10\times 9}=\frac{63}{90}$.

$\therefore$ $\frac{80}{90}$ or $\frac{8}{9}$ is the *greatest*,

$\frac{63}{90}$ or $\frac{49}{70}$ is the next, and

$\frac{54}{90}$ or $\frac{9}{15}$ is the *least*.

Exercise 225.

Find the greatest and the least in each of the following sets of fractions :—

1. $\frac{4}{5}, \frac{3}{4}, \frac{2}{3}.$ 2. $\frac{2}{8}, \frac{3}{10}, \frac{2}{15}, \frac{7}{20}.$ 3. $\frac{6}{14}, \frac{9}{28}, \frac{10}{16}.$

4. $\frac{7}{24}, \frac{1}{4}, \frac{6}{20}.$ 5. $\frac{1}{6}, \frac{7}{44}, \frac{23}{132}, \frac{2}{11}.$ 6. $\frac{19}{144}, \frac{12}{12}, \frac{3}{2}, \frac{5}{36}.$

Arrange each of the following sets of fractions (*a*) in *descending* order of magnitude, (*b*) in *ascending* order of magnitude :—

7. $\frac{5}{6}, \frac{11}{12}, \frac{1}{18}.$ 8. $\frac{2}{3}, \frac{5}{6}, \frac{17}{24}, \frac{12}{16}.$ 9. $\frac{3}{10}, \frac{1}{4}, \frac{3}{8}.$

10. $\frac{7}{12}, \frac{2}{3}, \frac{1}{6}.$ 11. $\frac{1}{3}, \frac{2}{8}, \frac{5}{12}, \frac{11}{16}.$ 12. $\frac{30}{50}, \frac{2}{3}, \frac{7}{15}, \frac{5}{12}.$

Compare the fractions in each of the following sets :—

13. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}.$ 14. $\frac{2}{4}, \frac{6}{8}, \frac{10}{12}.$ 15. $\frac{16}{24}, \frac{3}{5}, \frac{35}{56}.$

16. $\frac{7}{11}, \frac{6}{10}, \frac{7}{12}.$ 17. $\frac{8}{7}, \frac{9}{8}, \frac{7}{6}.$ 18. $\frac{5}{6}, \frac{6}{7}, \frac{7}{8}.$

4. Addition of Vulgar Fractions.

247 (*a*). The sum of fractions having a *common denominator* is a fraction of which the numerator is the sum of their numerators, and the denominator the common denominator.

Example.—Add together 5/7 and 3/7.

5/7 = 5 *sevenths.*	∴ 5/7+3/7=(5+3) *sevenths*
and 3/7 = 3 *sevenths,*	= 8 *sevenths* = 8/7 *Ans.*

Thus it is clear that fractions having the same denominator can easily be added.

(*b*) When the denominators of the fractions to be added are *different*, the addition cannot be effected directly,* since

* **To the Teacher.**—This might be illustrated as follows:—If we cut one orange into 4 equal parts and another into 5 equal parts, and are required to find the sum of 3 parts of the former and 4 of the latter, we can only say that there are 7 pieces altogether and cannot specify what portion of one orange the total is.

there will be little or no meaning in adding *unlike* things—fractions not expressed in terms of the *same* denomination. In such a case, the method to be followed is to make the fractions *like* by expressing them as fractions with a *common* denominator, and then add them as in the Example above.

Example 1.—Add 3/7 and 2/5.

Since the L. C.M. of 7 and 5 is 35,

$$\frac{3}{7}=\frac{3\times5}{7\times5}=\frac{15}{35}\text{; and }\frac{2}{5}=\frac{2\times7}{5\times7}=\frac{14}{35}$$

$\therefore$ 3/7 + 2/5 = 15/35 + 14/35 = 29/35. *Ans.*

Example 2.—Add together 2/3, 3/4, 4/5.

Since the L C.M of 3, 4, 5 is 60,

$$\frac{2}{3}=\frac{2\times20}{3\times20}=\frac{40}{60};\qquad \frac{2}{3}+\frac{3}{4}+\frac{4}{5}$$

$$\frac{3}{4}=\frac{3\times15}{4\times15}=\frac{45}{60};\qquad =\frac{40+45+48}{60}$$

$$\frac{4}{5}=\frac{4\times12}{5\times12}=\frac{48}{60}\qquad =\frac{133}{60}=2\frac{13}{60}.\quad \textit{Ans.}$$

NOTE.—The addition of 3/7 + 2/5 may be shown *graphically* thus :—Take two contiguous rectangles containing 35 squares each to represent *two* units as in Fig. 47 and let the shaded

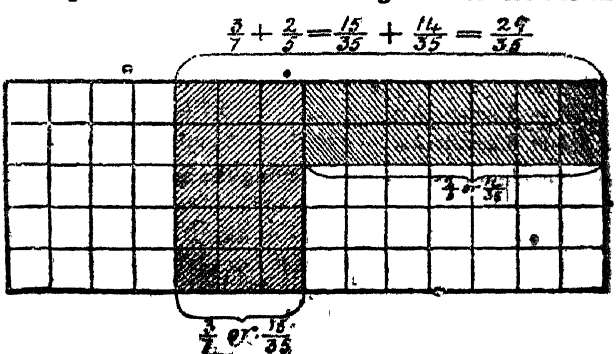

Fig 47.

portion of one of them (consisting of 3 columns of squares out of 7) denote 3/7 of a unit and the shaded portion of the other (con-

sisting of 2 rows of squares out of 5) denote 2/5 of a unit. It is clear that the two portions together contain $15 + 14$ squares or 29 squares out of the 35 squares in the unit. Hence it follows that $3/7 + 2/5 = 29/35$

Exercise 226.

Find the sum of each of the following groups of fractions and prove your result *graphically*:—

1. 2/5 and 1/7. 2. 3/8 and 1/4. 3. 3/10 and 13/20. 4. 2/8 and 1/6.

Exercise 227.

Perform the following additions:—

1. $\frac{3}{4} + \frac{1}{3}$. 2. $\frac{3}{8} + \frac{11}{16}$. 3. $\frac{1}{3} + \frac{2}{5}$. 4. $\frac{7}{12} + \frac{2}{5}$.

5. $\frac{5}{12} + \frac{2}{3}$. 6. $\frac{3}{10} + \frac{7}{15}$. 7. $\frac{4}{9} + \frac{5}{12}$. 8. $\frac{2}{3} + \frac{1}{6}$.

9. $\frac{4}{9} + \frac{5}{6} + \frac{3}{4}$. 10. $\frac{7}{14} + \frac{1}{3} + \frac{3}{4} + \frac{4}{5}$. 11. $\frac{3}{4} + \frac{4}{5} + \frac{5}{6}$.

12. $\frac{1}{2} + \frac{3}{4} + \frac{5}{6}$. 13. $\frac{3}{10} + \frac{13}{15} + \frac{1}{5} + \frac{4}{9}$. 14. $\frac{5}{14} + \frac{4}{12} + \frac{3}{7}$.

Exercise 228—(*Oral*).

1. $\frac{3}{4} + \frac{1}{3}$. $\frac{1}{6} + \frac{1}{8}$ 3. $\frac{2}{3} + \frac{1}{12}$. 4. $\frac{2}{3} + \frac{4}{9}$.

5. $\frac{5}{12} + \frac{3}{4}$. 6. $\frac{7}{6} + \frac{11}{18}$. 7. $2\frac{1}{2} + \frac{1}{6}$ 8. $1\frac{1}{4} + 2\frac{1}{3}$.

---

248. (*a*) The sum of mixed numbers may be obtained by finding (1) the sum of the integers and (2) the sum of the proper fractions in the mixed numbers, and adding the two sums together.

Example.—Find the value of $2\frac{1}{4} + 3\frac{2}{3} + 1\frac{5}{6}$.

$$2\tfrac{1}{4} + 3\tfrac{2}{3} + 1\tfrac{5}{6} = 2 + 3 + 1 + \tfrac{1}{4} + \tfrac{2}{3} + \tfrac{5}{6}$$

$$= 6 + \frac{3 + 8 + 10}{12} = 6 + \frac{21}{12} = 6 + 1\frac{9}{12} = 7\tfrac{3}{4}. \textit{ Ans.}$$

(*b*) Improper fractions may be reduced to mixed numbers, and proper fractions to their lowest terms.

Example.— Simplify $\frac{11}{6} + \frac{10}{8}$.

$$\frac{11}{6} + \frac{10}{8} = 1\frac{5}{6} + 1\frac{2}{8} = 1\frac{5}{6} + 1\frac{1}{4}$$

$$= 1 + 1 + \frac{5}{6} + \frac{1}{4} = 1 + 1 + \frac{10 + 3}{12}$$

$$= 2 + \frac{13}{12} = 2 + 1\frac{1}{12} = 3\frac{1}{12}. \quad \textit{Ans.}$$

Exercise 229.

(*a*) Find the value of—

1. $2 + 4\frac{1}{2} + \frac{1}{3}$.
2. $\frac{1}{3} + 2\frac{1}{6} + \frac{3}{4}$.
3. $3\frac{3}{4} + 2\frac{2}{3} + 8 + \frac{1}{12}$.
4. $7\frac{1}{4} + 6\frac{1}{6} + \frac{4}{15}$.
5. $1\frac{1}{7} + \frac{3}{14} + \frac{3}{12} + 3$.
6. $5\frac{1}{6} + 12\frac{1}{4} + 25\frac{7}{12}$.
7. $1\frac{1}{7} + 2\frac{1}{2} + \frac{5}{14}$.
8. $1\frac{2}{3} + 3\frac{1}{5} + 5\frac{1}{15}$.
9. $3\frac{13}{21} + 4\frac{6}{7} + 5\frac{4}{9}$.
10. $7\frac{1}{15} + 3\frac{7}{30} + 1\frac{11}{45}$.
11. $11\frac{2}{13} + 5\frac{7}{39} + 6\frac{2}{65} + 4\frac{3}{52}$.
12. $5\frac{1}{6} + 6\frac{7}{12} + 3\frac{5}{9} + 4\frac{17}{24}$.
13. $6\frac{1}{19} + 5\frac{13}{57} + 3\frac{18}{19} + 4\frac{16}{95}$.
14. $5\frac{1}{21} + 6\frac{33}{35} + 3\frac{17}{28} + 5\frac{1}{7}$.

(*b*) 1. What number exceeds $8\frac{6}{85}$ by $7\frac{5}{34}$?

2. Find the sum of the greatest and least of the fractions 7/18, 9/20 and 11/24.

3. The difference between two numbers is $109\,^3/_{14}$. If the *smaller* number is $30\,^{11}/_{28}$, what is the larger number?

(*c*) 1. If $x = 10\,^3/_{10}$, $y = 15\,^7/_{15}$, $z = 30\,^7/_{30}$, find the value of (i) $x + y$, (ii) $y + z$, (iii) $z + x$, (iv) $x + y + z$.

2. Find the value of x when $x - 10\,^{17}/_{35} = 7\,^{25}/_{42}$.

(*d*) 1. A has 1/3 of an estate, B has 1/6 of it, and C 3/8; what part of the estate have A, B and C together?

2. A owns 1/7 of a garden, B owns 3/14 more than A, and C has as much as A and B together. How much of the estate have all the three together?

5. Subtraction of Vulgar Fractions.

249. The *subtraction* of vulgar fractions is performed by the same method as the addition of vulgar fractions.

(a) The *difference* of fractions having a *common denominator* is a fraction of which the numerator is the difference of their numerators, and the denominator the common denominator.

Example.—Subtract 3/7 from 5/7.

5/7 = 5 sevenths;	$\therefore$ *5/7 — 3/7 = (5—3) sevenths*
and *3/7 = 3 sevenths.*	*= 2 sevenths = 2/7. Ans.*

(b) To find the difference of fractions whose denominators are *not* the same, we have first to express the given fractions as fractions with a *common denominator*, and then proceed with the subtraction as in the example given above.

Example 1.—Subtract 3/4 from 7/6.

Since the L.C.M. of 6 and 4 is 12,

$$\frac{7}{6}=\frac{7\times 2}{6\times 2}=\frac{14}{12},\qquad \therefore \frac{7}{6}-\frac{3}{4}=\frac{14}{12}-\frac{9}{12}$$

$$\frac{3}{4}=\frac{3\times 3}{4\times 3}=\frac{9}{12}.\qquad =\frac{5}{12}.$$

The subtraction of 3/4 from 7/6 can be graphically shown thus:—

In the following figure let the rectangle ABCD containing 6 rows of two squares each denote the unit or 6/6. Then AFED will denote 7/6 of the unit, and the shaded squares 9 in number will denote 9/12 or 3/4 of the unit. Hence the remaining 5 squares denote the difference obtained by subtracting 3/4 from 7/6 And as it denotes 5/12 of the unit, it follows that 7/6 — 3/4 = 5/12.

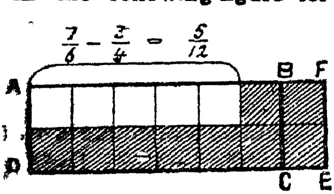

Fig. 48.

* *Example* 2. Find the value of $1 - \frac{3}{8}$.

$$1 - \frac{3}{8} = \frac{1}{1} - \frac{3}{8} = \frac{8}{8} - \frac{3}{8} = \frac{8-3}{8} = \frac{5}{8}. \quad \textit{Ans.}$$

Exercise 230.

Find the value of each of the following expressions and prove a *few* of your answers *graphically* :—

1. 5/8 — 3/16. 2. 5/4 — 7/8. 3. 5/6 — 7/12.
4. 3/4 — 1/5. 5. 4/5 — 3/4. 6. 2/3 — 3/7.
7. 7/5 — 2/3. 8. 7/12 — 1/4. 9. 3/4 — 4/9.
10. 3/2 — 4/5. 11. 1 — 7/16. 12. 2 — $1\frac{1}{5}$.

Exercise 231—*(Graphical)*.

1. Look at figure 49 below and find the value of $\frac{3}{4} - \frac{1}{6}$, 1/2 + 5/12, $\frac{1}{3} + \frac{1}{4} + \frac{1}{6}$, $\frac{2}{3} - \frac{1}{4}$, etc.

2. Taking suitable diagrams, find the value of 1/2 + 1/3, $\frac{2}{3} - \frac{1}{6}$, 1/2 + 1/5, etc.

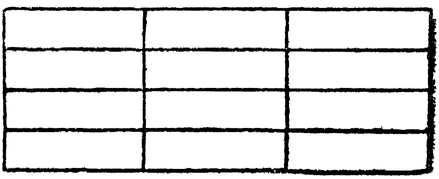

Fig. 49.

Example 3.—To find the value of $4\frac{1}{2} - 2\frac{1}{6}$.

$$4\tfrac{1}{2} - 2\tfrac{1}{6} = 4 + \tfrac{1}{2} - 2 - \tfrac{1}{6} = 4 - 2 + \tfrac{1}{2} - \tfrac{1}{6} = 2 + \frac{3-1}{6} = 2 + \tfrac{2}{6} = 2\tfrac{1}{3}. \quad \textit{Ans.}$$

Otherwise thus :—

$$4\tfrac{1}{2} - 2\tfrac{1}{6} = \tfrac{9}{2} - \tfrac{13}{6} = \frac{27-13}{6} = \tfrac{14}{6} = \tfrac{7}{3} = 2\tfrac{1}{3}. \quad \textit{Ans.}$$

---

* To the Teacher.—The student should be well drilled in *oral* exercises like the following :—1—$\frac{1}{8}$, 1—1/15, 1—1/39, 1—5/12, 1—8/25, 1—75/119, 2—1/14, 3—3/16, 3—25/49, 4—$2\frac{1}{5}$, 8—$3\frac{2}{8}$; and so on.

Example 4.—To find the value of—

(*a*) $\frac{1}{2} + {}^{1}/_{6} - \frac{2}{3}$. (*b*) ${}^{25}/_{4} - 2{}^{13}/_{16} + {}^{10}/_{32}$.

(*a*) $\frac{1}{2} + {}^{1}/_{6} - \frac{2}{3}$

$= \dfrac{3 + 1 - 4}{6}$

$= {}^{0}/_{6} = 0.$ *Ans.*

Note.—*Cipher* divided by any number is *cipher*. (See note under Art 87.)

(*b*) ${}^{25}/_{4} - 2{}^{13}/_{16} + {}^{10}/_{32}$

$= 6\frac{1}{4} - 2{}^{13}/_{16} + {}^{5}/_{16}$

$= 4 + \dfrac{4-13+5}{16} = 4 + \dfrac{9-13^{*}}{16}$

$= 3 + 1\dfrac{9-13}{16}$

$= 3 + \dfrac{16+9-13}{16} = 3{}^{12}/_{16} = 3\frac{3}{4}$ *Ans.*

Exercise 232.

(*a*) Simplify the following expressions:—

1. $3{}^{2}/_{5} - 2{}^{1}/_{7}$.
2. $3\frac{1}{4} - \frac{3}{4}$.
3. $3\frac{1}{3} - {}^{5}/_{6}$.
4. $17\frac{1}{4} - 12{}^{3}/_{5}$.
5. $2{}^{1}/_{5} - 1\frac{7}{8}$.
6. $3\frac{1}{2} - 1{}^{3}/_{5}$.
7. ${}^{5}/_{6} - \frac{1}{3} - {}^{5}/_{18}$.
8. $\frac{1}{4} - \frac{1}{8} + {}^{5}/_{32} + {}^{3}/_{32}$.
9. ${}^{3}/_{2} + \frac{3}{4} - {}^{19}/_{12}$.
10. $\frac{3}{4} - {}^{7}/_{12} + {}^{5}/_{6} - 1$.
11. ${}^{3}/_{5} - \frac{1}{2} - {}^{1}/_{10}$.
12. $\frac{1}{4} - {}^{1}/_{7} + {}^{5}/_{28}$.
13. ${}^{9}/_{4} + 1\frac{1}{8} - 3{}^{1}/_{6}$.
14. $2\frac{1}{2} + 3 - 4\frac{3}{4} + {}^{13}/_{4}$.
15. $3\frac{1}{4} + 13{}^{5}/_{6} + 17{}^{1}/_{12}$.
16. $13{}^{1}/_{7} - {}^{25}/_{6} - {}^{13}/_{4}$.
17. $2{}^{3}/_{10} - 10{}^{2}/_{15} + 8{}^{1}/_{12}$.
18. $\frac{1}{2} - 2{}^{3}/_{20} + {}^{17}/_{5}$.
19. ${}^{9}/_{8} + {}^{9}/_{4} - {}^{1}/_{6} - 3$.
20. $\frac{1}{3} - {}^{2}/_{27} + {}^{25}/_{54}$.
21. $4{}^{17}/_{36} - 5\frac{1}{2} + 1{}^{1}/_{9}$.

(*b*) 1. Take away the sum of $7\frac{1}{2}$, $3\frac{1}{4}$, and $6\frac{3}{4}$ from 100.

2. From the sum of $3\frac{1}{2}$, $2\frac{1}{3}$, and $5{}^{1}/_{6}$, take away the difference of $17\frac{1}{8}$ and $20{}^{3}/_{64}$.

---

* We cannot subtract 13 from 9; and so we take 1 from 4 and add it to $\dfrac{9-13}{16}$, getting $\dfrac{16+9-13}{16}$.

3. By how much does the sum of 3 and $2\frac{1}{4}$ exceed their difference?

4. Find the fraction which, when added to the sum of 3/4, 2/3 and 3/5 will produce 4.

5. The sum of two fractions is 30 and their difference is $2^{14}/_{15}$. Find the two fractions.

(c) 1. A has 3/8 of an estate, B has 1/12 of it, and C has the remainder. What fraction of the estate has C?

2. John has Rs. $4^5/_6$ and Samuel has Rs $3^1/_2$. If John gives to Samuel Re. $1^1/_4$ and Samuel gives to John Re. $1^7/_{12}$, how much has each of them now?

3. A man's age is $35^1/_2$; his wife is $7^3/_8$ years younger; and his son is $26^5/_6$ years younger than his wife. What is the son's age? And what is the sum of the ages of the three?

4. 1/3 of a garden belongs to A, 2/7 to B, and the remainder to C. How much of the garden has C more than A?

(d) 1. If $x = 130^5/_{18}$, $y = 76^{11}/_{27}$, $z = 49^7/_{36}$, find the value of (i) $x - y$, (ii) $y - z$; (iii) $x + y - z$.

2. Find the value of x when $x + 17^7/_{17} = 34^3/_{34}$.

---

250. To find *mentally* the sum or difference of two fractions having small numerators and denominators, multiply the numerator of each by the denominator of the other. The sum or difference of these products will be the numerator of the required fraction, and the product of the original denominators will be its denominator.

Example.—To find the sum and the difference of —

(a) 5/8 and 7/12. (b) 15/8 and 7/5.

(a) $\frac{5}{8} + \frac{7}{12} = \frac{60 + 56}{8 \times 12}$

$= \frac{116}{96} = \frac{29}{24} = 1\frac{5}{24}$. *Ans.*

Again, $\frac{5}{8} - \frac{7}{12} = \frac{60 - 56}{8 \times 12}$

$= 4/96 = 1/24$. *Ans.*

(b) $\frac{15}{8} + \frac{7}{5} = \frac{75 + 56}{8 \times 5}$

$= \frac{131}{40} = 3\frac{12}{40}$. *Ans.*

Again, $\frac{15}{8} - \frac{7}{8} = \frac{75 - 56}{8 \times 5}$

$= 19/40$. *Ans.*

Exercise 233—(*Oral*).

Find (*a*) the sum, (*b*) the difference of—

1. $\frac{3}{4}$ and $\frac{7}{10}$. 2. $\frac{1}{2}$ and $\frac{1}{6}$. 3. $\frac{7}{3}$ and $\frac{4}{9}$. 4. $\frac{4}{3}$ and $\frac{3}{4}$.

5. $\frac{5}{7}$ and $\frac{7}{12}$. 6. $\frac{7}{12}$ and $\frac{3}{10}$. 7. $\frac{3}{7}$ and $\frac{7}{3}$. 8. $\frac{5}{4}$ and $\frac{4}{5}$.

251. Simplify (*a*) $2\frac{1}{2} - (\frac{3}{4} - \frac{1}{3}) + \frac{1}{6}$;
(*b*) $2\frac{1}{2} - (\frac{3}{4} - \frac{1}{3} + \frac{1}{6})$.

(*a*)
$$2\frac{1}{2} - (\frac{3}{4} - \frac{1}{3}) + \frac{1}{6}$$
$$= 2\frac{1}{2} - (\frac{9-4}{12}) + \frac{1}{6}$$
$$= 2\frac{1}{2} - \frac{5}{12} + \frac{1}{6}$$
$$= 2 + \frac{6-5+2}{12}$$
$$= 2\frac{3}{12} = 2\frac{1}{4}.\quad \textit{Ans.}$$

(*b*)
$$2\frac{1}{2} + (\frac{3}{4} - \frac{1}{3} + \frac{1}{6})$$
$$= 2\frac{1}{2} = \frac{9-4+2}{12}$$
$$= 2\frac{1}{2} - \frac{7}{12}$$
$$= 1\frac{3}{2} - \frac{7}{12}$$
$$= 1 + \frac{18-7}{12} = 1\frac{11}{12}.\quad \textit{Ans.}$$

Exercise 234.

Simplify the following expressions:—

1. $\frac{7}{12} - (\frac{3}{16} + \frac{1}{8})$.
2. $\frac{7}{12} - (\frac{3}{16} - \frac{1}{8})$.
3. $12\frac{1}{2} - (8\frac{1}{3} - \frac{5}{12})$.
4. $100 - (25\frac{13}{15} + 5\frac{7}{20})$.
5. $2\frac{1}{3} - \frac{3}{4} - (\frac{1}{8} + \frac{1}{6})$.
6. $(\frac{1}{4} + \frac{3}{7}) - (\frac{1}{3} + \frac{1}{6})$.
7. $(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}) - \frac{7}{12}$.
8. $\frac{1}{2} - (\frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{15})$.
9. $(\frac{1}{2} - \frac{1}{3}) - (\frac{1}{4} - \frac{1}{5})$.
10. $2\frac{1}{2}(\frac{1}{3} + \frac{1}{2} + \frac{1}{6})$.
11. $\frac{1}{5} - (\frac{1}{2} - \frac{1}{3} - \frac{1}{6})$.
12. $3\frac{1}{4} - (1\frac{1}{2} - \frac{3}{8}) + \frac{3}{8}$.

6. Multiplication of Vulgar Fractions by Integers.

252. (1) To multiply a proper fraction $\frac{3}{5}$ by a whole number 3, is to find the sum of $\frac{3}{5}$ repeated 3 times. Thus—

$$\frac{3}{5} \times 3 = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{3 \times 3}{5} \text{ or } \frac{9}{5}.$$

Hence, to multiply a proper fraction by a whole number, we multiply the numerator by the whole number and retain the denominator unchanged.

(2) Similarly, to multiply an improper fraction by a whole number, we multiply the numerator by that number and let the denominator remain unchanged.

Example.—Multiply $\frac{25}{8}$ by 6.

$$\frac{25}{8} \times 6 = \frac{25 \times 6}{8} = \frac{25 \times 2 \times 3}{2 \times 4}$$

$$= \frac{25 \times 3}{4} = \frac{75}{4} = 18\frac{3}{4} \quad \textit{Ans.}$$

253. To multiply a mixed number by a whole number, we may reduce the mixed number to an improper fraction and multiply, or we may multiply the integral and fractional parts of the mixed number separately and add the two products.

Example.—Multiply $4\frac{1}{6}$ by 8.

$$4\frac{1}{6} \times 8 = \frac{25}{8} \times 8$$

$$= \frac{25 \times 8}{6} = \frac{25 \times 4}{3} = \frac{100}{3}$$

$= 33\frac{1}{3}$. *Ans.*

Or thus:—

$4 \times 8 = 32$;

$$\frac{1}{6} \times 8 = \frac{1 \times 8}{6} = \frac{4}{3} = 1\frac{1}{3}.$$

$\therefore 4\frac{1}{6} \times 8 = 32 + 1\frac{1}{3} = 33\frac{1}{3}$. *Ans.*

ercise 235.

(A) Multiply—

1. 7/12 separately by (*a*) 6, (*b*) 8, (*c*) 12.
2. 4/15 separately by (*a*) 10, (*b*) 20, (*c*) 30.

(B) Multiply (by two methods)—

1. $2\frac{1}{3}$ separately by (*a*) 4, (*b*) 6, (*c*) 9, (*d*) 19.
2. $12\frac{2}{5}$ separately by (*a*) 5, (*b*) 6, (*c*) 12.
3. $40\frac{71}{440}$ separately by (*a*) 44, (*b*) 220, (*c*) 440.

(C) Find, *correct to the unit*, the following products, by using the easier of the two methods available —

1. $234\frac{17}{35} \times 11$.   2. $308\frac{12}{49} \times 7$.   3. $1208\frac{3}{35} \times 15$.

4. $2054\frac{11}{34} \times 17$.   5. $750\frac{1}{267} \times 89$.   6. $625\frac{19}{256} \times 16$.

---

7. Division of Vulgar Fractions by Integers.

254. If we take half an orange and divide it into two equal parts, each of the parts is one-fourth of an orange. Again, if we take one-fourth of an orange and divide it into three equal parts, each of the parts is one-twelfth of an orange. Thus,

(1) $\frac{1}{2} \div 2 = \frac{1}{4}$; *i.e.*, $\frac{1}{2 \times 2}$.   (2) $\frac{1}{4} \div 3 = \frac{1}{12}$: *i.e.*, $\frac{1}{4 \times 3}$

From the above and other similar instances, we get the following rule:—

To divide a proper or improper fraction by an integer, multiply the denominator by the integer and let the numerator remain unchanged.

Example.—To divide (*a*) 3/5 by 4; (*b*) 125/4 by 15.

(*a*) $3/5 \div 4$

$= \frac{3}{5 \times 4}$

$= 3/20$. *Ans.*

(*b*) $125/4 \div 15$.

$= \frac{125}{4 \times 15} = \frac{5 \times 5 \times 5}{4 \times 3 \times 5}$

$= \frac{5 \times 5}{4 \times 3} = 25/12 = 2^1/_{12}$. *Ans.*

255. To divide a mixed number by an integer, we may reduce the mixed number to an improper fraction and divide.

Example.—Divide $16\frac{1}{4}$ by 5.

$16^1/_4 \div 5 = 65/4 \div 5 = \frac{65}{4 \times 5} = \frac{13}{4} = 3^1/_4$.

Or thus: $16\frac{1}{4} \div 5 = 3 + (1\frac{1}{4} \div 5) = 3 + 1/4 = 3\frac{1}{4}$. *Ans.*

Exercise 236.

(A) Divide—

1. 9/10 separately by (*a*) 3, (*b*) 6, (*c*) 12,

2. 18/25 separately by (*a*) 4, (*b*) 9, (*c*) 24.
3. 20/29 separately by (*a*) 15, (*b*) 10, (*c*) 18.

(*B*) Divide (by two methods where possible)—

1. $12\frac{1}{2}$ separately by (*a*) 10, (*b*) 4.
2. $16^{1}/_{5}$ separately by (*a*) 9, (*b*) 27.
3. $140^{2}/_{3}$ separately by (*a*) 7, (*b*) 20.
4. $156^{24}/_{121}$ separately by (*a*) 105, (*b*) 140, (*c*) 100.

(*C*) Find, correct to the *unit*, the value of—

1. $136778/_{27} \div 123$. 2. $112218^{161}/_{169} \div 55$.

---

8. Multiplication of Vulgar Fractions by Vulgar Fractions.

256. The definition of multiplication given in Art. 54 is inapplicable in cases in which the multiplier is a fraction. We therefore offer here a more comprehensive definition.

Definition.—To multiply any number by another number is to do to the former what is done to unity to produce the latter.

Suppose we have to multiply 5/7 by ¾. Now, 3/4 means that the unit is divided into 4 equal parts and that 3 of these parts are taken. Therefore, to multiply 5/7 by 3/4 we must divide 5/7 into 4 equal parts and take 3 of these parts, *i.e.*, we must divide 5/7 by 4 and multiply the quotient by 3.

$$\text{Thus } 5/7 \times 3/4 = \frac{5}{7 \times 4} \times 3 = \frac{5 \times 3}{7 \times 4} = 15/28.$$

Hence, to multiply one vulgar fraction by another, *we must multiply the numerators together for a new numerator and the denominators together for a new denominator.*

Note 1.—In multiplying one fraction by another, it is best to divide both numerator and denominator by any common factor before multiplying out, as in the following examples,—

Example.—Multiply 9/11 by 5/6.

$$\text{The product required} = 9/11 \times 5/6 = \frac{\overset{3}{\cancel{9}} \times 5}{11 \times \underset{2}{\cancel{6}}} = 15/22. \quad \textit{Ans}$$

In this example we cross out 9 and 6 which have a common factor 3 and put the quotients 3 and 2 by their side. This process is called, as already explained, cancelling *common factors.*

NOTE 2—We must reduce mixed numbers to improper fraction before multiplying.

Example.—Multiply $2\frac{1}{4}$ by $1\frac{1}{3}$.

The required product $= 2\frac{1}{4} \times 1\frac{1}{3} = 9/4 \times 4/3 = \dfrac{\overset{3}{\cancel{9}} \times \cancel{4}}{\cancel{4} \times \cancel{3}}$
$= 3.$ *Ans.*

NOTE 3—In simplifying fractions, if common factors cannot be found out by inspection, they must be found out by finding the G.C.M of the number in the numerator and denominator as in the following

Example.—$40\frac{7}{10} \times \frac{5}{1147} = \frac{407}{10} \times \frac{5}{1147}$.

Now G.C.M of 407 and 1147 is 37. Dividing them by 37, we have for the required product $\frac{11 \times 5}{10 \times 31}$ or $\frac{11}{2 \times 31}$ or $\frac{11}{62}$. *Ans.*

257. The multiplication of one vulgar fraction by another can be graphically represented as follows :—

Suppose it is required to find the product of 4/5 and 2/3.

Let a rectangular figure ABCD contain 15 squares as in the following diagram,—

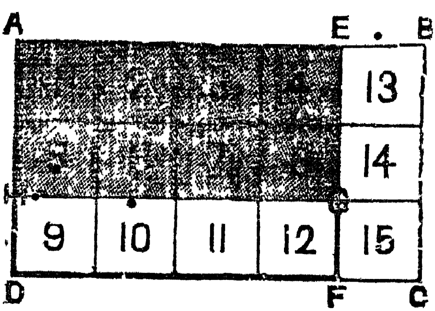

Fig. 50.

Here AEFD = 4/5 of ABCD and AEGH = 2/3 of AEFD.

$\therefore$ AEGH = 2/3 of 4/5 of ABCD.

$\therefore \frac{AEGH}{ABCD} = \frac{2}{3} \times \frac{4}{5}$.

But AEGH contains 8 squares, while ABCD contains 15 squares.

$\therefore 2/3 \times 4/5 = 8/15 = \frac{2 \times 4}{3 \times 5}$.

Exercise 237—(*Graphical.*)

Find the following products *graphically* :—

1. $3/4 \times 3/5$. 2. $2/3 \times 2/3$. 3. $1/2 \times 5/6$.

Exercise 238.

Find the following products :—

1. $4/5 \times 8\frac{3}{4}$. 2. $3\frac{1}{2} \times 6/5$. 3. $12\frac{3}{4} \times 6/17$.
4. $7\frac{1}{2} \times 2/15$. 5. $7/16 \times 5\frac{1}{3}$. 6. $2\,1/7 \times 3\frac{1}{2}$.
7. $13/17 \times 3/11$. 8. $2\,1/7 \times 3\frac{1}{4}$. 9. $\frac{2}{3} \times 3\frac{1}{4}$.
10. $2\,9/56 \times 140/53$. 11. $137/165 \times 3\,21/68$.
12. $5\,79/105 \times 91/453$.

258. The general rule for multiplying any number of vulgar fractions together is :—*Multiply all the numerators together for the numerator of the product, and all the denominators together for its denominator.*

Example 1.—Multiply together 3/11, 5/7 and 9/13,

The product required $= \dfrac{3 \times 5 \times 9}{11 \times 7 \times 13} = 135/1001$. *Ans.*

Example 2.—Multiply together 13/34, $2\frac{1}{5}$, $1\frac{12}{39}$ and 5.

The product required $= 13/34 \times 11/5 \times 51/39 \times 5/1 = 11/2 = 5\frac{1}{2}$. *Ans.*

Exercise 239.

(*A*) Find the following products :—

1. $2\frac{1}{4} \times 3\frac{1}{3} \times 5/6$. 2. $16\frac{1}{8} \times 3\,1/7 \times 84$.
3. $2\,3/26 \times 4/9 \times 39/44$. 4. $85/468 \times 2\,15/119 \times 5\,7/23$.
5. $5\frac{5}{12} \times \frac{1}{15} \times 1\frac{5}{13}$. 6. $4\frac{17}{25} \times 1\frac{19}{21} \times 1\frac{17}{18}$.
7. $570 \times \frac{62}{135} \times \frac{31}{190}$. 8 $3\frac{1}{3} \times 4\frac{1}{5} \times 5\frac{1}{6} \times 15$.

9. $7\frac{7}{8} \times 6\frac{6}{7} \times 5\frac{5}{6}$. 10. $14\frac{9}{20} \times 1\frac{9}{35} \times 1\frac{1}{127}$.

11. $1\frac{100}{121} \times 1\frac{53}{90} \times 1\frac{31}{68}$. 12. $\frac{7}{12} \times 5\frac{3}{8} \times 33 \times 1\frac{4}{11}$.

(*B*) 1. Multiply the sum of $3\frac{3}{4}$ and $2\frac{2}{3}$ by $1\frac{5}{21}$.

2. Multiply the sum of $4\frac{2}{3}$ and $2\frac{1}{2}$ by the difference of 6 and $4\frac{1}{2}$

3. Multiply the difference of $31\frac{5}{7}$ and $23\frac{3}{14}$ by the sum of $10^3/_{13}$ and $3^{10}/_{13}$.

4. Multiply 30 by the sum of $4^1/_{15}$ and $3^2/_{15}$.

5. From the product of $9^1/_{11}$ and $9/10$ subtract the product of $2\frac{2}{3}$ and $3\frac{1}{2}$.

6. Multiply $4\frac{1}{8}$ by $2\frac{1}{5}$ and subtract the product from 100.

(*C*) 1. (*a*) Given that the *circumference* of a circle is $3\frac{1}{7}$ times the *diameter*, find, *correct to a yard*, the circumference of a circular race course whose diameter is 100 yds.

(*b*) Also find the length of the *equator* of the earth, taking the diameter of the earth as 7924 miles.

2. I divide a certain number by $12\frac{3}{8}$ and get $26\frac{2}{3}$. What is the number?

3. Find the value of xy when $x = 4^1/_{17}$, $y = 7^9/_{23}$,

4. Find the value of $3^6/_{13}\, x$, when $x = {}^{14}/_{85}$.

5. Find the value of $^1/_7\, x, \sim {}^3/_{14}\, y$, when $x = 714^7/$[illegible] $y = 610$.

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9. Division of Vulgar Fractions by Vulgar Fractions.

259. To divide a number, say by 3, we may divide it, say by 4 times 3 or 12, and multiply the result by the same number 4. Similarly to divide a fraction, say 5/7, by any fraction, say 3/4, we may divide 5/7 by 4 times* 3/4 or 3, and multiply the result by 4.

* We take 4 times 3/4 so as to get a *whole number* for the product.

Thus, $\frac{5}{7} \div \frac{3}{4} = \left(\frac{5}{7} \div 3\right) \times 4 = \frac{5}{7 \times 3} \times 4 = \frac{5 \times 4}{7 \times 3}$ or $\frac{5}{7} \times \frac{4}{3}$.

From this example we deduce the following rule :—

To divide a number by a vulgar fraction, invert the divisor and multiply.

260. *To prove* **graphically** *that* $\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2}$.

Noting that the L C M, of 5 and 3 (in 3/5 and 2/3) is 15, draw on squared paper or plain paper) a line AB containing 15 small divisions to represent the unit. Take in AB, a line AC of 9 divisions to represent 9/15 or 3/5 of the unit, and a line AD containing 10 divisions to represent 10/15 or 2/3 of the unit.

Now it is clear that $\frac{3}{5} \div \frac{2}{3}$ **= 9 divisions ÷ 10 divisions**

$$= 9 \div 10 = \frac{9}{10} = \frac{3 \times 3}{5 \times 2} = \frac{3}{5} \times \frac{3}{2}.$$

Exercise 240—(*Graphical.*)

Prove *graphically* that— **1.** $3/4 \div 2/3 = 3/4 \times 3/2$.
2. $1/5 \div 3/4 = 1/5 \times 4/3$. **3.** $2/3 \div 3/7 = 2/3 \times 7/3$.

---

261. *Mixed numbers* should be reduced to *improper fractions* before dividing one fraction by another.

Example.—Divide (*a*) 3/4 by $1\frac{5}{6}$; (*b*) $7\frac{1}{2}$ by $1\frac{1}{4}$.

(*a*) $3/4 \div 1\frac{5}{6}$
$= 3/4 \times 11/6$
$= \frac{3 \times 3}{2 \times 11} = \frac{9}{22}$. *Ans.*

(*b*) $7\frac{1}{2} \div 1\frac{1}{4}$
$= 15/2 \div 5/4 = 15/2 \times 4/5$
$= 3 \times 2 = 6$. *Ans.*

Exercise 241.

(A) Find the value of—

1. $14/51 \div 7/17$	**2.** $12\frac{1}{2} \div 5/6$.	**3.** $1/5 \div 1/10$.
4. $3/10 \div 3\frac{1}{3}$.	**5.** $4\frac{1}{3} \div 2\frac{1}{4}$	**6.** $11\frac{1}{9} \div 16\frac{2}{3}$.
7. $14 \div 2\frac{1}{4}$.	**8.** $5\frac{63}{85} \div \frac{671}{119}$.	**9.** $\frac{177}{53} \div 8\frac{3}{7}$.
10. $6\frac{1}{4} \div 4\frac{1}{6}$	**11.** $166\frac{2}{3} \div 22\frac{2}{9}$	**12.** $44\frac{4}{9} \div 166\frac{2}{3}$.
13. $114\frac{2}{7} \div 23\frac{17}{21}$.	**14.** $47\frac{18}{21} \div 228\frac{4}{7}$	**15.** $1\frac{87}{100} \div \frac{119}{250}$.
16. $97\frac{3}{4} \div 143\frac{3}{4}$.	**17.** $10\frac{5}{22} \div 15$.	**18.** $87 \div 8\frac{2}{7}$.

(B) 1. Divide the sum of $3\frac{1}{4}$ and $5\frac{1}{2}$ by 100.

2. Divide the sum of $8\frac{1}{2}$ and $3\frac{1}{3}$ by their differance.

3. Divide the difference of 100 and $65\frac{1}{6}$ by the product of $3\frac{1}{3}$ and 9/10.

4. The product of two numbers is 1760; and one of them is $73\frac{1}{3}$; find the other.

5. The product of two numbers is $1\frac{8653}{10000}$ and one of them is $1\frac{23}{100}$; find the other number.

6. What number multiplied by $4\frac{129}{149}$ will give 145 for the product?

(C) 1. Find the value of x

when (a) $4\frac{1}{3}x = 39\frac{3}{4}$; (b) $\frac{7}{16}x = \frac{3}{8}$.

2. Show that $\frac{x}{y} = \frac{2}{5}$, when $x = 28\frac{2}{3}$, $y = 71\frac{2}{3}$.

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10. Simplification of Compound Vulgar Fractions.

262. Definitions—A simple fraction is one whose numerator and denominator are *both* whole numbers; thus 3/7 and 10/6 are *simple fractions.* **A compound fraction** is a *fraction* of a *fraction*; thus 5/6 of 7/8 and 2/3 of 5/8 of $\frac{3}{4}$ are *compound fractions.*

263. The word *of* in *compound fractions* may be replaced by the sign ×, since 5 *of* a quantity means 5 times the quantity, or 5 × the quantity.

Example 1.—Find the value of 3/4 *of* 5/9

$$\frac{3}{4} \text{ of } \frac{5}{9} = \frac{\cancel{3}}{4} \times \frac{5}{\cancel{9}_{3}} = \frac{5}{12}. \quad Ans.$$

Example 2.—Find the value of 2/5 of 3/7 of 9/11

$$2/5 \text{ of } 3/7 \text{ of } 9/11 = 2/5 \times 3/7 \times 9/11 = \frac{2\times3\times9}{5\times7\times11} = \frac{54}{385}.$$

It follows from the above examples that *any compound fraction can be reduced to a simple fraction by multiplying together the numerators of the several fractions for a new numerator, and the denominators together for a new denominator.*

NOTE 1.—It will be easily seen that 2/3 of 4/5, 2/3 × 4/5, 4/5 of 2/3, and 4/5 × 2/3 are all of the same value.

NOTE 2.—*Mixed numbers* must be reduced to *improper fractions* before a compound fraction is simplified.

$$\textit{Example.}\text{—}2\frac{1}{10}\times 8\times\frac{5}{84} = \frac{\cancel{21}}{\cancel{10}}\times\frac{\cancel{8}}{1}\times\frac{\cancel{5}}{\cancel{84}} = 1. \quad \textit{Ans.}$$

Exercise 242.

(A) Find the value of—

1. $\frac{5}{7}$ of $1\frac{5}{16}$.
2. $3\frac{1}{6}$ of $\frac{4}{19}$ of $\frac{2}{3}$.
3. $\frac{26}{15}$ of $\frac{9}{16}$ of $1\frac{1}{29}$.
4. $9\frac{5}{7}$ of $2\frac{15}{17}$.
5. $6\frac{6}{11}$ of $\frac{3}{18}$ of $\frac{11}{12}$.
6. $\frac{47}{84}$ of $\frac{7}{8}$ of $5\frac{1}{3}$.
7. $11\frac{3}{7}$ of $7\frac{3}{11}$ of $9\frac{5}{8}$.
8. $\frac{1}{17}$ of 48 of $21\frac{1}{4}$.

(B) Simplify the following expressions:—

1. $\frac{1}{9}$ of $22\frac{1}{2} + \frac{5}{7}$ of $41\frac{1}{4}$.
2. $2\frac{1}{3}$ of $50 - \frac{7}{12}$ of $6\frac{3}{4}$.
3. $1\frac{1}{4}$ of $3\frac{3}{11} + \frac{7}{11}$ of $\frac{3}{4}$.
4. $15 - 1\frac{1}{2} \times 2\frac{1}{3} \times 3\frac{1}{4}$.

(C) 1. A owns 3/4 of a house and gives 2/9 of his share to B. What fraction of the house does B get?

2. If 3/5 of a ship belongs to A and 1/4 of the remainder to B, what part of the ship is owned by B?

11. Simplification of Complex Vulgar Fractions.

264. **A complex fraction** is one whose numerator or denominator or both are fractions; thus $\frac{2\frac{2}{3}}{3}$, $\frac{2}{3\frac{1}{2}}$, $\frac{\frac{1}{4}}{2\frac{1}{3}}$ are *complex fractions.*

Example 1.—Reduce $\frac{7\frac{1}{2}}{6\frac{2}{3}}$ to the simplest form.

Solution

$\frac{7\frac{1}{2}}{6\frac{2}{3}}$ means $7\frac{1}{2} \div 6\frac{2}{3}$

$$\therefore \frac{7\frac{1}{2}}{6\frac{2}{3}} = \frac{15}{2} \div \frac{20}{3} = \frac{\overset{3}{\cancel{15}}}{2} \times \frac{3}{\underset{4}{\cancel{20}}} = \frac{9}{8} = 1\frac{1}{8} \quad \textit{Ans.}$$

Example 2.—Simplify $\frac{3\frac{1}{3} \times 4\frac{1}{2}}{35}$.

Solution.

$$\frac{3\frac{1}{3} \times 4\frac{1}{2}}{35} = \frac{10/3 \times 9/2}{35/1} = \frac{10 \times 9}{3 \times 2} \times \frac{1}{35}$$

$$= \frac{\overset{1}{\cancel{10}} \times \overset{3}{\cancel{9}} \times 1}{\cancel{3} \times \cancel{2} \times \underset{7}{\cancel{35}}} = \frac{3}{7}. \quad \textit{Ans.}$$

Example 3.—Find the value of $\frac{3^{3}/_{8}}{1^{4}/_{5} \times {}^{1}/_{7}}$

Solution.

$$\frac{3^{3}/_{8}}{1^{4}/_{5} \times 1/7} = \frac{27/8}{9/5 \times 1/7} = \frac{\overset{3}{\cancel{27}}}{8} \times \frac{5}{\cancel{9}} \times \frac{7}{1}$$

$$= \frac{3 \times 5 \times 7}{8} = \frac{105}{8} = 13^{1}/_{8}. \quad \textit{Ans.}$$

Exercise 243.

(*a*) Reduce the following *complex fractions* to their **simplest** form :—

1. $\frac{2^{1}/_{5}}{5^{5}/_{11}}$ 2. $\frac{4^{18}/_{25}}{1^{43}/_{75}}$. 3. $\frac{4^{24}/_{25}}{49^{3}/_{5}}$

4. $\dfrac{9^5/_9}{14\frac{1}{3}}$ 5. $\dfrac{20^5/_6}{12^1/_2}$ 6. $\dfrac{8^{29}/_{32}}{3^9/_{16}}$

7. $\dfrac{8^1/_{19}}{^{51}/_{57}}$. 8. $\dfrac{6\frac{1}{4}}{1^7/_{18}}$

(*b*) **Simplify the following** ***complex fractions*** **:—**

1. $\dfrac{5\frac{3}{4} \times 7\frac{1}{2}}{1\frac{7}{8}}$ 2. $\dfrac{12^3/_5 \times 8}{4^1/_5}$

3. $\dfrac{3^3/_7 \times 3\frac{1}{2}}{48}$ 4. $\dfrac{8\frac{1}{8}}{6\frac{1}{2} \times 2\frac{1}{3}}$

5. $\dfrac{6\frac{1}{4}}{5 \times 7\frac{3}{4}}$ 6. $\dfrac{3\frac{3}{8}}{^9/_{11} \times 4\frac{1}{8}}$

(*c*) **Find the value of—**

1. xy/z when $x=33\frac{1}{3}$, $y=6^3/_5$, $z=2^6/_7$.
2. x/yz when $x=182$, $y=15\frac{3}{4}$, $z=10^1/_9$.
3. ab/cd when $a=600$, $b=11/119$, $c=5/17$, $d=77$.

CHAPTER XLVII.

DRAWING TO SCALE.

265. Suppose we want to make a drawing on paper of a black-board 4 feet long and 3 feet broad. The drawing cannot, of course, be of the same *size* as the black-board though it may be of the same *shape*. We must make the drawing much smaller than the black-board by making each line in the drawing much smaller than the corresponding line in the black-board. For instance, we can draw a figure in which each foot in the length and in the breadth of the black-board is represented by 1 inch, or by $\frac{1}{2}$ an inch, or by 1 cm., and so on.

This operation is called *drawing to scale.* And if 1 inch in the drawing represents 1 foot in the object, the scale is said to be *a scale of* 1 *inch to* 1 *foot*

Exercise 244.—(*Practical*).

(*To be done on plain paper*)

(A) 1. Make a drawing of each of the following objects using a scale of (*a*) 1 inch to 1 foot, (*b*) ½'' to 1 foot, (*c*) 1 cm. to 1 foot ,—

(i) A black-board 5 ft. long and 3 ft. broad;

(ii) A wall map 6 ft. long and 4 ft. broad ,

(iii) A table 4 ft long and 2 ft. broad.

NOTE.—On each of the drawings mark the scale to which it is drawn. Thus . 'Scale: 1 inch to 1 foot' 'Scale ½'' to 1 foot'; 'Scale 1 cm to 1 foot', and so on,

2. Draw a plan of each of the following to the scale of 1 inch to 10 feet:—

(*a*) The floor of a room 30 feet by 20 feet ,

(*b*) A play-ground 80 feet by 40 feet ,

(*c*) A square field 85 feet each way

3. Make drawings of the following to the scale of 1 cm. to 20 yards.—

(*a*) A hall 120 yards by 60 yards ,

(*b*) A tank 200 yards by 150 yards ;

(*c*) A court-yard 160 yards square.

4. Draw a plan (taking a suitable scale) of each of the following. Mark the scale on each plan.

(*a*) A circular flower-bed 20 yards in diameter ;

(*b*) A circular race-course of 4 miles radius ;

(*c*) A garden 4½ miles square.

5. Make a drawing of the following to the sclo of ¼" to a foot ;—

(*a*) A room 20 ft. by 16 ft. ;

(*b*) A circular flower-bed of 18 ft. radius :

(*c*) A square plot of ground measuring 25 ft each way.

(B) 1. Draw (to the scale of 1 cm. to the foot) a plan of a class-room 20 feet by 16 feet with two doorways 4 ft. broad, one in the

middle of each of the lengthwise walls, and two windows 3 ft. broad, one in the middle of each of the breadthwise walls. [The doorways and windows may be shown by thick lines].*

2 A rectangular garden 120 yards by 100 yards has a circular tank of 20 yards radius just at the centre Draw a plan of the garden showing the tank. [Scale $\frac{1}{8}''$ to 10 yards.]

3. A circular garden of 200 yards radius has a tank 30 yds. square just in the centre Draw a plan of the garden and show the tank. [Scale 1 cm. to 20 yards]

4. Draw a plan of a garden 300 feet by 250 feet. [Scale 1 inch to 50 ft] And mark in it two pathways 25 ft. broad, running across the garden from the middle of each side.

5. Draw a plan of a wall 16 ft. long and 12 ft. high and mark in it two windows each 4 ft. high and 3 ft broad, the top of each window being 5 ft. from the top of the wall and the outer side of each window being 3 ft. from the nearer end of the wall. [Scale: 1 cm. to 1 foot.]

6. Draw a plan of the front of a building containing a room 24 ft. long and 12 ft broad in the middle and at each end a semi-circular room of 6 ft. radius. [Scale: $\frac{1}{4}''$ to $2'$.]

Exercise 245.

1. In a plan drawn to scale, a certain line measures $3''$. What is the length of the line represented by it, if the drawing has been made to a scale of (*a*) $1''$ to 1 foot (*b*) $1/2''$ to 1 yard, (*c*) $1/4''$ to a mile, (*d*) $1''$ to a metre, (*e*) $1''$ to 10 feet, (*f*) $1/2''$ to 50 yards.

2 The greatest breadth of India is 1,800 miles. If in a map of India the greatest breadth is $20''$, what is the scale of the map? And what actual distance is represented by a length of (*a*) $3''$, (*b*) $2\frac{1}{2}''$, (*c*) $3/4''$ on the map?

3 If, in the map of a field, 1 cm. represents 100 metres, what actual distance is denoted by (*a*) 2 cm (*b*) 5·5 cm. on the map? And what length on the map will denote an actual distance of (*c*) 240 metres, (*d*) 325 metres, (*e*) a kilometre.

4. On the map of India in your Atlas, find the distance (in a straight line) between the following pairs of towns :—

(*a*) Madras and Bombay, (*b*) Madras and Calcutta;
(*c*) Bombay and Calcutta; (*d*) Madras and Tuticorin.

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* To the Teacher,—The student should often be made to make a *rough* hand sketch and mark the given dimensions on it, before drawing a plan accurately to scale.

CHAPTER XLVIII.

MULTIPLICATION AND DIVISION OF COMPOUND QUANTITIES BY FRACTIONS.

1. Multiplication of Compound Quantities by Fractions.

266. *To multiply a compound quantity by a proper or improper fraction*, multiply the compound quantity by the numerator of the fraction and divide the product by the denominator, *or* divide the compound quantity by the denominator and multiply the quotient by the numerator.

Example—Multiply Rs. 13-4-6 by $\frac{2}{3}$.

$$\text{Rs. } 13\text{-}4\text{-}6 \times 2/3 = \frac{\text{Rs. } 13\text{-}4\text{-}6 \times 2}{3} = \frac{\text{Rs. } 26\text{-}9\text{-}0}{3} = \text{Rs. } 8\text{-}13\text{-}8.$$

Ans.

Or thus:—$\text{Rs. } 13\text{-}4\text{-}6 \times 2/3 = \dfrac{\text{Rs. } 13\text{-}4\text{-}6}{3} \times 2$

$$= \text{Rs. } 4\text{-}6\text{-}10 \times 2 = \text{Rs. } 8\text{-}13\text{-}8.$$

Ans.

267. *To multiply a compound quantity by a mixed number*, multiply the compound quantity separately by the integral part of the number and by the fractional part and add the results; or reduce the mixed number to an improper fraction and multiply.

Example.—Multiply £7-15-7½ by 2⁴/₅.

Solution.

1st method.—$£7\text{-}15\text{-}7\tfrac{1}{2} \times 2 = £15\text{-}11\text{-}3.$

$£7\text{-}15\text{-}7\tfrac{1}{2} \times \tfrac{4}{5} = £6\text{-}4\text{-}6.$

$\therefore £7\text{-}15\text{-}7\tfrac{1}{2} \times 2\tfrac{1}{5} = £21\text{-}15\text{-}9.$ *Ans.*

2nd method.—$£7\text{-}15\text{-}7\frac{1}{2} \times 2\frac{4}{5} = £7\text{-}15\text{-}7\frac{1}{2} \times \frac{14}{5}$

$$= \frac{£7\text{-}15\text{-}7\tfrac{1}{2}}{5} \times 14 = £1\text{-}11\text{-}1\tfrac{1}{2} \times 14 = £21\text{-}15\text{-}9. \textit{ Ans.}$$

Exercise 246.

Find the value of each of the following expressions and check your result by a rough estimate :—

1. £12-4-8 × 3/4.	2. £17-16-8 × 4/5.
3. Rs. 10-4-6 × 5/6.	4. Rs 9-10-0 × 7/12.
5. £1-14-3 × $2^2/_3$.	6. Rs. 4-7-4 × $3^1/_8$.
7. 4 cwt 3 qrs 7 lb. × 4/7	8. 3 mls. 2 fur. 22 yds. × 12/11.

2. Fraction of a Compound Quantity.

268. The value of 5/9 of 12 yds. 2 ft. 6 inches is the same thing as 12 yds. 2 ft. 6 in × 5/9.

Exercise 247.

(A) Find the value of—

1. 5/9 *of* 12 yds. 2 ft. 6 in.	2. 3/5 *of* Rs. 79-8-6.	
3. $6^2/_3$ *of* Rs 17-6-0.	4. 7/5 *of* 4 mds. 2 viss 10 pal.	
5. 13/12 *of* 7*s*. 6*d*.	6. 5/8 *of* £1-0-4	
7. 18/19 *of* 3 ft. 2 in.	8. $5^4/_5$ *of* 6 tons.	9. $6^3/_7$ of £5-5-0.

(B) 1. If a tree is worth Rs 25-7-6, what is the value *of* 7/9 of it ?

2. A owns 3/11 of a bandy worth Rs. 150-8-9. Find the value, *correct to a pie*, of A's share of the bandy.

3. A clerk's monthly salary is Rs. 37-8-0 Find, *correct to a pie*, his salary for 10/31 of a month

4 A man had £375-4-8. He gave 1/4 of it to A and 1/5 of the remainder to B. What sum had he after this ?

5. A purse contains Rs 1000, 5/12 of it belongs to A, 3/8 of the remainder to B, and what still remains to C. What fraction of the purse has C ? And what is its value ?

3. Division of Compound Quantities by Fractions.

269. *To divide a compound quantity by a vulgar fraction*, invert the fraction and multiply, taking care first to reduce the divisor, if a mixed number, to an improper fraction.

Example 1.—Divide Rs. 3-0-6 by $2^2/_3$.

Solution.

Rs. 3-0-6 ÷ $2\frac{2}{3}$ = Rs 3-0-6 × 3/8

= Rs. $\frac{3\text{-}0\text{-}6}{8}$ × 3 = Rs. 0-6-$0\frac{3}{4}$ × 3 = Rs. 1-2-$2\frac{1}{4}$. *Ans.*

Exercise 248.

Find the value of the following :—

1. Rs. 3-0-6 ÷ $2\frac{2}{3}$.
2. Rs 18-5-4 ÷ $3\frac{1}{2}$.
3. Rs. 2-3-0 ÷ 3/10.
4. £17-6-0 ÷ 7/15.
5. £20-2-0 ÷ $2\frac{1}{2}$.
6. £8-7-6 ÷ $3\frac{3}{4}$.

Exercise 249.

[*The answers are to be correct to a pie or a penny as the case may be.*]

1. If 1 viss of coffee costs Rs. 2-2-0, find the cost of $3\frac{3}{5}$ viss.
2. If Rs $45\frac{1}{2}$ = £2-15-6, find the value of Re 1.
3. If $3\frac{1}{2}$ yards of cloth costs Rs. 8-8-0, what will 1 yard cost?
4. Find the cost of $7\frac{1}{9}$ maunds of coffee at Rs. 5-4-3 a maund.

CHAPTER XLIX.

REDUCTION OF CONCRETE VULGAR FRACTIONS.

270. The process of reducing a *concrete vulgar fraction* (*i.e. a fractional part of a concrete quantity*) of one denomination to lower denominations, and of reducing a compound quantity to *the fractional part* of a higher denomination, will be made clear by the following examples :—

Example 1.—Reduce £$\frac{3}{16}$ to shillings and pence.

Solution.

£3/16 = 3/16 × 20*s*. = 15/4*s*. = $3\frac{3}{4}$*s*.;

3/4*s*. = 3/4 × 12*d* = 9*d*.

∴ £3/16 = 3*s* 9*d*. *Ans.*

Example 2.—Reduce 10 as. 8 pies to the fraction of a rupee.

Solution.

8 pies = $\frac{8}{12}$ a. = $\frac{2}{3}$ a.

∴ 10 as. 8 p. = $10\frac{2}{3}$ as.

= Rs. $\frac{10\frac{2}{3}}{16}$ = Rs. $\frac{10\frac{2}{3} \times 3}{16 \times 3}$ = Rs. $\frac{32}{16 \times 3}$ = Re. $\frac{2}{3}$. *Ans.*

Verification.—Example 1 may be proved by reducing the answer back to the fraction of £1, and Example 2 by reducing the answer to annas and pies.

Exercise 250.

Reduce to shillings and pence, or annas and pies, as the case may be, and verify your answers,—

1. £ 5/6.	2. £ 3/20.	3. £ 7/12.
4. £ 13/24.	5. Re. 1/3.	6. Re. 5/6.
7. Re. 13/24.	8. Re. 3/64.	9. £5/32.
10. £ 7/18.	11. Re. 5/18.	12. Re. 2/9.

Exercise 251.

Reduce to lower denominations, and verify each answer—

1. 3/8 cwt. 2. 7/16 md. 3. 3/16 mile. 4. 3/16 yd.

Exercise 252.

(*A*) Reduce to fractions of a pound *or* of a rupee, as the case may be, and verify your answers—

1. 1*s*. 3*d*.	2. 3*s* 4*d*.	3. 6*s*. 8*d*.	4. 3*d*.
5. 10*s*. 8*d*.	6. 5 as. 4 p.	7. 10 as. 8 p.	8. 15*s*.
9. 1 a. 4 p.	10. 2 as. 8 p.	11. 1 a. 6 p.	12. 2 as.
13. 9 as. 4 p.	14. 8*s*. $1\frac{1}{2}$*d*.	15. 6*s*. 10*d*.	16 $3\frac{1}{2}$*d*.

(*B*) Reduce—

1. (*a*) 3 qrs. 14 lb., (*b*) 1 qr 7 lb. to cwt.
2. (*a*) 2 ft 3 in., (*b*) 2 ft 4 in., (*c*) 1 ft. 9 in., to yards.
3. (*a*) 2 fur. 110 yds. (*b*) 7 fur. 165 yds. 1 ft., to miles
4. (*a*) 1 viss 8 pal. to maunds. (*b*) 6 pal 2 tolas to seers.

Exercise 253—(*Oral*.)

1. Reduce to shillings and pence—
   £$\frac{1}{3}$, £$\frac{3}{5}$, £$\frac{1}{8}$, £$\frac{1}{6}$, £$\frac{3}{8}$, £$\frac{5}{8}$, £$\frac{8}{15}$, £$\frac{7}{12}$.
2. Reduce to annas and pies—
   Re. $\frac{1}{8}$, Re. $\frac{2}{3}$, Re. $\frac{5}{8}$, Re. $\frac{7}{12}$, Re. $\frac{1}{9}$, Re. $\frac{7}{9}$.
3. Reduce to annas or shillings—
   4*d*., $4\frac{1}{2}$*d*., $3\frac{1}{3}$ p., $2\frac{2}{3}$ p., $3\frac{3}{4}$*d*., $2\frac{1}{2}$p.
4. Reduce to pounds or rupees—
   5 as. 6 p., 2*s*. 6*d*., 6 as., 6*s*. 8*d*, 13 as. 4 p, 16*s*. 8*d*., 1*s*. 8*d*., 10 as. 8 p., 7*s*. 6*d* 9 as. 4 p.

Exercise 254.

(*a*) Reduce to £s:—

1. £4. 7*s* 6*d*. 2. £1 6*s*. 8*d*. 3. £2. 17*s*. 7*d*.

(*b*) Reduce to Rs.:—

1. Rs 4 8 as. 2. Rs 5 5 as. 4 p.

3. Re 1 10 as 4 p 4. Re 1 2 as. 8 p.

(*c*) Reduce to Rs Annas, Pies:—

1. Rs. $5\frac{3}{8}$. 2. Rs. $4\frac{5}{6}$. 3. Rs. $4\frac{5}{12}$. 4. Rs. $1\frac{2}{3}$.

(*d*) Reduce to £s, shillings, pence:—

1. £$4\frac{7}{12}$. 2. £$3\frac{5}{16}$. 3. £$8\frac{1}{3}$. 4. £$10\frac{5}{8}$.

(*e*) Reduce to yards, feet, inches:—

1. $3\frac{5}{12}$ yards. 2. $4\frac{1}{6}$ yds. 3. $2\frac{3}{8}$ yards.

CHAPTER L.

BISECTION OF LINES AND ANGLES.

271. Bisection of Lines—

Exercise 255.—(*Practical*).

1. Find by folding, the middle point of a piece of thread and of the edge of a sheet of paper.

2. Find, by using the dividers, the middle points of a line by measuring off with them equal lengths from each end of the line and bisecting by eye the portion of the line lying between the two pin-pricks on the line.

3. Bisect a straight line by measuring its length and finding its half by calculation.

4. Bisect a straight line by guessing its middle point, and test your guess by measuring each of the two parts.

5. Repeat Exercises 1—4 above several times.

6. Divide a straight line (*a*) into 4 equal parts, (*b*) into 8 equal parts, by each of the methods of Exercises 1—4 above.

7. Bisect a line *by using compasses and ruler* thus: Draw a line AB. With centres A and B and radii greater than half of AB, describe circles to cut each other at C and D. Join CD cutting AB at P. Then P is the middle point of AB

8. Repeat the previous Exercise several times.

9. Divide a st. line (*a*) into 4 equal parts, (*b*) into 8 equal parts, by using the compasses and ruler and test by measuring.

272. Bisection of Angles.—

Exercise 256 —(*Practical.*)

1. Draw any angle on tracing paper and fold so that one arm of the angle may fall on the other. Then the crease of the fold bisects the angle.

2 Cut out a paper angle and fold it so that one edge containing the angle may fall on the other. Then the angle is bisected by the crease.

3 Measure the angle to be bisected by the protractor: find its half by calculation: and bisect the angle by using the protractor again.

4 Draw the bisector of an angle by guess and test your guess by measuring with the protractor.

5 Repeat Exercises 3 and 4 above several times.

6. Bisect an angle *by using compasses and ruler*, thus: Take any angle BAC. With centre A and any convenient radius draw an arc of a ⊙ cutting AB at D and AC at E. With centres D and E and radii greater than half of the chord DE, describe two circles to cut each other at F. Join AF. Then AF is the bisector of ∠BAC. Verify this by (1) measuring with the protractor each of the angles BAF and CAF. (2) By folding the angle BAC, so that AB falls on AC.

7. Repeat the previous Exercise several times.

8. Divide an angle (*a*) into 4 equal parts, (*b*) into 8 equal parts by the methods of Exercises 1, 2, 3 and 6 above.

CHAPTER LI.

PROBLEMS.

1. Unitary Method.

Model 1.—If 2 lb. of sugar cost 7 as. 6 p., what would be the cost of $^4/_5$ lb.?

Solution.

Cost of 2 lb. of sugar $= 7\frac{1}{2}$ as.

$\therefore$ cost of 1 lb. of sugar $= \frac{7\frac{1}{2}}{2}$ as. $= \frac{15}{2} \times \frac{1}{2}$ as.

$\therefore$ cost of 4/5 lb. of sugar $= 15/2 \times 1/2 \times 4/5$ as. $= 3$ as. *Ans.*

Model 2.—If the cost of $^4/_5$ of a measure of rice is 1 a. 6 ps., find the cost of $2^1/_2$ measures.

Solution.

Cost of 4/5 of a measure $= 1\frac{1}{2}$ as.

$\therefore$ cost of 1 measure $= \frac{1\frac{1}{2}}{4/5}$ as. $= 3/2 \times 5/4$ as.*

$\therefore$ cost of $2\frac{1}{2}$ measures $= 3/2 \times 5/4 \times 5/2$ as. $= 75/16$ as.

$= 4\frac{11}{16}$ as $\doteq$ 4 as. $8\frac{1}{4}$ pies. *Ans.*

Model 3.—If 2 mds. 32 srs. of sugar cost Rs. 7-5-4, how much sugar can be bought for Re. 1-13-4?

Solution.

2 mds. 32 srs. $= 2\frac{4}{5}$ mds; Rs. 7-5-4=Rs $7\frac{1}{3}$; Re. 1-13-4=Re. $1\frac{5}{6}$.

Now for Rs. $7\frac{1}{3}$ we can buy $2\frac{4}{5}$ mds.

$\therefore$ for Re. 1 we can buy $2\frac{4}{5}$ mds. $\div 7\frac{1}{3}$ or $\frac{14}{5} \times \frac{3}{22}$ mds.

$\therefore$ for Re. $1\frac{5}{6}$ we can buy $\frac{14}{5} \times \frac{3}{22} \times \frac{11}{6}$ mds. *or* $\frac{7}{10}$ mds. *or* 28 srs. *Ans.*

Exercise 257.

1. If 3 yards of silk cost Re. 1-9-0, what will be the cost of $\frac{3}{10}$ of a yard?

2. If $2\frac{1}{2}$ acres of land are worth Rs. 1,250, find the value of $\frac{7}{40}$ of an acre.

3. If a man can walk $22\frac{1}{2}$ miles in 5 hours, how far can he walk in 2 hrs. 20 minutes?

4. If $6\frac{3}{4}$ lb. of sugar cost Re. 1-8-0, how much will 4 lb. cost? (Answer to be correct to a pie.)

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* To the Teacher.—It may be pointed out to the pupil that in passing from the value of 4/5 of a measure to the value of *one* measure, we have to *divide* by 4/5, just as in passing from the value of 2 measures to the value of *one* measure we *divide* by 2.

5. If a man's wages for 6 days be 7*s*. 8*d*., find his wages, correct to a penny, for $3\frac{1}{2}$ days.

6. If $6\frac{3}{4}$ lb. of sugar cost Re. 1-6-6, how much will $3\frac{1}{5}$lb cost?

7. If a train runs $113\frac{1}{3}$ miles in 5 hrs. 40 m'n., in what time will it run 15 miles ?

8. If 4 viss 20 pal. of sago can be bought for Rs. 3-2-0, how much sago can be bought for Re. 1-4-0 ?

9. If a man saves Rs. 140 in $3\frac{1}{2}$ months, how much will he save in $2\frac{1}{2}$ months ?

10 If 14 quires of paper can be purchased for Re. 1-8-6, how many quires can be bought for Re 1-12-0 ?

11. If a gross of steel nibs cost 7 as. 8 pies. find the cost, correct to a pie, of $2\frac{1}{2}$ scores of nibs.

12. If the cost of 1 metre 8 dm. of lace be 5*s*. 6*d*., find the cost, correct to a penny, of 2m , 5 dm of lace

13. If the cost of a thousand bricks be Rs 8-8-0, find the cost correct to an anna, of 720 bricks.

14. Find, correct to a pie, the salary due to a servant for $13\frac{1}{2}$ days in the month of August, at Rs. 17-4-0 a month.

Model 4.—If 8 men can build a house in 36⅔ days, in how many days can 12 men build it ?

Solution.

8 men can build the house in $36\frac{2}{3}$ days

$\therefore$ 1 man can build the house in $36\frac{2}{3} \times 8$ days.

$\therefore$ 12 men can build the house in $\frac{36\frac{2}{3} \times 8}{12}$ days *or*

$\frac{110}{3} \times \frac{8}{1} \times \frac{1}{12}$ days *or* $\frac{220}{9}$ days *or* $24\frac{4}{9}$ days.

Exercise 258.

1. If 12 men can reap a field in $28\frac{1}{2}$ hours, in how many hours can 21 men reap it ?

2 If 6 horses can plough a field in $4\frac{1}{2}$ days, in how many days can 8 horses plough the same field ?

3. If 10 men can build a wall in $8\frac{1}{3}$ days, in how many days can 15 men build it ?

4. If 18 men can reap a field in $30\frac{1}{4}$ hours, in what time will 11 men reap it ?

5. How many men will be required to do in $7\frac{1}{2}$ days a work that 15 men can do in $5\frac{1}{2}$ days ?

6. If 24 oxen can plough a field in $12\frac{1}{2}$ days, how many oxen will be required for ploughing the same field in $18\frac{3}{4}$ days ?

7. A man can perform a journey in $7\frac{1}{3}$ hours, if he walks at $3\frac{1}{2}$ miles an hour. In how many hours can he perform the same journey, if his rate of walking be $2\frac{3}{4}$ miles an hour? [The answer to be correct to an hour.]

Model 5.—If x articles cost p rupees, what will be the cost of y articles ? Find the value of the answer, when $x = 2\frac{1}{2}$, $y = 4$, and $p = 6$.

Solution.

x articles cost p Rs.

$\therefore$ 1 article costs $\frac{p}{x}$ Rs.

$\therefore$ y articles cost $\frac{p}{x} \times \frac{y}{1}$ Rs. $= \frac{py}{x}$ Rs. *Ans.*

Again if $x = 2\frac{1}{2}$, $y = 4$, and $p = 6$,

then $\frac{py}{x}$ Rs. $= \frac{6 \times 4}{2\frac{1}{2}}$ Rs. $= \frac{6 \times 4 \times 2}{2\frac{1}{2} \times 2}$ Rs. $= \frac{48}{5}$ Rs.

$=$ Rs. $9\frac{3}{5} =$ Rs. $9\text{-}9\text{-}7\frac{1}{5}$ *Ans.*

Exercise 59.

1. If x pounds of coffee cost r rupees, find the cost of y pounds Find the value of the answer when (i) $x = 25$, $y = 7$, and $r = 37\frac{1}{2}$, (ii) $x = 7\frac{1}{2}$, $y = 4$, and $r = 15$.

2. If the cost of x oranges be l shillings, find the cost of y oranges. What is the value of the answer if (i) $x = 32$, $y = 4\frac{1}{2}$, and $l = 21$; (ii) $x = 80$, $y = 140$, and $l = 17\frac{1}{2}$?

3. If x men can build a house in m days, in how many days can y men build it? What is the value of the answer when (i) $x = 34$, $y = 25\frac{1}{2}$, $m = 6$, (ii) $x = 14$, $y = 60$, $m = 80$?

4. If a boy can read x lines in y minutes, how many lines can he read in z minutes? Find the value of your answer when (i) $x = 215$, $y = 3\frac{1}{2}$, $z = 2\frac{1}{10}$; (ii) $x = 360$, $y = 6\frac{2}{3}$, $z = 8\frac{1}{2}$.

2. Fractional Parts.

Model 1.—A has $\frac{2}{5}$ of an estate. B has $^5/_6$ of the remainder, and what still remains belongs to C. What part of the estate has C?

Solution.

A has 2/5 of the estate.

∴ the remainder is (1 — 2/5) or 3/5 of the estate,

Again, B has 5/6 of 3/5 or 1/2 of the estate.

∴ the remainder is (3/5 — 1/2) or 1/10 of the estate.

But this, by the question, belongs to C.

∴ C's share = 1/10 of the estate. *Ans.*

Exercise 260.

1. A owns 2/3 of a garden, B owns 5/8 of the remainder, and C owns the rest. What is C's share of the garden?

2. I have a ship: I give away 3/7 of it to A, 7/16 of the remainder to B, and what is still left to C. What part of the ship does C get?

3. A father gives away 1/3 of his property to his son and 3/8 of the remainder to his wife, and keeps the rest for himself. What part of his property does he retain for himself?

4. A man lost 1/4 of his money, then 1/6 of the remainder, and then 2/5 of what still remained. How much of the money remained with him at the end?

Model 2.—A has $\frac{2}{3}$ of an estate; he gives away $\frac{1}{3}$ of his share to B, and $\frac{1}{2}$ of it to C. What part of the estate has he still left?

Solution.

By the question, A has 2/3 of the estate.

He gives to B 1/3 of 2/3 or 2/9 of the estate.

He gives to C 1/2 of 2/3 or 1/3 of the estate.

∴ He gives to B and C together (2/9+1/3) or 5/9 of the estate.

∴ He has (2/3 — 5/9) or 1/9 of the estate still left. *Ans.*

Exercise 260—(*continued.*)

5. A man has 3/4 of a garden; he gives away 1/3 of his share to B and 1/2 to C. How much of the garden has he still left?

6. I own 5/12 of the cargo of a ship: I sell 2/3 of my share to one man, 1/8 of it to another, and what is still left to a third. How much of the cargo do I sell to the last?

7. A man has 2/5 of a ship, he sells 1/4 of his share to A and 5/6 of the remainder to B. What part of the ship has he still left?

8. I have 1/4 of an acre of land, I sell to A 1/8 of an acre, to B 3/10 of the remainder, and what is still left to C. How much land do I sell to C?

Model 3.—After spending 2/5 of my money, I have Rs. 144 left. What sum had I at first?

Solution.

I spend 2/5 of my money.

$\therefore$ the remainder = (1 — 2/5) or 3/5 of my money.

Now, by the question, 3/5 of my money = Rs 144

$\therefore$ the whole of my money = Rs 144 × 5/3
= Rs. 240. *Ans.*

Proof of the Answer.

I have Rs 240.

I spend 2/5 of this, *i.e*, Rs. 240 × 2/5 or Rs. 96.

$\therefore$ the remainder is Rs 240—Rs 96 = Rs 144 which is the amount given in the question. Hence the answer Rs 240 is correct.

Exercise 260—(*continued.*)

(*The Answers are to be verified*)

9. After spending 2/5 of my money, I had Rs. 240, what sum had I at first?

10. I spend 3/5 of my money and have £ 180 left; how much money had I at first?

11 A man spends 1/4 of his income and has Rs. 45 remaining; what is his income?

12. A man had a certain number of apples. He gave away 4/5 of the number and had 20 apples remaining. Find the number of apples he had at first.

---

3. Profit and Loss.

Model 1.—I buy 12 fruits at 4 as. 2 pies each ; the cost of carriage is 2 as. ; if I sell all the fruits for Rs. 3, what do I gain or lose ?

Solution.

The cost of 1 fruit is $4\frac{1}{6}$ as.

∴ the cost of 12 fruits is $4\frac{1}{6} \times 12$ as. = (48 + 2) as. = 50 as.

The cost of carriage is 2 as

∴ the total cost of the 12 fruits is 52 as

But the selling price is Rs. 3 or 48 as.

∴ the loss is 52 as.—48 as. or 4 as. *Ans*

Exercise 261.

1. I buy 16 books at 4 as. 3 pies each, postal charges come to 6 as ; if I sell all the books for Rs. 5, find my gain or loss.

2 A market woman bought 200 eggs at 3/4 *d* each, and sold them all for 15 shillings ; find her gain.

3. A merchant bought 1 cwt. of sugar at 5 as. 3 pies a lb. and paid 12 as for carriage, if he sold all the sugar for Rs 40, find his gain.

4. A merchant bought 2 maunds of coffee at $12\frac{1}{2}$ as. a seer and sold the whole for Rs. 30 4-0 ; find his loss.

Model 2—I buy 20 books for Rs. 3-4-0; the postal charges amount to 6 as. ; find my gain or loss, if I sell the books at 3 as. 6 pies each.

Solution.

The cost of 20 books is Rs. 3-4-0.

The postal charges amount to 6 as.

∴ the total cost of the 20 books is Rs 3-10-0.

Now the selling price of one book is $3\frac{1}{2}$ as.

∴ the selling price of 20 books is $3\frac{1}{2}$ as. × 20 or 70 as. or Rs. 4-6-0

∴ my gain is Rs. 4-6-0—Rs. 3-10-0 or 12 as. *Ans.*

Exercise 261—(*continued.*)

5. I buy 1 Cwt. of sugar for Rs. 19-4-0 and pay a duty of 12 as. What do I gain or lose by selling the sugar at 2 as. 9 pies per lb.?

6. I bought 2 reams of paper at Rs. 2-0-10 per ream and sold it at 3 as. $7\frac{1}{2}$ pies per quire, find my gain.

7. Bought 5 dozen apples at Rs 2-0-4 a dozen and sold them at 3 as. 4 pies an apple, what is the gain?

8. A merchant buys 2 cwt. 16 lb. of sugar for Rs. 80 and pays duty at $1\frac{1}{4}$ pies per lb If he sells all the sugar at 5 as per lb., find his loss.

Model 3 —I buy 240 apples at 2 as. 8 pies each and sell them at 2 as. 10 pies each; find my total gain.

Solution.

The cost price of 1 apple = $5^2/_3$ as.

∴ the cost price of 240 apples = $8/3 \times 240$ as. = 640 as.

The selling price of 1 apple = $2^5/_6$ as.

∴ the selling price of 240 apples = $17/6 \times 240$ as = 680 as.

∴ the gain = 680 as — 640 as = 40 as. = Rs. 2-8-0. *Ans.*

Another Method.

The cost price of 1 apple = 2 as. 8 pies

And the selling price of 1 apple = 2 as. 10 pies.

∴ the gain on 1 apple = 2 pies.

∴ the gain on 240 apples = 2×240 pies

$= \frac{2 \times 240}{12}$ as. = 40 as. = Rs. 2-8-0, as before. *Ans.*

Exercise 261—(*continued*)

(*To be done by two methods*)

9. I buy 36 apples at 4 as 3 pies each and sell them at 5 as. 2 pies each; find my total gain.

10. Bought 1 maund of sugar at 2 as. 6 pies a seer and sold it at 2 as 3 pies a seer, find the total gain or loss

11. Bought 10 reams of paper at 2 as a quire and sold the same at 2 as 3 pies a quire, find the total gain

12. Bought 12 gross of steel nibs at 1/2 pie a nib and sold them at 3/4 of a pie a nib, find the total gain.

Model 4 —I buy 1 cwt. of sugar for Rs. 10-8-0 and pay duty on it at $1\frac{1}{2}$ pies per lb. At what price per lb. must I sell the sugar so as to gain 14 as. by the bargain?

Solution.

Duty on 1 lb. $= 1\frac{1}{2}$ pies.

$\therefore$ duty on 112 lb. $=$

$$\frac{3}{2} \times 112 \text{ pies} = \frac{3}{2} \times \frac{112}{12} \text{ as.} = 14 \text{ as.}$$

And the cost price of 112 lb. is Rs. 10-8-0 or 168 as.

$\therefore$ the total cost of 112 lb. $=$ 168 as. $+$ 14 as $=$ 182 as. and the gain is 14 as.

$\therefore$ the selling price of 112 lb $=$ 196 as.

$\therefore$ the selling price of 1 lb. $= 196/112 = 7/4$ as.
$= 1\frac{3}{4}$ as. $=$ 1 a. 9 pies. *Ans.*

Exercise 261—(*continued*)

13. A man bought 1 cwt 8 lb. of sugar for £1; he paid duty on it at $3/8d.$ per lb At what price per lb. must he sell the sugar in order to gain $9s.$ $7d.$ on the whole?

14. A merchant buys 20 books at Rs. 2-8-0 each At what price must he sell each book so as to gain Rs 5 on the whole?

15. A man bought 180 fruits for Rs. 10 and sold them so as to gain Rs 3-2-0 on the whole, find the selling price of each fruit.

16. Bought 20 reams of paper at Rs. 2-4-0 a ream and sold it so as to lose Rs. 7-8-0 on the whole, find the selling price per quire.

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4. Income-Tax

***Model* 1.**—When the income-tax is 8 pies in the rupee, what income-tax will be paid by a gentleman whose income is Rs. 640?

Solution.

Income-tax on Re. 1 $=$ 8 pies.

$\therefore$ income-tax on Rs. 640 $= 8 \times 640$ pies $= \dfrac{8 \times 640}{12 \times 16}$ Rs.

$=$ Rs $80/3 =$ Rs. $26\frac{2}{3} =$ Rs 26-10-8 *Ans.*

Exercise 262.

1 What is the income-tax on Rs 360 at 4 pies per rupee?

2. Find the income-tax on Rs. 720 at 4 pies per rupee.

3. A gentleman's income is £500 per month, he pays income-tax at $5d.$ in the £, find the tax paid by him per month.

4 A gentleman, whose monthly income is £120, pays a tax of $4d.$ in the £. What tax does he pay per annum?

Model 2.—A gentleman, whose gross income is Rs. 550, pays an income-tax of 5 pies in the rupee; find his net income.

[NOTE.—The *gross income* of a man is his entire income. The *net income* is the amount of income that he has after paying the income-tax.]

Solution.

Income-tax on Re. 1 is 5 pies,

∴ the tax on Rs 500 is (5×500) pies $=$ 2,500 pies $=$ Rs 13-0-4.

∴ the net income = Rs. 500 — Rs. 13-0-4=Rs 486-15-8 *Ans.*

Exercise 262—(*continued.*)

5. A gentleman, whose gross income is Rs 200, pays an income-tax of 4 pies in the rupee, find his net income?

6. Find the net income of a person whose gross income is £240, supposing the income-tax to be 4*d* in the pound.

7. A man, whose gross income is Rs 480, pays an income-tax of 4½ pies in the rupee, find his net income

8. What will be the net income of a person, when his gross income is Rs 240-8-0 and the income-tax is 6 pies in the rupee?

Model 3.—When the income-tax is 6*d*. in the £, a gentleman pays a tax of £2-10-0; find his gross income.

Solution.

By the question, 6*d*. is the tax on 1 £.

∴ 1*d*. is the tax on 1/6 £.

∴ £2-10-0 or $50 \times 12d.$ is the tax on $1/6 \times 50 \times 12$£ or £100. *Ans.*

Exercise 262—(*continued*)

9. A gentleman pays a tax of Rs. 2 at the rate of 4 pies per rupee of his income; find his income.

10. What must be the income of a person who pays a tax of Rs. 12-8-0 at the rate of 5 pies per rupee?

11. If a man pays a monthly tax of 2 as. when the tax is 2 pies in the rupee, find his annual income

12. A man pays a tax of £2-5-0 per annum: find his monthly income, the rate of tax being 5*d* in the £.

---

5. Division of Property.

Model.—Divide Rs. 340-8-6 between A and B, so that A may have twice as much as B.

Solution.

B's share	= *once* B's share.
A's share	= *twice* B's share.
A's share + B's share	= 3 times B's share.
But A's share + B's share	= Rs. 340-8-6.
3 times B's share	= Rs 340-8-6.
B's share = Rs 340-8-6 ÷ 3.	= Rs 113-8-2.
A's share or twice B's share	= Rs 113-8-2 × 2
	= Rs 227-0-4;
and B's share	= Rs 113-8-2. } *Ans.*

Another Method.

Let B's share be x.

Then A's share $= 2x$

A's share + B's share $= 3x$

But A's share + B's share = Rs. 340-8-6.

$3x$ = Rs 340-8-6.

$x = \frac{\text{Rs } 340\text{-}8\text{-}6}{3}$ = Rs 113-8-2.

$2x$ = Rs 113-8-2 × 2 = Rs. 227-0-4,

That is, A's share is Rs 227-0-4; and B's share is Rs 113-8-2 } *Ans.*

Exercise 263.

(To be done by two methods.)

1. Divide Rs 345 between A and B, so that A may have twice as much as B.

2 A house and its furniture are worth £3,000, if the house is worth 4 times as much as the furniture, find the cost of the house and the furniture separately.

3 I bought a sheep and a bullock for Rs 55-8-3, find the cost of the bullock, supposing that it cost 10 times as much as the sheep

4. Divide Rs 84 between A and B, so that A may have $2\frac{1}{2}$ times as much as B.

5 Divide the number 35 6 into two parts, so that one part may be *thrice* the other.

6. A has 1/2 as much money as B. If A and B together have Rs 480, how much has each?

7. A ship and its cargo are together worth one lakh of rupees. the cargo is worth $4\frac{1}{3}$ times as much as the ship. Find the cost of the ship,

8. A tin of ghee weighs 100 seers, find *to the nearest seer*, the weight of the ghee, supposing the empty tin to weigh 1/15 as much as the ghee.

---

6. Revolution of Wheels.

Model 1.—The circumference of a wheel is 11 feet. How many revolutions will it make in a journey of 1 mile 2 furlongs?

Solution.

1 mile 2 fur. = 10 fur. = $10 \times 220 \times 3$ ft.

Now the circumference of the wheel = 11 ft.

∵ the length rolled over in 1 revolution = 11 ft.

∴ the number of revolutions required the number of times 11 feet is contained in 1 mile 2 fur. $= \frac{10 \times 220 \times 3}{11} = 660$ *Ans.*

Model 2.—The wheel of a carriage is 11 feet in circumference. In what distance will it make 600 revolutions?

Solution.

By the question, the circumference of the wheel is 11 ft.

distance for 1 revolution = 11 ft.

distance for 600 revolutions = 11×600 ft. $= \frac{11 \times 600}{3 \times 1760}$ miles = 5/4 miles = $1\frac{1}{4}$ miles = 1 mile 2 fur. *Ans.*

Model 3.—Find the circumference of a carriage wheel which makes 600 revolutions in a journey of 1 mile 2 furlongs.

Solution.

In 600 revolutions the wheel goes over 1 mi. 2 fur or 10×220 yds.

in 1 revolution it goes over $\frac{10 \times 220}{600}$ yds. or 11/3 yds. or 11ft.

the circumference of the wheel is 11 ft. *Ans.*

Exercise 264.

1. The circumference of a carriage wheel is 11 feet. How many revolutions will it make in a journey of 1 mile 4 furlongs?

2. How many revolutions will a carriage wheel 3 yards 2 feet in circumference make in a journey of 4 fur. 4 chains?

3. The circumference of a carriage wheel is 11 feet; in what distance will it make 900 revolutions?

4. A carriage wheel is 14 feet round; in what distance will it make 660 revolutions?

5. A wheel 3 yards 2 feet in circumference makes 450 revolutions Find the distance travelled by it.

6. Find the distance travelled by a wheel 10 feet in circumference when it makes 660 revolutions.

7. Find the circumference of the wheel of a carriage which makes 660 revolutions in a journey of 1 mile 2 furlongs,

8 What is the circumference of a carriage wheel which makes 880 revolutions in travelling $1\frac{1}{2}$ miles?

9. The fore-wheel of a carriage is 9 ft. in circumference and the hind-wheel 14 ft. 8 in. How many revolutions will the one make more than the other in 3 miles?

---

7. The Clock and the Calendar.

Model 1.—How many hours are there (*a*) from 8 a. m. to 3 p. m; (*b*) from 8 a. m. to 10 p. m.; (*c*) from 8 a. m. to 5 a.m.?

Solution.

(*a*) From 8 a.m. to 12 mid-day there are (12—8) or 4 hours.*

And from mid-day to 3 p.m., there are 3 hours.*

∴ from 8 p. m. to 3 p. m. there are 7 hours.* *Ans.*

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* To the Teacher—The pupil may first be taught to find out that there are 4 hours from 8 a m. to 12 mid-day by counting on the fingers beginning with 9 and saying *nine, ten, eleven, twelve*; and then he may be made to see that 4 can be got by subtracting 8 from 12.

Similarly to find the number of hours from 12 mid-day to 3 p m.. the pupil must be taught to begin counting from *one*.

(*b*) From 8 a.m. to 8 p.m. there are 12 hours.
And from 8 p m to 10 p.m there are 2 hours.
∴ from 8 a.m. to 10 p m. there are 14 hours. *Ans.*

(*c*) From 8 a m. to 8 p.m. there are 12 hours.
From 8 p m to 12 mid-night there are 4 hours.
And from 12 mid-night to 5 a m. there are 5 hours.
∴ from 8 a m. to 5 a.m there are 21 hours. *Ans.*

Exercise 265—(*Oral*).

1. How many hours are there (*a*) from 7 a m. to 12 mid-day ; (*b*) from 5 a.m. to 10 a.m.

2. How many hours are there (*a*) from 9 a. m to 3 p. m. ; (*b*) from 9 a.m. to 9 p.m. ; (*c*) from 10 a m. to 11 p.m. ?

3. How many hours are there (*a*) from 4 a. m. to 7 p.m. ; (*b*) from 6 a.m. to 12 mid-night : (*c*) from 9 a m. to 5 a.m.

4. How many hours are there (*a*) from 2 p. m. to 10 p. m. ; (*b*) from 2 p.m. to 12 mid-night. (*c*) from 1 p.m. to 3 a.m. ?

5. How many hours are there (*a*) from mid-night to 4 a.m. ; (*b*) from 8 p m. to 9 a.m. ; (*c*) from 5 p.m to 7 a.m. ?

6. A starts from a place P at 8 a .m. and travels towards another place Q at 5 miles an hour and reaches it at 4 p.m. If B starts from P at 9 a.m. and travels towards Q at 4 miles an hour, when will he reach Q ?

Model 2.—How many hours are there from 7 a.m. on Monday to 5 p.m. on Tuesday ?

Solution.

From 7 a. m. on Monday to 7 a. m. on Tuesday
there are 24 hrs.

From 7 a. m. on Tuesday to 12 noon on Tuesday
there are 5 hrs.

And from 12 noon on Tuesday to 5 p m. on Tuesday there are 5 hrs.

∴ from 7 a. m. on Monday to 5 p m on Tuesday
there are 34 hrs. *Ans.*

Exercise 265—*continued* (*Oral*).

7. How many hours are there (*a*) from 6 a m. on Monday to 4 p.m on Tuesday; (*b*) from 8 a m on Tuesday to 5 p m on Wednesday; (*c*) from 11 a.m. on Wednesday to 7 p.m. on Thursday

8. How many hours are there (*a*) from 6 p m on Monday to 12 mid-night on Wednesday, (*b*) from 4 p m on Friday to 11 p m on Saturday, (*c*) from 3 a m. on Saturday to 3 p.m. on Sunday.

9 How many hours are there (*a*) from 2 p m, on Monday to 5 a m. on Wednesday; (*b*) from 6 a m. on Tuesday to 12 mid-night on Thursday, (*c*) from 10 p m. on Wednesday to 7 a m. on Saturday

10. A train starts from a station M. at 8 p m. on Monday and reaches another station T 53 hours after starting. On what day and at what hour does it reach T?

11. A train leaves R at 5 a.m. on Saturday for Q a distance of 1,452 miles and travels at an average rate of 22 miles an hour including stoppages. On what day and at what hour does it reach Q?

Model 3.—If the *first* day of a month fall on a Monday, what other days of the month will fall on the same day? And what day of the week will be the 18th of the month?

Solution

Since there are 7 days in a week, the days of a month falling on the same week day as the first should be the days next to the 7th, 14th, 21st and 28th days.

Hence the days of the month required are the 8th, 15th, 22nd and 29th. *Ans.*

Again, since Monday is the first day of the month we say "Monday to Monday inclusive 15, Tuesday 16 Wednesday 17, Thursday 18". Thus it is seen that the 18th of the month falls on a Thursday. *Ans.*

Model 4.—If the first of January be a Wednesday, what day of the week will be the first of the next month February.

Solution.

Since January has 31 days, we say "from Wednesday to Wednesday inclusive 29 days; Thursday 30; Friday 31." Hence it is seen that the 1st of February will fall on Saturday. *Ans.*

Exercise 265 *continued.*—(*Oral.*)

12. If the 1st of a month falls on Saturday, what other days of the month will fall on the same week day ?

13. If a month begins on Wednesday, what day of the week will be (*a*) the 17th, (*b*) the 20th, (*e*) the 31st of the month ?

14. If the month of April begins on a Friday, what days of the month will fall on Monday ?

15. How many Saturdays are there in a month (*a*) of 28 days, (*b*) of 29 days, (*c*) of 30 days, (*d*) of 31 days, supposing the 1st day of the month to be (i) a Sunday, (ii) a Monday, (iii) a Tuesday, (iv) a Wednesday, (v) a Thursday, (vi) a Friday, (vii) a Saturday ?

16. If in a certain year the first of May falls on a Tuesday, on what day will (*a*) the 1st of June, (*b*) the 1st of July, (*c*) the first of August fall ?

17. If in a leap year the 1st of February falls on a Monday, on what day of the week will the 18th of March fall ?

18. If the 1st January 1916 fell on a Saturday, name the day of the week on which the 1st day of each of the other eleven months of the year fell ?

CHAPTER LII.

THE MARINER'S COMPASS.

273. The Mariner's Compass is an instrument consisting of a magnetised needle and a card on which 32 directions of the sky are marked. It is used to steer ships, by the needle indicating the absolute direction of the ship at any given time.

274. The four cardinal points :—

Draw a *horizontal* line AB from *left to right* and a *vertical* line CD *downwards* to cut AB *at right angles* at O. Then the four lines OB, OC, OA, OD will respectively denote the four directions *East, North, West, South.*

NOTE 1.—The contractions for East, North, West, South are E., N., W., S. respectively.

NOTE 2.—The four cardinal points may also be shown by folding a paper circle into 4 equal sectors or quadrants, unfolding it, and spreading it flat, so that the crease may show the four directions.

Exercise —What is the magnitude of the angle between any one of the above 4 directions and the next one?

275. The eight chief points of the sky:—

Cut out a paper circle, fold it into 8 equal sectors, spread it out flat and mark the creases as in Fig 51. You will see that there are 8 lines proceeding in eight different directions from the centre of the circle, whose names are indicated by the contraction E., N.-E., etc.

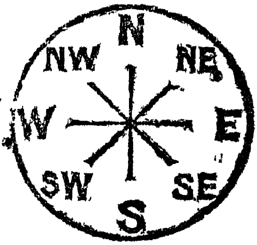

Fig. 51.

Exercise —What is the magnitude of the angle between any one of the above 8 directions and the next one? And of the angles between (1) N.-E. and S.-E., (2) N.-W. and S., (3) E. and N.-W., (4) S.-W. and N.-E.

276. The sixteen chief points of the Mariner's Compass.—

(*a*) In Fig. 52 the 16 *chief points* of the Mariner's Compass are shown. What is the magnitude of the angle between any two adjacent directions? And between the following directions? N.-E. and E., S.-E., S., S.-W. and W., S.-W.: N.N.-E. and W.; N. and S., S.-E.: and so on.

(*b*) Make a card-board model of Figure 52.

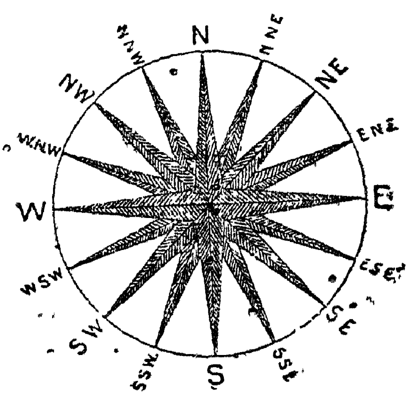

Fig. 52.

(*c*) In the Mariner's Compass there are 32 points marked. What is the magnitude of the angle between any two adjacent points?

REVISION EXAMPLES.

A.

1. Find the difference between *three thousand three hundred and thirty millions, and seventeen million seven hundred thousand and seventy-seven and three-fourths.* Express this difference in words according to the Indian System of Numeration.

2. How many *thousands* are there in 5 *lakhs*? How many *lakhs* in $2\frac{1}{2}$ *crores*?

3. If a metre=39·37079 inches, how many inches are there in (i) a *Kilometre*, (ii) in a *Decametre*, (iii) in a *centimetre*. (iv) in a *millimetre*?

4. If x, y, z denote the digits of a number of three digits taken in order from the *left*, express the value of the number in terms of x, y, z.

5. Express the value of $\frac{1}{10^3}+\frac{2}{10^5}+\frac{8}{10^7}$ as a decimal.

6. Represent the number 47 on squared paper taking 5 small squares to represent the unit.

7. Draw the *net* of a box *without lid* whose *length*, *breadth* and *height* are 8 cm., 5 cm., and 2 cm.

8. From any point in any line and on the same side of it draw two st. lines, so that the three acute angles formed thereby may be equal to each other.

9. Describe a ⊙ of 4″ diameter. From the centre O draw the radii OA, OB so that ∠ AOB may be 60°. Join AB; and from O drop the perpendicular OC on AB. Measure AB and OC and the angles at A, B and C.

10. Describe a ⊙ and draw within it two diameters cutting each *other obliquely*, and join their ends in order. Show by measurements that the figure so formed is a rectangle.

11. Prove *graphically* by means of small squares (each of which denotes the unit) taken on squared paper that

(1) $(5 + 3)^2 = 5^2 + 2 \times 5 \times 3 + 3^2$.

(2) $2 \times 3 \times 4 = 3 \times 2 \times 4$.

(3) $4\ (10 - 4 - 2) = 4 \times 10 - 4^2 - 4 \times 2$.

12. In a school of 321 children the fee income for a certain year was Rs. 789; the salary of the teachers amounted to Rs. 3,000; the rent for the building was Rs. 250; and other expenses came to Rs. 489. What was the average cost to the manager of educating a child during that year?

13. The product of two numbers is 77762223. If one of them is 7777, find the other.

14. Divide 5659·5 by 1848 using factors.

15. Find in lakhs of rupees the cost of 324 acres of land at Rs. 1,525 per acre.

16. Taking the speed of the earth in its orbit round the sun to be 19·5 miles per second, find in millions of miles its speed per day.

17. Find, by the shortest method, the value of—

(*a*) 2345×125. (*b*) $87650 \div 25$.

(*c*) $1205^5/_{16} \times 30$. (*d*) 5608×997.

(*e*) $50469 \div 125$. (*f*) $37605^{11}/_{20} \div 12$.

(*g*) Rs. 2-15-6 $\times$ 36. (*h*) $45\ 00625 \times 696$.

18. Find the *divisor* when the *dividend* is Rs. 209-3-9, the *quotient* is Rs. 3 4-3 and the *remainder* is 3 as. 9 pies.

19. If 27 times a number increased by 27·839 be 100, what is 33 times the number decreased by 419?

20. Find, by the shortest method, the value of—

(1) $1686 \times 35 + 2314 \times 35$; (2) $2076 \times 49 - 1576 \times 49$.

(3) $725 \times 126 - 725 \times 88 + 725 \times 72$.

21. Find the difference between the local values of (*a*) the two 8's in 12803·0586 and (*b*) the two 65's in 26500 104365.

22. If 3 ciphers be added to a number, it is increased in value by 3401595. Find the number.

23. Fill up the omitted items in the following table of statistics of a certain railway in the first quarter of the years 1921 and 1922 :—

Month.	No. of passengers carried in 1921	No. of passengers carried in 1922	Increase or Decrease in 1922 over 1921, Increase being denoted by *plus* and Decrease by *minus*.
January	7,50,235	... .	+ 35,428
February	8,77,054	7,99,849	... .
March	....	6 90,450	— 1,00,048
Total for the 3 months	....	. . . .	....

24. How often does a child's heart beat in 6 years of 365 days each, supposing that the number of beats is 140 a minute during the first 3 years and 120 during the next 3 years ?

25. A boat can sail in still water at 6·7 miles an hour. If it sails up a river whose speed is 3·2 miles an hour, how far will it sail up in 12 hrs. ? And how far will it sail down the river in the same time ?

26. Find, without multiplication, the value of 625 × 625 ÷ 125 × 125 × 125.

27. I buy a bottle of liquid medicine containing 8 ounces. If a dose is 10 minims and I take 3 doses a day, for how many days will the bottle of medicine last me ?

28. How many tea-spoonfuls are there in 1 lb. of liquid medicine ? And how many table-spoonfuls ?

29. Reduce (*a*) 4 fur. 5 chains 20 yds. to yards ; (*b*) 4052 links to fur., chains and links.

30. Fill up the omitted items in the following table of statistics of a certain railway for 3 years :—

Year.	Gross Earnings	Working Expenses.	Net Earnings.
1921	Rs. 75,20,304-9-8	Rs 48,44,105-6-10	Rs ....
1922	Rs-...............	Rs 50,00,750-4-0	Rs. 39,48,015-6-4
1923	Rs. 69,74,005-0-9	Rs.....................	Rs 29,49,800-5-5
Total for the 3 yrs	Rs..................	Rs..................	Rs......... ....

31. How many times is £55-11-3½ contained in £2712-13-6, and how much remains over ?

32. A man walks at the rate of 96 steps per minute and each step is 2 ft. 10 in. long ; how many minutes will he take to walk 4 miles 63 chains 6 yds. ?

33. Two pieces of cloth of the same length cost Rs. 30-10-0 and Rs. 55-5 0 respectively; if the price of the second is Rs. 2-12-3 per yard, what is the price of the first per yard ?

34. A gentleman has in his drawer 20 piles of rupees, each containing 15. His servant steals them and puts in their place 20 piles, each consisting of 14 half-anna pieces with a rupee at the top. How much does the gentleman lose ?

What would have been his loss if he had 15 piles of 20 rupees each ?

35. Divide 27 mds. 4 vis. 4 srs. of sugar into 48 equal parts and find the value of one of these parts at 3 as. 6 pies a seer.

36. I have to send to London £42-10-0 through a Bank, when £1 ≙ Rs. 16. If the Bank charges Rs. 5 for sending the money, what sum shall I have to pay the Bank ?

37. A merchant buys 51 yards of cloth at Re. 1-8-0 a yard and sells it at Re. 1-12-0 a yard. If in selling he uses

a yard-measure which is 2 inches too short, what is his gain ?

38. I exchange an old copper vessel weighing $24^1/_2$ seers for a new one weighing 20 seers. If the price per seer of the old vessel is 6 as. and of the new vessel $10^1/_2$ as., what sum should I pay the bazaarman ?

39. Which is the cheaper kind of coffee, 5 vis. for Rs. 11-15-4 or 3 vis. 10 pal. for Rs. 8-9-7 ?

40. Which is the cheaper kind of cloth, one costing Rs. 50 and lasting for $2^1/_2$ years, or one costing Rs. 40 and lasting for $1^1/_2$ years ?

41. Which is the cheaper carpet, one which is 7 cubits by 2 cubits and costs Rs 10, or one which is 6 cubits by 2·5 cubits and costs Rs. 11 ?

42. Prove, *without actual division*, that 19404 is divisible by 396.

43. By what digit must the asterisk (star mark) in 235*78 be replaced so that the number may be divisible by 11 ?

44. Show, *without actual division*, that the remainder after dividing 435088 by 25 is 13.

45. By resolution into *prime* factors, show that 45036 is both a *perfect square* and a *perfect cube*.

46 Find the sum of three consecutive numbers whose product is 7980.

47. Find the largest area that can divide both a *cawni* and an *acre* exactly

48. Taking the *kilometre* as 3388 ft., and the *knot* as 5082 ft., show that the least integral number of miles containing an exact number of *knots* or *kilometres* is 77. Find the number of *knots* and *kilometres* in 77 miles.

49. What is the greatest length of the square bricks required for paving the floor of a room 20 ft. 8 inches by 13 ft. 4 inches ? And what will be their cost at Rs. 15 per hundred ?

70. Divide the sum of 1 m, 2 dm. 3 cm. 4 mm. and 4 m. 3 dm. 2 cm. 1 mm. by the difference between 3 m. 3 dm. 1 cm. 1 mm. and 3 m. 2 dm. 1 cm.

71. If 1 mile = 1·6093 Km., find in Km., etc., the length of 56 miles.

72. A man buys 13·45 metres of cloth for 32·74 francs, 8·04 m. for 25·40 fr., and 3·51 m. for 12·02 fr. What is the average price of a metre of cloth ?

73. A man walks along a road first 2345 metres, then 68 Decametres, then 82 Hectometres, then 8 Kilometres. How much further will he have to walk to make up 20 Kilometres ?

74. Prove *graphically* that (*a*) 1 sq. ft. = 144 sq. inches, (*b*) 1 sq. metre = 100 sq. decimetres.

75. Find the length of a side of a *square* which has the same area as a *rectangle* whose length is 60 cm. and breadth 15 cm.

76. Find the length of an edge of a *cube* which has the same volume as a *cuboid* 9 in. × 8 in. × 3 in.

77. Find the cost of varnishing the six faces of a *cubical* block of wood whose edge is 1 ft. 3 inches at 8 pies per sq. foot. Also find the weight of the block at ·2 palams per cubic inch.

78. A rectangular cistern 5 ft. long, 4 ft. broad and 3 ft. deep is full of water. If a cubic foot of water weighs 1000 oz., find the weight of the water in tons, etc.

79. From a tin sheet 3 ft. square a piece 10 inches by 8 inches is cut off. What fraction of the whole sheet does this piece form ?

80. From a cuboidal block of wood 20 cm. × 12 cm. × 8 cm. a 4 cm. cube is cut off. What fraction of the whole block is left ?

81. If apples are bought at Rs. 21-8-3 a hundred and are sold at Rs. 3-2-6 a dozen, making a profit of Rs. 12-12-0, find the number of apples bought and sold.

82. If in a common year the first of February falls on a Sunday, on what day of the week will the 14th April of the same year fall ?

83. If the 1st January 1924 fell on a Tuesday, name the day of the week on which each of the next 6 months fell.

84. If a block of a certain kind of wood 1 ft. by 10 in. by 9 in. weighs 4 vis. 1 sr., how much will a block of the same wood 1 ft. 6 in. by 1 ft. 3 in. by 8 in. weigh ?

(*Ex.* 85—88 *are to be solved by using x, and answers to be verified.*)

85. I take a number, subtract 7 from it, multiply the remainder by 4, and get 52. What is the number ?

86 If a cipher be affixed to a certain number it is increased in value by 2997. What is that number ?

87. If the sum of four consecutive numbers is 4106, find the numbers.

88. The sum of two quantities is 150 m. 6 cm. and their difference is 85 m. 4 dm. 8 cm. What are the two quantities ?

89. Draw a horizontal line AB and from points C, D and E in it, draw vertical lines to represent the heights of 3 mountains which are 5,400 ft., 6,200 ft., and 4,800 ft. high. Scale : 1 cm. to 1000 ft.]

90. Draw a vertical line AB and from points C, D, E and F in it, draw horizontal lines to represent the lengths of 4 rivers which are 800 miles, 575 miles, 625 miles, and 550 miles. [Scale : 1 cm. to 50 miles.]

91. A plan of a district is drawn to the scale of 1 inch to 10 miles. Find in miles the distance between two places which are 4·2 inches apart on the plan. Find also the distance on the plan between two other places which are 39 miles distant from each other.

92. The scale of a map in an Atlas is 1 : 6,000,000. How many *miles* is this to an *inch* ? And how many *Kilometres* to a *centimetre* ?

93 Draw a plan of a circular race course 320 yards in diameter on any convenient scale. Draw two radii OA, OB at an angle of 60° and join AB. Measure AB on the plan and hence determine the distance between two posts fixed on the course at points corresponding to A and B on the plan.

94. From a town P two roads proceed at right angles to each other. A town Q is situated on one of the roads at a distance of 24 miles from P and another town R is situated on the other road at a distance of 10 miles from P. Find by means of a plan drawn to scale the straight line distance between Q and R.

95 If in Fig. 53 AD represents 30 yds., what length is denoted by each of the other 4 lines?

96. If the same figure represents a rectangular field (scale: 1 to 1000), find its *perimeter** and *diagonal* in feet and inches.

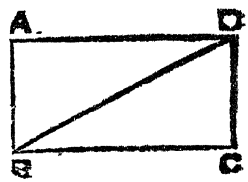

Fig. 53.

B.

1. Show that a clock striking only hours, strikes 156 times a day. How often does it strike in a leap year?

2. The value of a franc is $9\frac{1}{2}d.$ and of a dollar 4*s* 2*d.* Express the sum of 480 francs and 150 dollars in English money. Also express the answer in Indian money taking £1 as equivalent to Rs. 15.

3. The mean solar year consists of thirty-one million five hundred and fifty-six thousand nine hundred and twenty-six seconds. Express this in days, hours, minutes and seconds.

4. (*a*) When the moon is eclipsed, the earth is between it and the sun. Find the distance of the sun from the moon at a lunar eclipse, the distance of the moon from the earth being 238793 miles and the distance of the earth from the sun 91430000 miles.

---

* Sum of the four sides.

(*b*) When the sun is eclipsed, the moon is between it and the earth. Find the distance of the moon from the sun at a solar eclipse.

5. (*a*) Find the sum of the complements of the angles 31° 42′ 53″ and 65° 43′ 21″

(*b*) Find the difference between the supplements of the angles 134° 31′ 28″ and 89° 13′ 53″.

6. One Kilogram of gunpowder costs 9s. and the charge for a gun is 6 grams. How many charges will £1-2-6 worth of powder furnish? [1 Kilogram = 1,000 grams.]

7. If one chain of wire weighs 1·2 maunds and one mile of it costs Rs. 200, how much will 200 mds. of wire cost?

8. If £100 = 2042 German Marks, find, to the nearest penny, the value of 1,200 Marks.

9. A cooly works for 8 hrs. in the day and $2\frac{1}{2}$ hrs. in the night on five of the week days (Monday to Friday) and for only 4 hrs. in the day on Saturday. If his weekly wages be Rs. 8-10-0, find his wages per hour of work done in the night, given that he is paid at the rate of 2 as. per hour of work done in the day.

10. Find by the shortest method the sum of seven consecutive odd numbers of which 1001 is the middle one.

11. A labourer pledges a jewel with a money-lender and borrows from him Rs. 3, undertaking to repay the loan with interest at 6 pies per month per rupee. If he wants to redeem the jewel 2 years 8 months after the date of the loan, what sum will he have to pay the money-lender?

12. Fill up the blanks in the following sums.—

Magic Square.

	6·4	2·9
		7·8
		4·3

Addition.

```
  * 4 9 3
  5 * 5 7
  4 9 * 6
  -------
1 5 4 9 *
```

Multiplication.

```
 * 4 * 5
       9
 -------
22 * 6 *
```

Division.

(*a*)

```
9)28 * 7 *
   * 1 * 4—3
```

(*b*)

```
8)33 * * 1
   * 2 0 *—5
```

13. Fill up the blanks in the following *short division* sums:—

(*a*)	(*b*)
)3542	8)* * * * * *
7)****—Remainder 5	7)* * * * *—6
***—Remainder 2	5)* * * *—5
Complete remainder 17	513—3

14. A man starts from a place P at 8 a.m. and travels towards another place Q at 5 miles an hour and reaches it at 4 p.m. If another person starts from P at 9 a.m. and travels towards Q at 4 miles an hour, when will he reach Q?

15. A certain sum of money was divided equally among 80 men and another sum equal to the first was divided equally among a certain number of women. If each man received 8 as. 6 pies and each woman 6 as. 8 pies, find the number of women.

16. The distance between two towns, which are known to be 60 miles apart, measures $3\frac{3}{4}$ inches on a map. Find the scale of the map. [State the answer as so many miles to an inch.]

17. A man is 56 years old. Twenty years ago he was *thrice* as old as his son then was. What is the son's present age? And what was the age of the father when the son was born?

18. Find the weight of a goods train consisting of two engines weighing 40·4 tons each, 12 empty wagons weighing 6·2 tons each, and 48 loaded wagons each of which contains goods weighing 10 tons. Find also the total length of the train in furlongs, supposing the length of each engine and each wagon including the couplings to be 30 ft. and 21 ft. respectively.

19. A railway passenger counts the telegraph posts on the line as he passes them. If they are 88 yds. apart and the train is going at 48 miles an hour, how many will he pass per minute?

20. The velocity of light is 186,330 miles per second. The distance of the sun from the earth is 92,900,000 miles. Find, to the nearest second, the time taken by light to travel from the sun to the earth.

21. Explain how the following rule is got:—'The number of *annas* in the price of a viss is equal to *twice* the number of *rupees* in the price of a maund.'

Apply this rule to find the price of a viss of a substance, when a maund of it costs (*a*) Rs. 4; (*b*) Rs. 21; (*c*) Rs. 3-2 as.

22. Three persons A, B and C go out on a pleasure trip. A pays all the travelling expenses amounting to Rs. 4-10-0; B pays Rs. 9 12-0 for their food; and C pays all the other expenses amounting to Rs. 2-14-0. If the total expenses are to be paid by the three in equal proportions, find how much should be paid by each of A and C to B.

23. A publisher sells books to a bookseller at Rs. 5 a copy, but allows 25 copies to be counted as 24. If the bookseller sells each of the 25 copies at Rs. 6-4-0, what profit per rupee does he make?

24. Find how many revolutions a minute the wheel of a railway engine makes when the speed of the train is 36 miles an hour, given that the circumference of the wheel is 220 inches.

25. A bicyclist rides first up-hill for 15 miles at 12 miles an hour, then on level ground for 20 miles at 15 miles an hour, and then down-hill for 12 miles at 18 miles an hour. How long will the whole journey take? And what will be the time taken for the return journey?

26. Express a speed of 60 miles per hour in feet per second.

27. A person started from X at a certain hour and travelled towards Y at 6 miles an hour and reached Y at 7-35 p.m. If the distance from X to Y was 32 miles, when did he start from X?

28. A gramophone together with 10 plates of music costs Rs. 150, and with 15 plates Rs. $162\frac{1}{2}$. What will be the cost of the gramophone with 25 plates?

29. A vessel contains 4 seers of milk mixed with 2 seers of water. What fraction of the whole quantity is the milk? If we add 2 seers of water and 2 seers of milk to this mixture, what fraction of the resulting mixture is the milk contained in it?

30. A millionaire's income is said to be at the rate of 5 pence a second. What is it for a month of 30 days?

31. Find the sum of all the numbers between 400 and 600 which are divisible by 19.

32. Express in crores the product of 16, 64, 125, and 625.

33. Find the value of $\dfrac{x^2 + y^2}{l^2 - m^2}$ when $x = 97$, $y = 103$, $l = 9409$ and $m = 10609$.

34. Find the difference between $\dfrac{2}{5} + \dfrac{7}{10}$ and $\dfrac{2+7}{5+10}$.

35. Four fruits are placed on the ground in a line at a distance of 20 yards from each other and a cup is placed in the same line at a distance of 30 yards from the first fruit. A boy starts from the cup, runs up to the first fruit and carries it to the cup, then the second fruit, and so on to the fourth. How many yards does he run on the whole? And what time does he take, supposing he runs at $2\frac{1}{2}$ yds. a second and halts at each fruit for $\frac{1}{2}$ a second?

36. The charge for an urgent telegram is one rupee 8 as. for the first 12 words and 2 annas for every additional word. Find the number of words in an urgent telegram which costs Rs. 3-8-0.

37. How many telegraphic posts will be required for wires extending 73 miles 3 fur. 16 poles, supposing the interval between any two posts to be 99 ft.? [A pole = $5\frac{1}{2}$ yds.]

38. Find the *prime* factors of 496, and hence find all the numbers that will exactly divide 496 (including unity and excluding 496). And show that the sum of all these factors is 496.

39. Taking the diameter of the earth as 7912 miles, the distance of the moon from the earth as 59 times the radius of the earth, and the distance of the sun from the earth as 399 times the distance of the moon, find in millions of miles the distance of the sun from the earth.

40. A loaded waggon weighs 2 tons 3 cwt. 3 qrs. 9 lb.; the waggon by itself weighs 18 cwt. 3 qrs. 14 lb.; the load consists of 215 packages, each of the same weight. Find the weight of each package.

41. A ball of string contains 120 ft.; how many pieces each $5^1/_2$ ft. long can be cut from it, and what length will be left?

42. How many times round a garden 100 ft. long and 76 ft. broad will make a walk of 3 miles?

43. Which is the faster and by how many yards per minute, a torpedo boat with a speed of 30 knots, or a motor-car travelling 35 at miles an hour? [A knot = 6080 ft.]

44. The South Indian Railway system consists of (i) 446 miles of broad gauge (or 5′ 6″ gauge), (ii) 1218 miles of metre gauge (or 3′ 3⅜″), and (iii) 99 miles of narrow gauge (or 2′ 6″ gauge).

If the average cost of laying the line be 1¼ lakhs of rupees per mile of the broad gauge, 1 lakh of rupees per mile of the metre gauge, and ¾ of a lakh of rupees per mile of the narrow gauge, find (correct to a crore of rupees) the total cost of laying the whole length.

45. Thirty pipes each 20 ft. long are fitted together, each pipe overlapping the next by 2 inches. Find the length from end to end.

46. Re-write the following *multiplication* or *division* sums, supplying the missing figures :—

(*a*) 7)*****	(*b*) 4*0*	(*c*) 3*84	(*d*) 70**
6949—4	12	*	4
	**096	26272	28*56

47. By taking four consecutive numbers of three digits, show that the product of the first and third together with the product of the second and fourth is one less than double the product of the second and third.

48. A boy was born on the 11th May 1908. When will he have just completed his 18th year? His 21st year?

49. If in a certain leap year the 1st of May fell on a Monday, on what day of the week did the 1st of the four previous months fall?

50. From the *perpetual* calendar on the next page, find the day of the week corresponding to (*a*) April 15, 1910, (*b*) June 12, 1875, (*c*) September 27, 1900, (*d*) January 1, 1916.

Jan	A	B	C	D	E	F	G
Feb	D	E	F	G	A	B	C
Mar	D	E	F	G	A	B	C
April	G	A	B	C	D	E	F
May	B	C	D	E	F	G	A
June	E	F	G	A	B	C	D
July	G	A	B	C	D	E	F
Aug	C	D	E	F	G	A	B
Sep	F	G	A	B	C	D	E
Oct	A	B	C	D	E	F	G
Nov	D	E	F	G	A	B	C
Dec	F	G	A	B	C	D	E
First day of month	Sat.	Fri	Thu	Wed.	Tue.	Mon.	Sun

A perpetual calendar is one that can be used perpetually or over a wide range of years The one given here covers a period of 150 years from 1851 to 2000.

Method of using the Calendar.—Suppose we wish to find the day of the week corresponding to (say) the 21st August, 1915 Find the letter of 1915 It is B Then find B opposite August in the months Then below it you find Sunday which is the 1st day of the month, from which you see that the required day is Saturday.

NOTE:—In leap years use the first letter for January and February and the second letter for the other months

1851 E	1876 BA	1901 E	1926 B	[illegible] F	1976 GB
1852 DC	1877 G	1902 D	1927 A	1952 ED	1977 A
1853 B	1878 F	1903 C	1928 GF	1953 C	1978 G
1854 A	1879 E	1904 BA	1929 E	1954 B	1979 F
1855 G	1880 DC	1905 G	1930 D	1955 A	1980 ED
1856 FE	1881 B	1906 F	1931 C	1956 GF	1981 C
1857 D	1882 A	1907 E	1932 BA	1957 E	1982 B
1858 C	1883 G	1908 DC	1933 G	1958 D	1983 A
1859 B	1884 FE	1909 B	1934 F	1959 C	1984 GF
1860 AG	1885 D	1910 A	1935 E	1960 BA	1985 E
1861 F	1886 C	1911 G	1936 DC	1961 G	1986 D
1862 E	1887 B	1912 FE	1937 B	1962 F	1987 C
1863 D	1888 AG	1913 D	1938 A	1963 [illegible]	1988 BA
1864 CB	1889 F	1914 C	1939 G	1964 DC	1989 G
1865 A	1890 E	1915 B	1940 FE	1965 B	1990 [illegible]
1866 G	1891 D	1916 AG	1941 D	1966 A	1991 F
1867 F	1892 CB	1917 F	1942 C	1967 G	1992 DC
1868 ED	1893 A	1918 E	1943 B	1968 FE	1993 B
1869 C	1894 G	1919 D	1944 AG	1969 D	1994 A
1870 B	1895 F	1920 CB	1945 F	1970 C	1995 G
1871 A	1896 ED	1921 A	1946 E	1971 B	1996 FE
1872 GF	1897 C	1922 G	1947 D	1972 AG	1997 D
1873 E	1898 B	1923 F	1948 CB	1973 F	1998 C
1874 D	1899 A	1924 ED	1949 A	1974 E	1999 B
1875 C	1900 G	1925 C	1950 G	1975 D	2000 AG

ANSWERS.

Exercise 1.

(A) (*a*) 1 Seven lakhs seventy-six thousand seven hundred and five 2. Forty-one lakhs seventy-five thousand nine hundred and sixty 3. Twenty lakhs 4 Sixteen lakhs and nine. 5. One crore eighty-five lakhs and seventy-six thousand. 6. Six crores forty lakhs five thousand and four. (*b*) Sixty-four millions five thousand and four. 7 One hundred and sixty-five crores. 8. Eleven crores eleven lakhs eleven thousand and eleven. (*b*) 1. Seven-hundred and seventy-six thousand seven hundred and six. 2 Four millions one hundred and seventy-five thousand nine hundred and sixty. 3 Two millions 4. one million six hundred thousand and nine. 5. Eighteen million five hundred and seventy-six thousand. 6 Sixty-four million five thousand and four. 7. One thousand six hundred and fifty millions. 8. One hundred and eleven million one hundred and eleven thousand and eleven.

(*c*) In the place of 10 thousands, of thousands, of hundreds, of tens and of units respectively. 4 ten thousands, 4 thousands, 4 hundreds, 4 tens, 4 units, respectively.

Exercise 2.

1. Greatest 9999, least, 1000. 2. Greatest, 8653, least, 3568. 3. Greatest, 69997. least, 60007. 4. 443, 434, 344.

Exercise 3.

(*a*) 1. 1,00,00 000. 2 2,00.000. 3. 11,000,000. 4. 12,80,606. 5. 3.75,00,930 6. 7,005,490.006. 7. 120,00,40,990. 8. 90,090,019.

(*b*) 1. 100 lakhs; 10 millions. 2. 10 lakhs 3. 100 thousands. 4. 200 lakhs, 2 crores. 5. 3000 lakhs: 30 crores. 6. 900 lakhs, 9 crores.

Exercise 16.

(*b*) 1. 158. 2. 798. (*c*) 155. (*d*) 171. (*e*) 150, 227.

Exercise 17.

(*a*) 1. 200000. 2 17203. 3 11490. 4. 204576. 5. 254401. 6. 65058007 (*b*) 4545095.

Exercise 18.

(A) 1. (*a*) 9561. (*b*) 9581. 2. (*a*) 15547. (*b*) 9869. 3 (*a*) 22933. (*b*) 18132.

(B) 1. 87245. 2. 155959. 3. 12624. 4 87306. 5 81028. 6 142459. 7 92673. 5. 81577. 9. 58902. 10. 28551

(C) Rows: 723, 536, 1162, 906 1182; Grand Total 4509.
Columns: 979, 808, 1738, 1484, Grand Total 4509.

Exercise 21.

(b) 1, $16\frac{1}{4}$. 2 $42\frac{1}{2}$.

Exercise 22.

(a) 1 $70\frac{3}{4}$. 2 $244\frac{1}{2}$ 3 $68\frac{1}{2}$. 4 2347. 5 $7437\frac{1}{4}$.
(b) 36031647

Exercise 25.

(a) 1 153 2 110 0 3. 626 8. 4. 2000·0. 5. 241 9 6 196·5. (b) 469·5

Exercise 29

(e) 1. Females: 154 millions; excess: 7 millions.
2. 215000 miles.

Exercise 30.

(a) 1 190309. 2. 110995. 3. 388889. 4. 425463. (b) 1039416. (c) 1 86 2. 16132 3 106789. 4. 45678. (d) 1 5669. 2. 183335, (e) 13280 feet. (f) 1. 17795. 2. 1878 3. 2510. 4. 10611. 5. 1777.

Exercise 34.

1. $621\frac{1}{2}$. 2. 407. 3. $349\frac{1}{4}$. 4. $848\frac{3}{4}$. 5. $45\frac{1}{4}$. 6. $100\frac{1}{4}$. 7. $82807\frac{1}{2}$. 8. $235\frac{1}{2}$.

Exercise 37.

(a) 1 9 5 2. 52 8. 3. 103 3. 4. 12 8. 5. 1625·7. 6 24 1. 7 62 6. (b) 1. 105 5. 2. 153·9. (c 49 3; 33·7. (d) 359 8.

Exercise 38.

1. 90001. 2. 9990. 3. 983335. 4. 829460.

Exercise 44.

1. 9000 2. 45000. 3. 6400. 4. 12000. 5. 100000. 6 140000. 7. 128000. 8. 7000.

Exercise 45.

(A) 1. 39420. 2. 17376 3 1328000. 4. 96400000.
(B) 1 6666666606. 2. 44444444445. 3. 36144.
4. 2217600. 5. 45600000. 6. 166320000.

Exercise 46.

(A) 1 94976. 2. 89535. 3. 303030. 4. 104976. 5. 231361.
6 207936.

(B) 1. 39312. 2 128960. 3 225700. 4 826281. 5. 475896.
6 4032048. 7. 4102050 8 193600. 9 56325025.

(C) 1. 10080000. 2. 113400000. 3 46690000. 4. 11902500
5. 20250000. 6. 6432040000.

Exercise 47.

1 543600 2. 937500000 3 1417500000. 4. 8400000000.
5. 4173281 6. 8365427.

Exercise 48.

1 7245. 2 316260 3 254448 4. 786984

Exercise 49

1. 5. 2 0 3 4. 4. 7 5. 6 6. 3. 7. 3. 8. 1.

Exercise 50

(A) 1. 6105 2 23313 3 88830 4. 40061. 5. 500067.
6. 5555555505 7. 33333333334.

(B) 1 9835595 2 5026005 3 17784441. 4. 5332114.
5. 9247081. 6. 1764840100

Exercise 52

(A) 1 348 2 625. 3 297. 4 900 5 1290 6. 2512.
7 5588 8 1288 9. 2700 10 2560. 11. 10250. 12 20700.

(B) 1 720 2. 2520 3 6720 4. 945. 5. 3840. 6 1540.

Exercise 53.

1. 575 2 714. 3. 425. 4. 882 5. 648 6. 864.
7. 882. 8 840.

Exercise 54—(*Oral*)

1. 144000. 2. 90000. 3. 2160000000. 4. 25600000.
5. 72000000 6. 1320000000 7 200000000 8 100000000.

Exercise 55.

(b) 625; 441.

(c) 1. 324. 2 484. 3 529. 4 361 5. 289. 6. 1089. 7. 1225. 8. 1849. 9. 2704. 10 784. 11. 2209. 12. 4096.

Exercise 56.

(a) 1. 21025 2. 16384 3 62500. 4. 108160000.
(b) 1 15625. 2 1061208 3. 10648000. 4. 68921. 5. 238328.

Exercise 58—(*Oral*).

1. $11\frac{3}{4}$. 2. $46\frac{1}{2}$. 3. $2?2\frac{1}{2}$. 4. $227\frac{1}{2}$. 5. $191\frac{1}{4}$. 6. 2200. 7. $176\frac{1}{4}$. 8. 1575.

Exercise 60.

$2\frac{1}{4}$; $6\frac{1}{4}$, $12\frac{1}{4}$; $20\frac{1}{4}$; $30\frac{1}{4}$; $42\frac{1}{4}$; $56\frac{1}{4}$; $72\frac{1}{4}$; $90\frac{1}{4}$; $110\frac{1}{4}$.

Exercise 61.

1. $1951\frac{1}{2}$. 2. $5282\frac{1}{2}$. 3. 1682.
4. $14703\frac{1}{2}$. 5. $8887\frac{1}{2}$. 6. 5212.

Exercise 64.

1. 77·1 2. 354 0. 3 128. 4. 1226. 5. 259 6. 6. 12132.

Exercise 67-B.

1. 669—9. 2 64053—6 yds. 3 3033—Rs. 7.
4. 142857 pins—0 pins. 5 £13050—£5. 6. 2008—4 mds.

Exercise 68.

1. 304—0. 2. 512—2358. 3. 3480—103. 4. 987654321—0. 5. 123456789—0. 6. 90090090—10. 7. 5155—113225. 8. 367458—0. 9. 1575—0.

Exercise 69.

1. 32—1204. 2 1506—401. 3. 815—0.
4. 304—100. 5. 229—137005. 6. 243000—11111.
7. 58177—132433. 8. 703—74210.

Exercise 71.

1. 9. 2. 5. 3. 5. 4. 7. 5. 3. 6. 8.

Exercise 72

1. 102—44. 2 1395—103. 3. 612—40.
4. 392—1617. 5. 149—124. 6. 17777—486.

Exercise 78.

(a) 1. $79\frac{1}{4}$—0. 2. $2\cdot4\frac{3}{4}$—0. 3. $143\frac{1}{2}$—$\frac{3}{4}$.
4. $21\frac{1}{4}$—0. 5. $75\frac{1}{4}$—$\frac{1}{4}$. 6. $886\frac{1}{2}$—$\frac{3}{4}$.
(b) 1. $42\frac{1}{4}$—0. 2. $120\frac{1}{2}$—0. 3. $42\frac{3}{4}$—0. 4. $27\frac{1}{2}$—0.

Exercise 77.

(a) 1 9 1 2 2 4. 3 5. 4 6060 6—0.
5 7304 5 6 107 5 —·8.
(b) 1 3 6 – 1 5 2. 4—31 5 3 20 7—0. 4. 120 4—0.
(c) 42 2—0.

Exercise 79—(1—4 *Oral*).

5 42. 6 305. 7. 71 8 202

Exercise 80.

1. 180 2 4910 3 1201 4 1429 5. 11111. 6 83333.
7 625000 8 13333 9 106 10 1923

Exercise 82.

(a) 1 8 2 20. (b) 1 18 2 2 3. 12 4 25 5 62.
6. 145. 7. 25 8 21 9 41 10 9. 11 17 12. $35\frac{1}{2}$.
13 7. 14. $5\frac{1}{3}$. 15 126

Exercise 83.

1 145. 2 861. 3. 6569. 4. 4371. 5. 1953. 6 1199.
7 3911. 8 3779.

Exercise 84—(*Oral*).

1. 48. 2 60. 3 4 4 24 5 360 6. 450. 7. 7500.
8. 200 9. 0. 10. 36

Exercise 85.

1. £25 2 36 pins 3 16 miles 4. 7 7 lakhs.

Exercise 86

1. 3 terms. $+15\frac{1}{4}$, + 9, — 4 2 One term: 8 (6 + 4) ÷ 12.
3. 3 terms: 12, —3, + 4 (8+2 4 2 terms: — 5, + 3 (6+5).
5. One term : (4+8) (3 — $1\frac{1}{2}$). 6 2 terms·—(4+1)(4—1),+100.

Exercise 87.

1. 140. 2. $2\frac{1}{4}$. 3 6. 4. 10. 5 100. 6. 32. 7. 0.

Exercise 88.

1. 7 6. 2. ·5. 3 9 4 5. 5. 2 3. 6. 8·75. 7. 4.
8. 4·5. 9. 4.

Exercise 93

(A) 1. 9. 2 No; it must be greater than Rs 6 the least and less than Rs 14 the greatest of the given numbers

(B) 1 Rs 8 2 8 as. 4 pies. 3. $10\frac{1}{2}$. 4. $8\frac{3}{4}$. 5. 10 miles. 6. 7 7 lakhs

(C) 1 12 years 6 months 2 8 5 cm. 3 6 ft 10 inches.

Exercise 94.

(A) 1 203 2 129 2 3 $147\frac{1}{4}$. 4. $221\frac{3}{4}$.

(B) 1 454 fruits. 2 120 29. 3. (a Rs 40097. (b) Rs 38013 (c) Rs 39055 4 532.

Exercise 95

(a) 1 4245 2 12075 3 174950 4. 54345. 5. 756000. 6 222000. 7. 1226596 8 60753924. 9. 3033125. 10 969240 11 786448 12 3782514.

(b) 1 108864 2 9487044. 3. 5389395 4 3013848. 5. 66719172 6 116698320

(c) 1 1489537016. 2. 1136864832. 3. 302930775. 4. 69279766452. 5. 6348749680 6. 40463614268. 7. 499064758192 8. 105948519424. 9. 5262877270455.

(d) 1 71 — 10 2 12 — 106 3. 347 — 15. 4. 85. 5. 3042 6 19 — 470. 7 101 — 100. 8. 1519. 9 86 — 571.

Exercise 96

1. (a) 23 (b) 3941. $3706\frac{1}{2}$ (c) 729963
2. (a) 15, $22\frac{1}{2}$, 9 6 (b) $1780\frac{1}{4}$ 3881 2.
3. (a) 26, $32\frac{3}{4}$, 17 6 (b, 3400, $4054\frac{1}{4}$, 1777 5
4. (a) 17, 41. (b) 1279, $3358\frac{1}{2}$, 2390 4 (c) 3000, 2063·3.
5. (a) $15\frac{1}{2}$. (b) $794\frac{1}{4}$. (c) 584 8 (d 202.
6. (a) 37 (b) 1 $3267\frac{1}{2}$. 2 2827 4 (c 599·7.
7. (a) Rs 60 (b) Rs 1207. (c) Rs $1115\frac{1}{4}$
8. (a) 8, 3, $7\frac{1}{2}$ (b) 255 $128\frac{1}{3}$, 165, $198\frac{1}{3}$ (c) 123456789.
9. (a 180 (b 1 37185 2 102108 3. 347328 4. 16281.
10. (a 101. (b 30475. (c) 369405 (d) 870480 (e) 20120.
11. (a) 9 times, 3. (b) 43 times, 11. (c) 17 times, Rr. 35.
12. (a) 2, 3. (b) 1. 74 2 61
13. (a) 3, 5. (b) 1. 2. 2. 40.

14. (*a*) 8. (*b*) 75. (*c*) 2589. (*d*) 175 7.
15 (*a*) 6. (*b*) 256 (*c*) 125.
16 (*a*) 8, 6, 3 (*b*) **1**, 21 — 35 — 42. **2** 10 — $757\frac{1}{2}$ — 51.
17. (*a*) Rs 160. (*b*) Rs. 9000 (*c*) 960000 miles.
(*d*) 3372 rupees
18. (*a*) 240 (*b*) 2160 (*c*) 1815. (*d*) 5898. (*e*) 525.
19. (*a*) 30 (*b*) 70. (*c*) 84. (*d*) 33.
20. (*a*) 6, 8 10 (*b*) **1**. 481 123, 213. **2** $42\frac{3}{4}$, $105\frac{1}{4}$. $20\frac{1}{2}$.
3. 90 5 210·3, 399 7.
21. (*b*) 34444, 32222 (*c*) 1193·7, 666 6. (*d*) $967\frac{1}{2}$. $732\frac{1}{2}$.
22. (*a*) Rs. 36 (*b*) **1**. $1826\frac{3}{4}$ rupees. **2.** Rs. 859 9.
(*c*) 647 rupees
23. (*a*) A, Rs 53, (*b*) A, Rs. $316\frac{1}{2}$; (*c*) A, Rs. 410 3;
B, Rs 26. B, Rs. $98\frac{1}{4}$ B Rs 246 5.
24. (*a*) Rs. 120 (*b*) **1**, $226\frac{1}{2}$. **2**. 3145 9 (*c*) 72930.
25 (*a*) Re $\frac{3}{4}$ (*b*) Rs. $72\frac{3}{4}$ (*c*) 150 tins (*d*) 30 miles.
(*e*) 15 miles (*f*) Rs. $15\frac{1}{2}$ (*g*) 505 **26** (*a*) Rs. 9.
(*b*) Rs 315 (*c*) 60 pairs (*d*) Rs 107. (*e*) 80 photos.
27. (*a*) $6\frac{3}{4}$ (*b*) 315 (*c*) 2800. (*d*) $40\frac{1}{432}$ (*e*) 8.
(*f*) 500. (*g*) 16415. **28** (*b*) 128. (*c*) 12. (*d*) 1655.
29. (*a*) Rs 36, Rs 54. (*b*) Rs $15\frac{1}{2}$, Rs 31, Rs $46\frac{1}{2}$.
(*c*) Rs. 175: Rs 245. (*d*) Rs. $52\frac{1}{2}$. Rs. 35, Rs $17\frac{1}{2}$.
(*e*) A, £ 150-12-6, B, £ 180-15-0, C, £ 391-12-6.

Exercise 101.

1. 78817 pies, 24264 pies. **2.** 14108 qr as : 7057 qr as.
3. 560 as. 2016 pies **4** Rs. 21-11-6, Rs 16-6-7, Rs 52-1-4.
5. 8160*d*, 3958*d* 36210*d*. **6.** 16320*q*.; 34992*q* **7.** 5160*d*.
3000 four-pence **8** £75-8 9, £30-3-6 **9** £177-16-1 £37-15-0.
10. 161 gui. 2*s* 9*d*., 101016*g* **11.** 384 oz, 1184 oz.;
1920 oz **12** 3072 dr, 69712 oz **13** 3552oz, 2240 lb
14. 4704 lb., 1760 oz. **15.** 2 qr 4 lb 4 oz, 2 cwt 1 qr 12lb.
16. 1 ton 6 cwt 3 qr 5 lb., 1 ton 2 cwt. 16 lb **17** 30400 grs;
2 lb. **18**. 15201 grs ; 5 lb. 1 dr. 2 scr. **18** (*a*) 19,720 grs.
31,680 grs **18.** (*b*) 2 lb 2 oz. 2 dr 2 grs, 1 lb 1 oz. 2 dr.
44 grs **19** 1080 dwt 5760 grs. 10565 grs. **20.** 4 oz.
8 dwt. 1 gr.; 2 lb., 3 lb.; 5 dwt. **20.** (*a*) 2720 minims;

5178 minims. 20 (b) 2 oz. 0 dr. 40 minims; 3 oz.; 5 oz.
20. (c) 8, 16, 44 tea spoons. 20 (d) 2, 6, 13 table spoons.
20. (e) 7, $20\frac{1}{3}$; $8\frac{3}{4}$; $6\frac{1}{4}$ tea-spoons.
21. 960 tolas, 1600 srs. 22. 2049 tolas.
23. 6525 palams, 1784 palams. 24. 7401 tolas.
25. 2 mds., 1 md. 1 v. 1 sr 1 pal.
26. 4 mds., 2 v. 4 pal 2 tolas, 5 mds. 2 v. 4 sr 4 pal 2 tolas.
27. 25864 mea; 6400 mea. 28. 53006 ol.
29 4 kal 5 mar. 7 mea, 2 par. 3 mar.
30. 89 mar 5 mea 4 ol.; 2 gar 55 par, 2 mar. 7 mea.
31. 976 gills, 188 gills.
32 15 gals 2 qt 1 pt, 3 gals 3 qt. 1 gill.
33. 2272 pints; 10240 gills. 34 192 pks 1 gal 3 pt. 1 gill; 168 qr. 6 bush, 1 pk 0 gal. 1 qt 1 pt 2 gills
35. 7860 ft, 10560 ft. 36 187920 in; 63360 in.
37 1 mi. 4 fur 137 yds. 2 ft 4 in, 11 mi. 2 fur. 201 yds 1 ft.
38 38 mi. 1364 yds. 5 in. 23 miles 672 yds 1 ft.
38—A 67320 inches; 38—B. 2 mi. 5 ch. 20 yds. 2 ft.
39. 233280 sq. ft; 166680 sq. ft.
40 74926 sq ft., 3136320 sq in.
41. 154 sq yds. 2 sq ft 123 sq in, 3 ac 3891 sq yds. 6 sq. ft.
42 30465 sq yds., 57600 sq ft.
43. 105720 sq ft, 115200 sq. ft.
44. 1 caw 17 gr 1600 sq. ft 45. 166 ac 3125 sq. yds.
46, 127872 cub in; 40 cub yds 825 cub in.; 3 cub yds. 23 cub ft. 288 cub in., 498000 cub in.
47. 864000 sec, 3008 min 48 20775 min; 615600 sec.
49. 1 day 4 hrs. 6 sec., 3 wks. 5 days 6 hrs. 7 min.
50. 10 hrs 5 min, 56 sec, 5 wks. 51. 14730 sec. 352800 sec.
52. 11284 min; 16205 min. 53 120°; 197°; 360°.
54. 1 rt. angle 35 deg.; 3 rt angles 78 deg.; 4 rt. angles 40 deg; 3 rt. angles. 55. 5° 50', 42° 28'.

Exercise 102.

1. 725 pies. 2. 846 pies. 3. 970 *d.* 4. 966 *d.* 5 488 pies.
6. 168 as. 7. 1022 pies 8. 1325 *d.* 9. 542 tol. 10 140 lb.
11. Rs. 4-6-6. 12. Rs 6-4-10 13. £ 2-0-5. 14. £ 2-10-6.
15. £ 6-0-3, 16. Rs 7-8-8. 17. 425 cents. 18. 16 ac. 70 cents. 19. 62'. 20. 110°. 21. 180°. 22. 3 rt. angles 30°.

Exercise 103.

(A)

	RS	AS	P.		RS.	AS	P.		RS.	AS	P.
1.	2,431	14	9	2.	307	4	3	3.	3,846	13	9
4.	3,832	4	$5\frac{1}{4}$	5.	1,086	2	$8\frac{1}{2}$	6.	3,547	2	$2\frac{1}{4}$

(B)

	RS	AS.	P		RS.	AS.	P.		RS.	AS	P.
1.	2207	15	3	2.	1368	3	4	3	500	5	11
4.	1270	8	6	5.	856	11	2	6	206	2	11
7.	783	6	4	8.	959	11	7.				

(C) (i)

	£	s	d.		£	s.	d.		£	s.	d.
1.	7,969	9	2	2.	112	10	$0\frac{3}{4}$	3.	1,045	5	5
4.	2,000	0	0	5.	1,865	7	$4\frac{3}{4}$	6.	2,291	0	$5\frac{1}{4}$
7.	1,357	9	4	8.	86	8	5	9.	5,525	19	$0\frac{3}{4}$

(ii)

	Days.	Hrs.	Min.	Sec		Days	Hrs.	Min.	Sec.
1.	1	23	21	$26\frac{1}{2}$	2.	3	7	1	$52\frac{1}{4}$
3.	0	1	34	36	4.	33	1	53	$16\frac{1}{2}$
5.	5	1	48	4	6.	10	18	5	50
7.	7	9	5	57	8.	10	10	30	36
9.	4	18	20	$44\frac{1}{4}$					

(iii)

	Vis.	Sr.	Pal.	Tol.		Vis	Sr.	Pal.	Tol
1.	120	1	6	1	2.	68	2	2	$2\frac{1}{2}$
3.	45	4	0	2	4.	195	1	1	$2\frac{3}{4}$
5.	71	1	7	$1\frac{3}{4}$	6	34	0	6	1
7.	70	0	4	$1\frac{1}{2}$	8	47	1	7	$2\frac{3}{4}$
9.	206	4	1	$1\frac{1}{4}$					

(iv)

	Kal.	Mar.	Mea	Ol.		Kal.	Mar	Mea.	Ol.		Kal.	Mar	Mea	Ol.
1.	26	0	0	0	2.	1	8	5	0	3.	0	2	5	3
4.	20	7	5	4	5.	6	9	3	3	6.	17	5	1	1
7.	9	9	4	1	8	14	6	7	2					

(v)

	tons.	cwt.	qr.	lb.	oz.		tons.	cwt	qr.	lb.	oz.
1.	0	17	2	2	9	2.	2	8	2	3	0
3.	9	11	1	0	0	4.	4	9	3	7	0
5.	1	12	3	26	0	6.	5	4	3	16	5
7.	5	0	2	21	9	8.	3	6	0	6	15
9.	2	2	1	25	12						

		lb	oz	dwt.	gr		lb	oz	dwt.	gr.		lb	oz	dwt.	gr.
(vi)	1	15	3	9	3	2.	13	11	3	0	3.	2	0	10	11
	4.	17	3	13	0	5	11	2	13	8	6	17	2	14	5
	7.	13	11	3	9	8.	6	2	4	16					

		lb.	oz	dr	scr	gr.		lb	oz.	dr	scr.	gr.
(vii)	1	58	9	1	0	0	2	4	5	5	2	0
	3.	0	3	3	1	18	4	3	7	7	2	0
	5	14	11	4	1	15	6.	12	5	2	0	18
	7	10	3	3	1	17	8.	17	5	7	2	9
	9.	6	11	7	1	19						

		mi.	fur	yds.	ft.	in.		mi.	fur	yds.	ft.	in
(viii)	1	23	4	186	0	0	2.	18	4	145	0	0
	3.	2	2	63	0	0	4	0	0	58	0	8
	5	5	3	16	1	10	6	16	4	114	1	11
	7	9	1	72	0	11	8.	12	3	89	2	9
	9	10	7	58	2	3						

		ac	ro	yds	ft		ac	ro	yds	ft		ac.	ro	yds	ft.
(ix)	1.	27	1	388	0	2	25	2	661	0	3.	3	1	1138	8
	4	3	0	294	0	5	11	0	62	0	6	9	3	98	2
	7.	11	0	564	1	8	13	0	1048	2	9	14	1	709	3

		c	yds	ft	in		c.	yds	ft.	in		c.	yds	ft.	in.
(x)	1.		16	20	1343	2		13	16	15	3.		19	0	194
	4.		29	10	1004	5.		13	22	955	6.		17	1	1716
	7.		25	10	1282	8.		22	12	331.					

		rt. angles	deg.	min	sec.		rt angles	deg.	min.	sec.
(xi)	1.	10	88	18	12	2	10	14	43	0
	3.	2	27	58	1	4	9	26	31	5
	5.	2	63	7	39	6.	6	10	23	9
	7.	5	0	57	30					

Exercise 104.

(*a*) 1. Rs. 4-1-2. 2. Rs. 2-4-6. 3. Rs 5-9-3.
4. £306-5-1. 5. £120-13-7. 6. £530-13-10½.
7. £12-8-12-6. 8. £59-4-9-10. 9. 17 tons 3 qrs. 4¼ lbs.
10. 3 tons 3 cwt. 3 qrs. 24 lbs. 11. 2 right ∠s 4° 41′.
12 14° 43′ 49″.

(b) 1. 296 lb 3 oz. 0 dwt. 3 grains.
2. 2404 lb. 4 oz 6 dwt. 7 gr. 3. 1 oz. 0 dr. 1 scr 19 gr.
4. 5 vis. 1 sr. 6 pal. 5. 4 cand 19 mds. 1 vis. 3 seers.
6. 11 gar 395 mar 2 mea. 3 ol. 7. 7 mar. 5 ol.
8. 4 miles 7 fur 15 yds. 1 ft.
9. 1 day 21 hrs. 43 min. 10 sec.
10. 2003 sq yds. 2 sq. ft. 11. 3 rt. ∠s 41° 10'.

Exercise 105.

	Rs	a	p.		Rs.	a	p		Rs.	a.	p.
1.	486	15	8	2.	100	0	0	3.	309	0	$1\frac{1}{2}$
	162	4	11		51	8	9		108	0	5
	324	10	9		48	7	3		200	15	$8\frac{1}{2}$

Exercise 106.

Rs. a p. Rs. a. p.

(A) 1. (a) 21 13 0. (b) 59 15 9.
2. (a) 3,089 1 0 (b) 10,295 14 0.
3. (a) 67,126 2 0. (b) 1,76,206 1 3.
4. (a) £2,09,917 8 9. 5 (a) £14,614 0 0.
(b) £5,51,246 12 1. (b) £25,270 0 10.
6. (a) 4 tons 11 cwt 11 lb.
(b) 12 tons 2 cwt. 3 qrs. 20 lb.
7. (a) 18 tons 6 cwt 3 qrs 6 lb. 12 oz.
(b) 48 tons 18 cwt. 0 qr. 18 lb. 0 oz.
8. (a) 1 can, 18 mds. 4 vis 33 pal.
(b) 5 can. 8 mds. 6 vis 13 pal.
9. (a) 4 can. 2 mds 4 viss 9 pal. 1 tola.
(b) 24 can. 15 mds 1 vis. 16 pal. 0 tol.
10 (a) 77 lb. 11 oz (b) 132 lb. 5oz 10 dwt.
11. (a) 126 days 6 hrs 5 min.
(b) 58 dys. 22 hrs. 2 min 20 sec.
12 (a) 14 mls. 2 fur. 42 yds. 2 ft.
(b) 57 mls 0 fur 170 yds. 2 ft.
13. (a) 5 mls 6 fur. 202 yds. 1 ft.
(b) 6 mls. 6 fur. 162 yds. 2 ft. 2 in.
14. (a) 488 gar 47 par.
(b) 382 gar. 58 par. 0 msr. 6 mea.

		Rs. a. p.		Rs. a. p.
(B) 1.	(a)	61 2 2	(b)	152 13 5
		£ s. d.		£. s. d.
2.	(a)	20 2 5	(b)	100 12 1
3.	(a)	401 11 7	(b)	803 3 2
		Rs. a. p.		Rs a. p.
4.	(a)	53 14 $1\frac{1}{2}$	(b)	143 11 0
		£ s. p.		£. s. d.
5.	(a)	78162 12 $1\frac{1}{2}$	(b)	38598 18 4
6.	(a)	6446 18 10	(b)	24922 15 $7\frac{1}{2}$
7.	(a)	Rs. 13,45,51,822-14-8.	(b)	Rs. 6,52,65,707-3-7.

Exercise 107.

1. Rs. 4-9-7. 2. £64-3-8. 3. £ 13-3-4.
4. £ 175-0-7. 5. 4 cwt. 2 qr. 1 lb. 6. 1 mile 2 fur. 5 yds.
7. 2 days 14 min. $10^{20}/_{32}$ sec. 8. 2 mds. 5 vis. 1 sr. $5\frac{1}{3}$ pal.
9. 12 as, 9 p. 10. 1 yd. 2 ft. 2 in. 11. 148 yds. 2 ft $11^{8}/_{11}$ in.
12. 9 as. 4 p. 13. 8 as. 1 p. 14. 2 rt. ∠s 20° 54′
15. £1 Rs 8-7 2, 16 80 years. 17. 8 as. 1 pie.
18. Rs $1\text{-}0\text{-}7^{12}/_{17}$ 19. 2s. 2d. 20. 3 lb. 8 oz.
21. Rs. 10-0-9. 22. Rs 605-8-5. 23. (a) Rs. 10,010-0-2.
(b) Rs. 6,250-0-1, Remainder 249
(c) Rs. 1,750-0-0, Remainder 1,250. 24 (a) £100,100-8-$11\frac{1}{3}$.
(b) £39,200-3-6. (c) £27,300-2-$5\frac{1}{4}$.

Exercise 108.

	Quotient.	Remainder.		Quotient.	Remainder.
1.	Re. 0-7-3.	Re. 0-0-8.	6.	Re. 2-3-5.	3 as.
2.	Rs. 3-11-6.	Re. 0-1-6.	7.	£10-0-$9^{1}/_{2}$	Nil.
3.	£ 2-7-$11^{1}/_{2}$.	9 d.	8.	Re. 1-4-3.	Re. 1-2-6.
4.	Rs. 4-13-9.	1 a. 1 p.	9.	1 cwt. 2 qr 9 lb.	
5.	20 yds. 4 in.	3 in.		1 oz 13 dr.	2 oz. 1 dr.

Exercise 109.

1. Rs. 3-2-5. 2. Rs. 2-3-7. 3. £ 1-2-7. 4. £ 5-0-8.
5. Rs. 0-2-8. 6. £ 0-12-$0^{1}/_{2}$. 7. £ 11-11-9. 8. £ 42-6-2.

Exercise 110.

(a) 1. 3 times. 2. 6 times. 3. 12 times. 4. 11 times.
5. 10 times. 6. 400 times. 7. 21 times: Rem. 3 pies.
8. 32 times. 9. 9 times; Rem. 1 a. 11 p.

10. 3 times; Rem. 8 *s*. 3 *d*. 11. 65 times; Rem Re 1-11-1, 12. 48 times; Rem. £ 1-8-0. 13 15 times.
14. 16 times, Rem. 2 cwt 10 lb 15 21 times, 16. 128 times.
17. 10 times, Rem. 150 yds. 18. 33 times. 19. 35 times.
(*b*) 1. 12 times. 2. 11 times, 8 inches over.

Exercise 111.

1. £3-1-6 2. Rs. 6-14-7. 3 Rs. 7-11-10. £11-2-6.
5. £4-18-8 6. Rs. 4-12-5. 7 Rs. 1821-7-10. 8 Rs 226-4-7.

Exercise 112

1. Rs 174-4-11; Rs. 500-8-8. 2. C has £447-19-6.
3. D has Rs. 1924-1-0. 4. James, £779-11-0; Joseph £738-4-0.
5. Whole property. Rs. 10660-6-0. Difference Rs. 5740-3-6.
6. A. £69-10-8, B. £86-18-4. C. £191-4-4 7. Rs. 40,000.
8. They are equal, each being Rs. 590-7-5.
9. A. Rs. 24456-9-3, B. Rs. 58913-7-0. C. Rs. 116629-15-9.

Exercise 113.

1. (*a*) Rs 69-14-0 (*b*) Rs 89-9-0. (*c*) Rs. 68-12-9.
2. £552-9-6. 3. £8,249-17-6½

Exercise 114.

1. Re 1-2-6. 2. 10 as. 6 p. 3. 2 as. 6 p. 4. 60.
5. Re. 1-8-6.

Exercise 115.

1. 6 mls. 4 fur 88 yds. 2. Rs. 21-4-8. 3. £166-17-9.
4. 2 mds. 5 vis. 30 18/21 pal 5. Rs. 2,624-12-6.

Exercise 116.

407 yds. 2. 5*s*. 4*d*. 3. 3 as. 6 p. 4. 6 bulls.

Exercise 117.

1. 12 as. 11 pies. 2. £4-5-1. 3. 1 cwt. nearly.
4. 4 pies nearly.

Exercise 118.

(A) 1. £299-18-5, £500-6-11.
2. (*a*) £249-10-8½, £450-9-3½.
(*b*) 50 mls. 1 fur 50 yds; 70 mls 5 fur. 50 yds
(*c*) 37 mds 1 vis 18 srs., 62 mds 6 vis. 22 srs.

(B) 1. £34-19-0. £52-5-5. 2. 22 miles 5 fur., 25 miles 7 fur.
3 House, Rs. 4,799-11-9; Garden, Rs. 5,200-4-3.
4. Rs. 302-12-0, Rs. 362-12-0. 5. Rs. 96-0-3, Rs. 104-7-9.

Exercise 119.

1. £15-4-6. 2. Rs. 42-3-6. 3. 11 mds, 7 vis. 2 srs.
4. £26-6-2.

Exercise 120.

1. 4 as 6 ps. 2. 5 as. 5 ps., Rs. 3-6-2. 3. £4-9-3.
4. Rs. 1,931-9-0; Rs. 2-14-10.
5 (*a*) Rs 262-8-0, (*b*) Rs 3,150-8-0, Rs. 8-9-8.
6. 2 days 18 hrs 40 min. 7. 50.500 gui.
8. 1,10,930, 2 oz. 9. 10 lb.
10. Henry has 252 coins more than Joseph.
11 258 (or 259). 12 Rs. 14-8-6 13. 39, Re 1-4-6.
14. Rs 6-0-7. 15. 2*s*. $5\frac{1}{2}d$. 16 11 seconds.

Exercise 122.

A.		B.		C	
1.	Rs 614-8-6.	1	£1,607-2-9.	1	Rs 977-8-0.
2.	Rs 449-2-9	2.	£215-16-0	2	Rs 880-0-0.
3	£314-18-0.	3	Rs 168-0-0	3	£4,230-0-0.
4.	£5,323-3-4.	4	Rs 125-0-0.	4	Rs. 37,450-0-0.
5	Rs 571-2-8	5	Rs $54\text{-}15\text{-}11\frac{1}{4}$.	5.	£2,404-8-9.
6	Rs 326-4-0.	6	£274-17-4.	6	£31,375-19-1.

Exercise 123.

1 Rs. 277-9-9. 2. Rs $89\text{-}11\text{-}10\frac{1}{2}$ 3 £$83\text{-}17\text{-}4\frac{1}{2}$.
4. £33-19-6. 5. Rs 78-11-10. 6. £22-14-4.

Exercise 124.

(*a*) 1. Rs 20-11-8 (= Rs 15-12-9 + Rs. 3-7-0 + Re 1-6-8 + 1 a. 3 p.) 2 Rs. 46-11-3 (= Rs. 33-4-0 + Rs 4-7-3 + Rs 9-0-0). 3 £40-17-6 (=£28-2-0+£12-0-0+14*s*. 2*d*. + 1*s* 4*d*). 4. £3,291-5-6 (= £2,263-2-6 + £902-8-0 + £79-4-0 + £46-11-0).
5 Rs 6-3-8 (= Re 1-2-0 + Rs 4-2-8+4 as 3p. + 10 as 9 p.).
6 Rs 8-7-6 (= Re. 1-12-0 + Rs 6-4-0+4 as 9 p + 2as. 9p.).
7. Rs 52-10-8 (=Rs 31-14-0 + Rs 4-2-0 + Rs. 16-10-8).
8 11 as 9 p. (= 9 as 5 p + 1 a. 3 p + 1 a 3 p).
(*b*) Rs 58-12-11 (= Rs 20-13-0 + Rs. 36-1-6 + Re. 1-14-5).

Exercise 126.

1 Rs 2-7-0 (=Rs 5-3-2 — 10 as 2 p —Re. 1-14-5).
2 Re 1-15-1 (= Rs 3-4-6 — 4 as. — 8 as. 5 p).
3 4 as 8 p (= 4 as 2 p. + 6 as + 15 as — Re. 1-4-6).
4. Rs 237-14-8 (= Rs 1000 — Rs 631-14-0 — Rs 86-7-4 — Rs 43-12-0).

Exercise 128

5 65. 6. 10. 7 (i) 5 odd numbers. (ii) 10 odd numbers (iii) 10 (iv) 11. 8. 17,255. 9. 175. 10. 20 11 9680.

Exercise 131.

1 $2^1/_2$ 2 $3^1/_2$. 3 $2^4/_7$ 4. $2^2/_3$. 5. $1^4/_5$. 6. $2^1/_7$.

Exercise 132.

(a) 1. $4^1/_2$. 2. $5^2/_3$ 3 $2^5/_8$. 4. $7^{15}/_{16}$ 5. $2^5/_{16}$.
(c) 1 $1^5/_{16}$ 2 $2^1/_2$ 3 $1^1/_5$. 4. $7^1/_2$.

Exercise 133.

1 $1^5/_{16}$ 2 $4^{55}/_{72}$ 3 $2^{11}/_{40}$ 4 $9^7/_9$ 5. $4^4/_7$. 6. $1^{40}/_{77}$.
7. $1^{13}/_{15}$. 8. $1^{46}/_{55}$ 9 $1^{13}/_{15}$ 10. 3. 11. $4^{16}/_{21}$. 12. $1^1/_5$.

Exercise 134.

(C) 1 $11^1/_2$. 2 12 9 3 $23^1/_2$. 4. 7. 5. 48·2.
6 10 7 $16^1/_2$. 8. 10 9 92 4. 10. 4·3.
11 3/4. 12 0. 13. 30. 14. 63.

Exercise 135

1. $2^2/_3$. 2. $9^3/_7$ 3. $20^1/_8$. 4. 14. 5. $4^1/_2$.

Exercise 137.

1 32. 2. 35. 3. 8 4 36. 5 25 2. 6. 1053 bags.
7 59·5 mls. 8. 800 9. 184·8. 10. 123456.

Exercise 138.

1 11. 2 66 3 15. 4 54. 5 160. 6. 14.
7. 9 8. 1500 9. 18.

Exercise 139.

1. 422 2. 72 3. Rs 72. 4 Rs. 3,750. 5. 114·2.
6 $1,4031^1/_2$.

Exercise 140.

1. 41, 59 2 $20^1/_2$, $23^1/_2$ 4 Horse, Rs 410, Cow, Rs 90 5. House, Rs 2,385, Garden, Rs. 1,665.

Exercise 141.

1. 9. 2. 13. 3. 5. 4. 7. 5 12 6 6 7 $6^1/_2$ 8 $5^1/_2$.

Exercise 142.

1. 75 , 150 2. (a) 120 , 50 (b) 125 ; 345. (c) $151^1/_2$. $200^1/_2$ (d) 80 4 ; 122 4 (e) 456 7 ; 654 3

Exercise 143.

Answers are given correct to the unit.)

1. Rs 26 2. Rs 1313 3 583 miles 4. Rs 34.
5. Rs. yz/x ; Rs 1057

Exercise 144.

1. 16 hours 2. 3 days 3 45 days. 4 xy/z ; 100 days.

Exercise 145.

1. Rs 38 2. Rs 10 3 150 miles. 4 114 mangoes.

Exercise 149.

(a) 1000, 10000, 625, 32, 200, 4000 (b) 10^2, 10^3, 10^5, 10^6, 10^7.
(c) 3^3, 5^4, 4^3 (d) 4×10^3, 6×10^4, 11×10^5.
(f) 1 430211 2 10765 3 20186. 4 90990.

Exercise 153.

(a) 1 ·7 2. 4 4. 3. 30 71 4 1 123 5. 75 004.
6. 8 8. 7. ·06. 8 009. 9. ·000001. 10. ·0002. 11. ·101001.
12. ·001001. 13. 845. 14. ·555. 15. ·707. 16. 23·8.
17. 50·0081 18. 4·00057.

(b) 1. Seven-tenths. 2. Nine-tenths. 3. One-tenth.
4. Two-hundredths. 5. One-tenth and three-hundredths.
6. Four-tenths and five-hundredths. 7 Two, and three-tenths. 8. Fourteen, and four-hundredths and five ten-thousandths 9 One-tenth, nine-thousandths and five-millionths 10. One hundred and twenty and seven-hundred thousandths.

Exercise 154.

	Descending order.	*Ascending order.*
(b) 1.	·41, ·4, ·35 ,	·35, ·4, ·41.
2.	·12, ·05, ·043 ;	·043, ·05, ·12.
3.	·51, ·50, ·405;	·405. ·50, ·51
4.	1·43, 1·1, 1 089 ;	1·089, 1·1, 1·43.
5.	·01, ·003, ·0011 ;	·0011, ·003, ·01.
6.	·4336, ·3, ·0345 ;	·0345, ·3, 4336.

(c) 0001 , ·99999.

Exercise 156.

(*a*) 1. 195 069. 2. 243 551. 3. 5361·8. 4 22 8.
(*b*) 1. 60 98, 2 72 26. 4. 2.
(*c*) ·666. (*d*) 1. 9 106. 2 9 576. (*e*) 1.

Exercise 157.

(*a*) 1. 1·308. 2. 11 112 3 ·819. 4 108 99.
5 2 7036. 6 11 451 7. ·056.
(*b*) 1441 4895 (*c*) 6 285 (*d*) 9·055 (*e*) 1 326. (*f*) ·9951.
g) 1·0046 (*h*) 1 02 (*i*) 33·3 (*k*) 3 495 (*l*) (i) ·536.
ii) 1 023, (iii) 1 976. (*m*) 1. 7·044. 2 10 7.

Exercise 158.

(*a*) 1 41 314 2 40 406 3. 1·26 4. 212 636.
5. 3 34. (*b*) 1248 156 (*c*) 114 227, (*d*) (i) 4026 (ii) 16·374.
(*e*) (i) 242. (ii) 89 445.

Exercise 159.

(*b*) 1. 37·6 2. 108 5. 3. 450. (*c*) 1. 1581·42.
2. 401 8.

Exercise 160.

(*a*) 1. 33 372. 2 48 52 3. 37·52. 4. 46·002.
5. 90 81. 6. 138. 7 29 939, 8. 70·77. 9. 728·1.
(*b*) (i) 1440·4. (ii) 2160 6, (*c*) ·32. (*d*) 368 042.

Exercise 161.

(*a*) 1. 533 088. 2. 371·856 3. 1466 199. 4. 1882 53.
5 165 5. 6. 1471·808. 7 158·704. 8. 1 665, 8. 1481·9.
10. 7048·5. 11. 9363·6 12. ·4606. 13. ·0184. 14. 1304.
15. ·1248,
(*b*) (i) 18·432. (ii) 34 56 (iii) 467 712. (*c*) (i) ·5442.
(ii) ·1496. (*d*) (i) 2663 5. (ii) 59 76. (*e*) 132 3.
(*f*) (1) 17·8. (2) ·855.

Exercise 162.

1. 94 152. 2 649·44. 3. 1955·36. 4. 1 1515.
5. 64·32. 6. 338·921.

Exercise 163.

1. 3321·6. 2. 1206 72. 3. 325689. 4. 18126. 5. 5·88.
6. ·135756 7. 9·6 8. 19 608. 9. 14410 131.

Exercise 164.

(*b*) 1. 1 324. 2. 2·005. 3 ·3275. (*c*) (i) 2·279. (ii) ·0189.

Exercise 165.

(a) 1. 15 129. 2. 49·609. 3. 12 509. 4. 100 63.
5. 82·3423. 6. 18·32. 7 1·47092 8. 10·1827.
9. 16 674. (b) (i) 30 72. (ii) 11·52.

Exercise 166.

1. 1·0036. 2. 25 039. 3 10·0137 4. 8·018.
5. 61·029. 6. 2·0202 7. ·25902. 8. 4·0048.
9. 20 0469.

Exercise 167.

1. ·212. 2. ·2609. 3. ·15706. 4. ·3201.
5 36406. 6. ·56256. 7. 5018002. 8. 29502.
9. 607.

Exercise 168

1. ·0909. 2. ·0333 3. ·057002. 4. ·01372.
5. ·0137. 6. 00909. 7. 000206. 8. 0006006.
9. ·000209.

Exercise 169.

2. 1·2125. 2. 6·275. 3. 6 275. 4. 3·885.
5. ·15625. 6. ·015375. 7. 005975. 8. ·0205.
9. ·00083.

Exercise 170.

1. 1 125. 2. 3 75. 3. ·75. 4. 3 5. 5. ·1875.
6. ·125. 7. 3125. 8. 3 75.

Exercise 171.

(A) 1. 42 5. 2. 3 09008. 3. 4 070009.
4. ·0074. 5. ·001027 6. 00364.
7. ·30909625. 8. ·0625. 9 5·4.
(B) 1. 1 025. 2. 10 5. 3. 5 04. 4. 204.
(C) 15.

Exercise 172.

1. 2·304. 2 4 85. 3. 4 046. 4. 11 04. 5 ·1234.
6. 2 005. 7. 045. 8. 0412. 9. ·0145.

Exercise 173.

1. 3·14. 2. 4 05. 3. 12 16. 4. 4·25 5*. ·15625
6. ·675. 7. 1 625. 8. 1 075. 9. 3 125.

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* In the sum, correct 424 into 256.

Exercise 174.

(A) 1. 4 54. 2. 0145. 3. 425. 4. 1 05.
5. 3 03125. 6. ·104.

(B) 1. 12 04. 2. 31 4. 3. 00335. 4. 1 056.
5. 2 022. 6. 500 5.

Exercise 175.

1. 1·302 ..... 2. 5·234...... 3. ·475...... 4. 2 071.....
5. ·015...... 6. ·030...... 7. 2 2272..... 8. 1071.....
9. 1 4814 .... 10. 07428 ..... 11. 2·0480...... 12. 8 0662....
13. ·20937 .... 14. 36 34444 .... 15. 04722......

Exercise 178.

2. (*a*) 1 Km. = 1000 m. (*b*) 1 m = 001 Km. (*c*) 1 m = 100 cm. (*d*) 1 cm. = 01 m. (*e*) 1 cm = 10 mm. (*f*) 1 mm. = ·1 cm. 3. 1 Dm. = 393 7079 in.; 1 Hm = 3937·079 in., 1 Km. = 39370 79 in. 4. 1 dm. = 3 937 in.; 1 cm. = ·3937 in.: 1 mm = 03937 in, 1 Dm. = 393 7 in, 1 Hm. = 3937 in, 1 Km. = 39370 in. 5. (*a*) 50 mm. 54 mm.; 25 cm. (*b*) ·4 cm., 3 4 cm., 3 2 dm (*c*) ·007 m., 1 003 ms., 2 06 ms. 6. (*a*) 9 cm. 5 mm (*b*) 9 cm 4 mm 7. (*a*) 3 cm 4 mm. (*b*) 2 cm. 6 mm 8. (*a*) 8 2 cm. (*b*) 1·8 cm. (*c*) 7 cm. 2 mm. 9. (*a*) 7 cm. 5 mm. (*b*) 7 cm. (*c*) 52 cm. 10. (*a*) 1 cm 6 mm. (*b*) 1 cm. 8 mm. (*c*) 2 cm. 9 mm. 11. 8 pieces; 1 cm. 2 mm. 12. (*a*) 43 cm. 2 mm (*b*) 42 cm. (*c*) 1 cm. 5 mm. (*d*) 1 cm. 8·4 mm. 13. 6 cm. 7 2 mm. 14. 38 cm. 7 mm. 15. 7 pieces; 9 cm. 16. 6 lengths.

Exercise 179.

(2) (*a*) 12". (*b*) 1 6". 3. 39370", 40,000". Difference = 630".
4. 11 1/9 yards 5. (*a*) 1840"; (*b*) 5040" 6. 39 6".
7. 997 35 yds. or 1,000 yds. nearly 8. (*a*) 39 38". (*b*) 39 6".
9. 39 283".

Exercise 180.

(*A*) 1. 45°, 30°, 110°. .. 2. 1/3, 1/2, 1/4, 1/5, 1/6, 1/9, 5/36.
3. 1 1/3, 1 1/4, 2 1/2, 3 1/3.

(*B*) 1. 180°, 360°. 2. 360°, 6°, 30°. 3. 30°, 75°. 4. 6°.

Exercise 184.

(*a*) 1. 2, 3, 4, 8. 2. 2, 3, 4, 8, 9. 3. 2, 3, 5 9, 11.
4. 2, 5, 9. 5. 5, 11. 6. 3, 9. 7. 2, 3, 4, 8.
8. By none

(*b*) 1. 2, 8. 2. 1. 3. 4. 4. 3.

Exercise 185.

1. $2 \times 3^3 \times 5$. 2. $2^2 \times 3 \times 19$. 3. 3×11^2.
4. $2^2 \times 11 \times 17$ 5. $7^2 \times 11$. 6. $2^2 \times 3^2 \times 5 \times 11$.
7. $2 \times 5^3 \times 7$ 8. $5 \times 3 \times 7 \times 11^2$. 9. $2 \times 3^3 \times 5 \times 11$.
10. $3^2 \times 5 \times 11 \times 13$ 11. $2 \times 3^2 \times 5^2 \times 7 \times 11 \times 13$.
12. $2^2 \times 5^2 \times 7^2 \times 11^2$.

Exercise 186.

(*A*) 1. 12 2. 3. 3. 3 4. 6 5. 5. 6. 31. 7. 77, 8. 2.
(*B* 1. 5. 2. 2. 3. 6. 4. 15.

Exercise 189.

4. 10, 15, 20, 25, 30, 35, 40, 45 (mult. of 5); 14, 21, 28, 35, 42, 49, (mult. of 7), 18, 27, 36, 45. (mult. of 9); 20, 30 40 (mult. of 10); 24, 36 48 (mult of 12). 5. 9. 36, 45, 54; 10, 40 50, 60; 11, 33, 44, 55. 6. 120, 144, 168, 192. 7. Rs. 140, Rs. 175, Rs 210, Rs 245. 8. 75, 100, 125, 150 inches; 3, 4, 5, 6.

Exercise 190.

6. $130°$. 7. $\angle QOR = 120°$, $\angle ROS = 60°$, $\angle SOP = 120°$.

Exercise 192

(A) 2. 75°, 80°, 105°, 135°. 3. 150°, 120°, 105°, 60°, 30°, 10°. 4. 115°. 5. 45°, 135°, 6. 81°, 99°.

(B 2. 50°, 20°, 55° $67\frac{1}{2}$°. 3. 30°, 45°, 42°, 53°, 54°. 18°, $66\frac{1}{2}$°. 4. 18°, 72°, 5. 35°, 55°.

Exercise 193.

1. 2,3,6, 6. 2. 7. 7. 3. 2,3,6, 6 4. 3,5,15, 15. 5. 2,4, 4 6. 2,3,6; 6 7. 3, 3 8. 7, 7. 9. 3,5,3×5; 3×5. 10. 2; 2^2, 2^2. 11. 3, 3^2, 3^3, 3^3. 12. 7; 7.

Exercise 195

1. 10, 5,7. 2. 9, 5, 11. 3. 15, 7,4. 4. 14; 5,3. 5. 7, 9,2,7. 6. 6, 5,7,14. 8. 11; 6,5,8. 7. 3×5, 1,2. 9. 3×11; 7,3.

Exercise 196.

1. 2×3^2. 2. $2^2 \times 3^3$. 3. 7×11.
4. $2 \times 5 \times 7^2$. 5. 3×5^2. 6. 3×5.
7. $3 \times 5 \times 7$. 8. $5 \times 7 \times 11$.

Exercise 197.

(*a*) 1. 20, 2, 5. 2. 15, 6, 35. 3. 50, 2, 3. 4. 10; 35, 6. 5. 45, 11, 10. 6. 95, 15, 4. (*b*) 1. 30, 4, 15, 25. 2. 75, 6, 2, 9 3. 105, 7, 35, 11. 4. 55, 15, 9, 22. 5. 56, 9, 27, 70 6. 95, 35, 6, 40.

Exercise 199.

(A) 1. 13. 2. 15. 3. 24. 4 31. 5. 18. 6. 61.
7 33 8. 42. 9. 41.

Exercise 200.

(A) 1. 3. 2 12. 3. 2. 4. 21 5. 42. 6. 323.

Exercise 201.

1. Rs. 2-8-0 2. 4 as. 8 p. 3. 9 pies. 4. Rs. 5-7-6.
5. 11*d*. 6 £2-5-0. 7. 5*s*. 6*d*. 8. 1 hr. 15 min.
9 5 vis 8 srs 2 pal. 10. 8 as 6p 11 8*d*. 12. 1 qr. 8 lb.
13. (*a*) 1 cm 5 mm. (*b*) 7 m. (*c*) 6 cm. (*d*) 4·2 cm.

Exercise 202.

1 3. 2 18. 3. 28. 4 33. 5. 45. 6. 6 ft 3 in. 7. 80 ft.
8. 8 in , 26. 9. Rs 60-4-0 10. 14 ft. 11. 15 mrkls; 26.
12. 4 cubits; 18 men.

Exercise 206

1. 42. 2. 30. 3. 16. 4 125 5. 450. 6. 882. 7. 360.

Exercise 207.

1. 120. 2 270. 3 288. 4 144. 5 105. 6. 1,200.
7. 360. 8. 6,048. 9 945. 10 4,500. 11. 1,260. 12. 216.
13. 72. 14. 72 15. 1·470 16. 27,000.

Exercise 208.

(3) .10 153 (5) 15.

Exercise 209.

1. 27761. 2. 30504. 3. 16497. 4. 22932. 5. 223200.
6. 331200. 7 130680. 8. 32400. 9. 157080. 10 118400.
11 214775. 12. 343125.

Exercise 210.

(*a*) 1. 672. 2. 7200. 3. 27324. 4. 5040. 5. 1260.
6 1890. 7. 7200 8. 180. 9. 720 10 21945.
11. 2640. 12 7560 (*b*) 840. (*c*) (i) 120; (ii) 315.

Exercise 211.

1. Rs. 52-8-0. 2 Rs. 26-8-8. 3. Rs. 15-2-3.
4. Rs. 180-7-6. 5. £876-14-11. 6. £13-10-0.
7 49-10-0. 8. 140 hrs. 9. 54 mds. 3 vis 2 pals.
10. Rs. 89-4-0. 11 £2-0-0. 12. 41 cwt 2 qrs 4 lb.
13. (*a*) 3 cm. (*b*) 21 metres. (*c*) 19656 cm. (*d*) 81 9 metres.

Exercise 212.

1. 252, 504, 1008, 2016. 2. 420. 3. 62. 4. 62, 122; 182; 242. 5. Rs. 30. 6. Re. 0-5-0. 7 294 ft. 8 422; 842; 1262. 9. 606, 726. 10. 1200; 9600. 11. 75. 12. 774400 sq yds. 13. 40 ft ; 4 and 3. 14. 1 hr, A, 6; B, 4, and C, 3. 15. 120.

Exercise 214.

(B) 1. 10 in. 2. 180·1 in 3. 96; 40 ft.

Exercise 215.

(A) 1. $\frac{3}{8}$. 2. $\frac{6}{21}$. 3. $\frac{1}{100}$. 4. $\frac{31}{20}$. 5. $\frac{9}{12}$. 6. $2\frac{1}{2}$.

(B) 1. Two-thirds. 2. Seven-elevenths. 3. Thirteen-eighths 4. One hundred and one-thousand firsts. 5. Seven and fourteen thirty-eighths.

Exercise 217.

(*a*) 1. $\frac{18}{24}$. 2. $\frac{10}{24}$. 3. $\frac{21}{24}$. 4. $\frac{48}{24}$. 5. $\frac{16}{24}$. 6. $\frac{30}{24}$.

(*b*) 1. $\frac{48}{64}$. 2. $\frac{48}{88}$. 3. $\frac{48}{24}$. 4. $\frac{48}{90}$. 5. $\frac{48}{63}$. 6. $\frac{48}{45}$.

(*d*) 1. 100; 28. 2. 30. 3. 84; 78. 4. 165; 165. 5. 5. 6. 400 : 400.

(*e*) 105 sevenths; 205 ninths. (*f*) 154 elevenths.

Exercise 219

(*a*) 1. $\frac{3}{4}$. 2. $\frac{7}{6}$. 3. $\frac{4}{5}$. 4. $\frac{4}{5}$. 5. $\frac{3}{7}$. 6. $\frac{5}{8}$. 7. $\frac{3}{11}$. 8. $\frac{7}{100}$. 9. $\frac{7}{22}$. 10 $\frac{11}{12}$.

(*b*) 1. $\frac{5}{17}$. 2. $\frac{1}{18}$. 3. $\frac{9}{17}$. 4. $\frac{2}{13}$. 5. $\frac{9}{11}$. 6. $\frac{2144}{1659}$. 7. $\frac{11}{21}$. 8. $\frac{13}{120}$.

Exercise 220.

9. $\frac{3991}{480}$. 10. $\frac{1127}{16}$. 11. $\frac{24079}{120}$. 12. $\frac{719}{205}$.

Exercise 221.

11. $27\frac{7}{11}$. 12. $\frac{25}{1}$. 13. $13\frac{1}{18}$. 14. $20\frac{7}{120}$. 15. $9\frac{37}{1000}$.

Exercise 222.

5. $121\frac{4}{7}$. 6. $50\frac{8}{9}$. 7. $95\frac{53}{69}$. 8. $703\frac{9}{14}$. 9. $550\frac{16}{19}$. 10. $73\frac{5505}{5728}$.

Exercise 223

(*a*) $\frac{360}{960}$, $\frac{400}{960}$, $\frac{660}{960}$. (*b*) $\frac{120}{320}$, $\frac{120}{1680}$, $\frac{120}{252}$.

(*c*) $\frac{81}{108}$, $94\frac{1}{2}/108$, $\frac{90}{108}$. (*d*) 1. $\frac{20}{30}$, $\frac{18}{30}$, $\frac{21}{30}$.

2. $\frac{80}{110}, \frac{65}{110}, \frac{33}{110}$. 3. $\frac{18}{24}, \frac{15}{24}, \frac{14}{24}$. 4. $\frac{32}{60}, \frac{42}{60}, \frac{33}{60}$. 5. $\frac{30}{60}, \frac{40}{60}, \frac{45}{60}, \frac{48}{60}$. 6. $\frac{105}{315}, \frac{189}{315}, \frac{225}{315}, \frac{245}{315}$. 7. $\frac{48}{96}, \frac{64}{96}, \frac{72}{96}$. 8. $\frac{1470}{1680}, \frac{360}{1680}, \frac{720}{1680}, \frac{765}{1680}$. 9. $\frac{297}{528}, \frac{272}{528}, \frac{242}{528}, \frac{220}{528}$.

Exercise 225.

The gretest and the least fractions are as follows :—

1. $\frac{4}{5}$. $\frac{2}{3}$. 2. $\frac{7}{20}$. $\frac{2}{15}$. 3. $\frac{10}{16}$. $\frac{9}{28}$. 4 $\frac{6}{20}$. $\frac{1}{4}$.

5. $\frac{2}{11}$. $\frac{7}{44}$. 6. $\frac{3}{2}$, $\frac{19}{144}$.

7. (a) $\frac{11}{12}$. $\frac{5}{6}$. $\frac{1}{18}$; (b) $\frac{1}{18}$. $\frac{5}{6}$ $\frac{11}{12}$

8. (a) $\frac{5}{6}, \frac{12}{16}, \frac{17}{24}, \frac{2}{3}$; (b) $\frac{2}{3}, \frac{17}{24}, \frac{12}{16}, \frac{5}{6}$.

9. (a) $\frac{3}{8}, \frac{3}{10}, \frac{1}{4}$; (b) $\frac{1}{4}, \frac{3}{10}, \frac{3}{8}$.

10. (a) $\frac{2}{3}, \frac{7}{12}, \frac{1}{6}$; (b) $\frac{1}{6}, \frac{7}{12}, \frac{2}{3}$.

11. (a) $\frac{11}{16}, \frac{5}{12}, \frac{1}{3}, \frac{2}{8}$; (b) $\frac{2}{8}, \frac{1}{3}, \frac{5}{12}, \frac{11}{16}$.

12. (a) $\frac{2}{3}, \frac{30}{50}, \frac{7}{15}, \frac{5}{12}$; (b) $\frac{5}{12}, \frac{7}{15}, \frac{30}{50}, \frac{2}{3}$.

The fractions in descending order of magnitude are ,—

13. $\frac{7}{8}, \frac{5}{6}, \frac{3}{4}, \frac{1}{2}$. 14. $\frac{10}{12}, \frac{6}{8}, \frac{2}{4}$.

15. $\frac{16}{24}, \frac{35}{56}, \frac{3}{5}$. 16 $\frac{7}{11}, \frac{6}{10}, \frac{7}{12}$.

17. $\frac{7}{6}, \frac{8}{7}, \frac{9}{8}$. 18. $\frac{7}{8}, \frac{6}{7}$ $\frac{5}{6}$.

Exercise 226.

1. $\frac{19}{35}$. 2. $\frac{5}{8}$. 3. $\frac{19}{20}$. 4. $\frac{10}{24}$.

Exercise 227.

1. $1\frac{1}{12}$. 2. $1\frac{1}{16}$. 3. $\frac{11}{15}$. 4. $\frac{59}{60}$. 5. $1\frac{1}{12}$.
6. $\frac{23}{30}$. 7. $\frac{31}{36}$. 8. $\frac{5}{6}$. 9. $2\frac{1}{36}$. 10. $2\frac{23}{60}$.
11. $2\frac{23}{60}$ 12. $2\frac{1}{12}$. 13. $1\frac{73}{90}$. 14. $1\frac{5}{42}$.

Exercise 228—(*Oral*).

1. $1\frac{1}{12}$. 2. $\frac{7}{24}$. 3. $\frac{3}{4}$. 4. $1\frac{1}{9}$. 5. $1\frac{1}{6}$. 6. $1\frac{7}{9}$.
7. $2\frac{2}{3}$. 8. $3\frac{7}{12}$.

Exercise 229.

1. $6\frac{5}{6}$. 2. $3\frac{1}{4}$. 3 $14\frac{1}{2}$ 4. $13\frac{41}{60}$. 5. $4\frac{17}{28}$. 6. 43.
7. 4. 8. $9\frac{14}{15}$ 9. $13\frac{58}{63}$. 10. $11\frac{49}{90}$. 11. $26\frac{329}{780}$.
12. $20\frac{1}{72}$. 13. $19\frac{113}{285}$. 14. $20\frac{311}{420}$.

(*b*) 1. $15\frac{37}{170}$. 2. $\frac{61}{72}$. 3. $139\frac{17}{28}$. (*c*) 1. (i) $25\frac{23}{30}$
(ii) $45\frac{7}{10}$ (iii) $40\frac{8}{15}$ (iv) 56. 2. $x = 18\frac{17}{210}$.
(*d*) 1. $\frac{7}{8}$. 2. The whole garden.

Exercise 230.

1. $\frac{7}{16}$. 2. $\frac{3}{8}$. 3. $\frac{1}{4}$. 4. $\frac{11}{20}$. 5. $\frac{1}{20}$. 6. $\frac{8}{21}$.
7. $\frac{11}{15}$ 8. $\frac{1}{3}$. 9. $\frac{11}{36}$. 10. $\frac{7}{10}$. 11. $\frac{9}{16}$. 12. $\frac{4}{5}$.

Exercise 232.

(*a*) 1. $1\frac{11}{35}$. 2. $2\frac{1}{2}$. 3. $2\frac{1}{2}$, 4. $4\frac{13}{20}$. 5. $\frac{13}{40}$.
6. $1\frac{9}{10}$. 7. $\frac{2}{9}$. 8. $\frac{3}{8}$. 9. $\frac{2}{3}$ 10. 0. 11. 0.
12. $\frac{2}{7}$. 13. $\frac{5}{24}$. 14. 4. 15 $34\frac{1}{6}$. 16. $4\frac{61}{84}$.
17. $\frac{1}{4}$. 18. $1\frac{3}{4}$. 19. $\frac{5}{24}$. 20. $\frac{13}{18}$. 21. $\frac{1}{12}$.

(*b*) 1. $82\frac{1}{2}$ 2 $9\frac{41}{64}$. 3. 6 4. $2\frac{11}{60}$. 5. $15\frac{7}{15}$ & $13\frac{8}{15}$.

(*c*) 1. $\frac{13}{24}$. 2 John $5\frac{1}{6}$, Samuel $3\frac{1}{6}$.
3. Son's age $17\frac{7}{24}$; total $80\frac{11}{12}$. 4. $\frac{8}{21}$; $\frac{1}{21}$.
(*d*) 1. $53\frac{47}{54}$. 2. $27\frac{3}{108}$. 3. $157\frac{53}{108}$. 4. $16\frac{10}{17}$.

Exercise 234.

1. $\frac{13}{48}$. 2. $\frac{25}{48}$. 3. $4\frac{7}{12}$. 4. $68\frac{19}{20}$. 5. $1\frac{7}{24}$.
6. $\frac{1}{28}$. 7. $\frac{1}{2}$. 8.—$\frac{11}{60}$. 9. $\frac{7}{60}$. 10. $1\frac{1}{2}$.
11. $\frac{1}{5}$. 12. $2\frac{1}{2}$.

Exercise 235.

(A) 1. (*a*) $3\frac{1}{2}$. (*b*) $4\frac{2}{3}$. (*c*) 7.

2. (*a*) $2\frac{2}{3}$. (*b*) $5\frac{1}{3}$. (*c*) 8.

(B) 1. (*a*) $9\frac{1}{3}$. (*b*) 14. (*c*) 21. (*d*) $44\frac{1}{3}$.

2. (*a*) $69\frac{1}{9}$. (*b*) $73\frac{1}{3}$. (*c*) $146\frac{2}{3}$.

3. (*a*) $1767\frac{1}{5}$. (*b*) 8336. (*c*) 17672.

(C) 1. 2585. 2. $2157\frac{5}{7}$. 3. $18121\frac{2}{7}$.
4. $34923\frac{1}{2}$. 5. $66750\frac{1}{3}$. 6. $10001\frac{3}{16}$.

Exercise 236.

(A) 1. (*a*) $\frac{3}{10}$. (*b*) $\frac{3}{20}$. (*c*) $\frac{3}{40}$.

2. (*a*) $\frac{9}{50}$. (*b*) $\frac{2}{25}$. (*c*) $\frac{3}{200}$.

3. (*a*) $\frac{4}{87}$. (*b*) $\frac{2}{29}$. (*c*) $\frac{10}{201}$.

(B) 1. (*a*) $1\frac{1}{4}$. (*b*) $3\frac{1}{8}$.

2. (*a*) $1\frac{4}{5}$. (*b*) $\frac{3}{5}$.

3 (*a*) $20\frac{2}{21}$. (*b*) $7\frac{1}{30}$.

4. (*a*) $1\frac{59}{121}$. (*b*) $1\frac{14}{121}$. (*c*) $1\frac{68}{121}$.

(C) 1. $111\frac{16}{81}$. 2. $204\frac{6}{69}$.

Exercise 238.

1. 3. 2. $4\frac{1}{5}$. 3. $4\frac{1}{2}$. 4. 1. 5. $2\frac{1}{3}$. 6. $7\frac{1}{2}$.
7. $\frac{39}{187}$. 8. $6\frac{27}{28}$. 9. $2\frac{1}{6}$ 10. $5\frac{75}{106}$.
11. $2\frac{559}{748}$ 12. $1\frac{49}{315}$.

Exercise 239.

(A) 1. $6\frac{1}{4}$. 2. 4312. 3. $\frac{5}{6}$. 4. $2\frac{3}{56}$.
5. $\frac{1}{2}$. 6. $17\frac{1}{3}$. 7. $42\frac{32}{45}$. 8. 1085. 9. 315.
10. $33\frac{3487}{12225}$. 11. $4\frac{9}{40}$. 12. $141\frac{3}{32}$.

(B) 1. $7\frac{7}{8}$. 2. $10\frac{3}{4}$. 3. 119. 4. 216.
5. $-1\frac{5}{33}$. 6. $90\frac{37}{40}$.

(C) 1. (*a*) 314 yds. (*b*) 24904. 2. 336.
3. 30. 4. $3\frac{6}{7}$ 5. $28\frac{33}{56}$.

Exercise 241.

(A) 1. $\frac{2}{3}$. 2. 15. 3. 2. 4. $\frac{9}{100}$. 5. 2.
6. $\frac{2}{3}$. 7. $6\frac{2}{9}$. 8. $1\frac{1}{55}$. 9. $\frac{21}{52}$. 10. $1\frac{1}{2}$.
11. $7\frac{1}{2}$. 12. $\frac{4}{15}$. 13. $4\frac{4}{5}$. 14. $\frac{67}{320}$. 15. $3\frac{13}{14}$.
16. $\frac{19}{25}$. 17 $\frac{15}{22}$. 18. $10\frac{1}{2}$.

(B) 1. $\frac{7}{80}$. 2. $2\frac{9}{31}$. 3. $11\frac{11}{18}$. 4. 24.
5. $1\frac{6353}{12300}$. 6. $\frac{5}{149}$.

(C) 1. (*a*) $9\frac{9}{52}$. (*b*) $\frac{6}{7}$.

Exercise 242.

(A) 1. $\frac{15}{16}$. 2. $\frac{4}{9}$. 3. $1\frac{1}{116}$. 4. 28. 5. 1.
6. $2\frac{11}{18}$. 7. 800. 8. 60.

(B) 1. $20\frac{5}{28}$. 2. $112\frac{35}{48}$. 3. $4\frac{25}{44}$. 4. $11\frac{5}{24}$.

(C) 1. $\frac{1}{6}$. 2. $\frac{1}{10}$.

Exercise 243.

(*a*) 1. $\frac{121}{300}$. 2. 3. 3. $\frac{1}{10}$. 4. $\frac{2}{3}$. 5. $1\frac{2}{3}$.
6. $2\frac{1}{2}$. 7. 9. 8. $4\frac{1}{2}$.

(*b*) 1. 23. 2. 24. 3. $\frac{1}{4}$. 4. $\frac{15}{28}$. 5. $\frac{5}{31}$. 6. 1.

(*c*) 1. 77. 2. $1\frac{1}{7}$. 3. $2\frac{22}{49}$.

Exercise 245.

1. (*a*) 3 feet; (*b*) 6 yds; (*c*) 12 miles; (*d*) 3 metres; (*e*) 30 feet. (*f*) 300 yds. 2. 1 in. = 90 miles. (*a*) 270 miles; (*b*) 225 miles; (*c*) 67½ miles. 3. (*a*) 200 metres; (*b*) 550 metres; (*c*) 2·4 cm. (*d*) 3·25 cm.; (*e*) 10 cm.

Exercise 246.

1. £ 9-3-6. 2. £ 14-5-4 3. Rs 8-9-1. 4. Rs. 5-9-10. 5. £ 4-11-4. 6. Rs 13-14-11. 7. 2 cwt. 3 qrs. 8. 3 miles 4 fur 104 yds.

Exercise 247.

(A) 1. 7 yds $4\frac{2}{3}$ in. 2 Rs. 47-11-6. 3. Rs. 115-13-4.
4. 5 mds. 7 viss. 38 pal. 5. 8 s. $1\frac{1}{2}$d 6. 12s $8\frac{1}{2}$ d.
7. 3 ft 8 35 tons 4 cwt 9 £ 33-15-0.

(B) 1 Rs 19-12-$11\frac{1}{3}$. 2 Rs 41-0-11. 3. Rs. 22-1-7.
4 £ 234-10-5. 5 Rs 364-9-4.

Exercise 248

1. Re 1-2-$2^{1}/_{4}$ 2 Rs 5-3-$9^{5}/_{7}$. 3 Rs 7-4-8.
4 £ 37-1-$5^{1}/_{7}$. 5 £ 8-0-$9^{3}/_{5}$ 6. £2-4-8

Exercise 249.

1. Rs. 7-10-5. 2. 1 s 3 d 3. Rs. 2-6-10. 4. Rs. 39-3-$2\frac{1}{3}$.

Exercise 250.

1. 16 s 8 d. 2. 3 s 3 11s. 8 d 4. 10 s. 10 d. 5. 5 as. 4p.
6 13 as. 4 p 7 8as. 8p. 8 9 pies 9 3 s. $1\frac{1}{3}$ d. 10. 7s. $9\frac{1}{3}$d.
11. 4 as $5\frac{1}{3}$ p. 12. 3 as $6\frac{2}{3}$ p

Exercise 251.

1. 1 qr. 14 lb. 2 3 v. 2 srs 4 pal. 3. 1. fur 20 poles. 4. $6\frac{2}{3}$ in.

Exercise 252.

A. 1. £ 1/16. 2. £ 1/6. 3. £ 1/3. 4. £ 1/80. 5. £ 8/15.
6 Re. 1/3. 7. Re. 2/3. 8. £ 3/4. 9 Re. 1/12. 10. Re. 1/6.
11 Re. 3/32 12 Re. 1/8 13 Re. 7/12. 14. £ 13/32.
15 £ 41/120. 16. £ 7/480

B. 1 (*a*) 7/8 (*b*) 5/16. 2 (*a*) 3/4 (*b*) 7/9 (*c*) 7/12. 3. (*a*) 5/16. (*b*) 1279/1320. 4. (*a*) 3/20 (*b*) 5/6.

Exercise 254.

(*a*) 1. £$4\frac{3}{8}$. 2 £$1\frac{1}{3}$. 3. £$2\frac{7}{8}$.

(*b*) 1 Rs $4\frac{1}{2}$. 2 Rs. $5\frac{1}{3}$ 3. Re. $1\frac{31}{48}$. 4. Re. $1\frac{1}{6}$.

(*c*) 1. Rs. 5-6-0. 2. Rs. 4-13-4 3 Rs. 4-6-8. 4. Re. 1-10-8.

(*d*) 1. £4-11-8. 2. £3-6-3 3. £8-6-8. 4. £10-12-6.

(*e*) 1. 3 yds. 1 foot, 3 in. 2. 4 yds. 6 in. 3. 2 yds. 1 ft. $1\frac{1}{3}$ in.

Exercise 257.

1. 2 as. 6 p. 2 Rs 87-8-0 3 $10\frac{1}{2}$ miles
4. 14 as. 3 p 5. 4 *s* 6 *d*. 6. 10 as 8 p
7. 45 min. 8. 1 vis 32 pal. 9 Rs. 100.
10. 16 quires 11. 2 as 8 p. 12. 7 *s*. 8 *d*
13. Rs. 6-2-0. 14. Rs 7-8-2.

Exercise 258

1. $16\frac{2}{7}$ hrs. 2 $3\frac{3}{8}$ days 3. $5\frac{5}{9}$ days. 4. $49\frac{1}{2}$ hrs.
5. 11 men. 6 16 days. 7. $9\frac{2}{3}$ hrs.

Exercise 259

1. Rs. $\frac{yr}{x}$. (i) Rs 10-8-0; (ii) Rs 8.

2. $\frac{ly}{x}$ s; (i) $2\frac{61}{64}$s or 2 s $11\frac{7}{16}$ d (ii) £1-10-$7\frac{1}{2}$

3. $\frac{mx}{y}$ days; (i) 8 days; (ii) $18\frac{2}{3}$ days

4. $\frac{xz}{y}$ lines: (i) 129 lines; (ii) 459 lines.

Exercise 260.

1. 5/24 2. 9/28. 3. 5/12. 4. 3/8 5 7/8.
6. 1/18. 7. 1/40. 8 7/16. 9. Rs 400. 10. £450.
11. Rs. 60. 12. 100.

Exercise 261.

1. Gain 6 as. 2. 2 s 6 d, 3 Rs. 2-8-0. 4. Rs. 30-12-0
5. Loss 12 as 6. Rs 2-7-4. 7. Rs. 2-6-4 8 Rs 6-9-0.
9. Rs 2-1-0. 10. Loss 10 as. 11 10 as 12 Rs 2-4-0.
13. 3 1/8 d. 14. Rs. 2-12-0. 15. 1 a 2 p. 16. 1 a 6 p.

Exercise 262.

1. Rs 7-8-2. 2. Rs. 15. 3. £10-8-4. 4. 24.
5. Rs. 195-13-4. 6. £236. 7. Rs. 468-12-0. 8 Rs 232-15-9.
9. Rs. 96. 10. Rs. 480. 11. Rs. 144. 12 £9.

Exercise 263.

1. A, Rs. 230; B, Rs 115. 2. House, £2400: Furniture £600. 3. Rs. 50-7-6, 4. A Rs. 60: B, Rs 24
5. 26·7; 8·9. 6. A, Rs. 160, B, Rs 320. 7. Rs 1875.
8. 94 seers.

Exercise 264.

1. 720. 2 264. 3. 1 mi. 7 fur. 4. 1 mi. 6 fur
5 $7^1/_2$ fur. 6. 1 mi 2 fur. 7. 10 ft. 8. 3 yds.
9 2 yds 2 ft

Exercise 265.

6 7 p. m 7. (a) 34 hrs, (b) 33 hrs.; (c) 32 hrs.
8 (a) 54 hrs.; (b) 31 hr.; (c) 36 hrs. 9. (a) 39 hrs.,
(b) 66 hrs., (e) 57 hrs. 10 Thursday, 1 a. m
11. Monday, 7 p. m. 12 8th, 15th, 22nd, and 29th.
13. (a) Friday, (b) Monday, (c) Friday 14. 4th,
11th, 18th, and 25th
15. (a) (i) 4, (ii) 4; (iii) 4; (iv) 4, (v) 4, (vi) 4, (vii) 4
(b) (i) 4, (ii) 4; (iii) 4, 4, (v) 4, (vi) 4 (vii) 5.
(e) (i) 4, (ii) 4, (iii) 4 (iv) 4, (v) 4, (vi) 5, (vii) 5.
(d) (i) 4, (ii) 4; (iii) 4; (iv) 4; (v) 5 (vi) 5, (vii) 5.
16 (a) Friday. (b) Sunday (c) Wednesday. 17 Friday.
18. Tuesday, Wednesday, Saturday, Monday Thursday; Saturday, Tuesday, Friday, Sunday, Wednesday: Friday.

REVISION EXAMPLES.

A

1. $3312299922^1/_4$, three hundred and thirty-one crores twenty two lakhs ninety-nine thousand nine hundred and twenty two and one-fourth. 2. 500 *thousands* 3. (i) 39370 79 inches, (ii) 393 7079 inches; (iii) ·3937079 inch. (iv) 03937079 inch. 4 $100x + 10y + z$. 5. 0010208.
9. AB = 2", OC = 2 4", ∠A = 60°, ∠B = 60°; ∠C = 90°.
12. Rs 14-1-8 13. 9999. 14. ·0625. 15. Rs. 4 941 lakhs. 16 1 6848 millions of miles. 17. (a) 293125. (b) 3506. (c) $4017/_{96}$ (d) 5591176. (e) 403—94. (f) $3133^{181}/_{240}$ (g) Rs 106-12-0 (h) 31320 87 18. 64. 19. 100. 20 (1) 140000. (2) 24500, (3) 79750. 21. (a) 799·992. (b) 6499 999935 22. 3405. 23. 785663, —77205, 790498. 24. 136,656000 times 25 42 miles; 118 3 miles 26 1562500. 27 8128 days 28. 96 tea-spoonfuls, 24 table-spoonfuls 29 (a) 1010 yds., (b) 4 fur. 52 links. 30 Rs. 26,76,199-2-10, Rs 89,48,765-10-4; Rs. 40,24,204-11-4; Rs. 234,43,075-4-9; Rs. 138,69,060-6-2;

Rs. 95,74,914-14-7. **31.** 48 times; £45-11-6. **32.** 93 men. **33.** Re. 1-4-6. **34.** Rs. 271-4-0, Rs. 276-1-6. **35.** 4 vis. 3 ssr. Rs 4-10-9. **36.** Rs. 685. **37.** Rs. 14. **38** Rs. 3-15-0. **39.** The latter. **40.** The former. **41.** The former. **43.** (*a*) 2, 5, or 8; (*b*) 2; (*c*) 3; (*d*) 2, 5, or 8. **45.** 216^2, 36^3. **46** 60 **47.** 40 yds. **48.** 80 knots, 120 Km **49.** 8 inches; Rs 93. **50.** Rs. 12-8-0, 16 shillings, 25 half-rupees; 20 francs. **51.** (*a*) 25 sq. ft.; 25 and 16. (*b*) 54 sq ft. **52** Rs. 343-8-0. **53.** 3600. **56** $\frac{71}{693}$. **57.** $3\frac{1}{4}$. **58** $\frac{7}{11}$; $\frac{11}{7}$. **59.** (*a*) $8\frac{53}{121}$. (*b*) 3. (*c* 94 905. (*d*) $2\frac{1}{10}$. (*e*) £5-9-8. **60.** (*a*) $1\overset{\frown}{2}$. (*b*) 18. **61.** *a*) Rs. 959-8-6 (*b*) £281-4-1. (*c*) £67-19-$4\frac{1}{2}$. **62** Rs 24-3-$9\frac{1}{2}$. **65** (*a*) 43 27 dm.; (*b*) 432 7 cm. (*c*) 4327 mm. **66** (*a*) 14087 cm. (*b*) 1 4087 Hm (*c*) 14087 Km. **67.** (*a*) 105 4 Dm. (*b*) 1054 m. (*c*) 10540 dm **68** Km 1-9-6-0-9-4-9. **69.** Km. 4-6-9-8-5-4-0; Dm 4-0-1-5-8$\frac{8}{13}$. **70.** 55·1 **71.** 90 Km. 1 Hm. 2 Dm. 8 dm. **72.** 2 8064 francs. **73.** 775 metres. **75.** 30 cm. **76.** 6 in. **77** 6 as. 3 p.; 16 vis 4 srs. 3 pal. **78.** 1 ton 13 cwt. 1 qr. 26 lb. **79.** 5/81. **80.** 19/20. **81.** 240 apples. **82.** Tuesday. **83.** Friday, Saturday, Tuesday, Thursday, Sunday, Tuesday. **84.** 1 md. 2 srs. **85.** 20. **85.** 333. **87.** 1025, 1026, 1027, 1028. **88.** 117 m. 7 dm. 7 cm.; 85 m. 4 dm. 8 cm. **91.** 42 miles, 3·9 inches. **92.** 95 miles nearly; 60 Km. **93.** 160 yds. **94.** 26 miles.

B

2 £50-5-0, Rs. 753-12-0. **3.** 365 days 5 hrs. 48 min 46 sec. **4** (*a*) 91668793 miles (*b*) 91191207 miles. **5.** (*a*) 81° 33′ 46″, (*b*) 45° 17′ 35″. **6.** 625. **7.** Rs 416-10-8. **8.** £58-15-4. **9** 4 as. **10.** 7007. **11.** Rs. 6.

12.

5 7	6 4	2 9
2 2	5 0	7 8
7 1	3 6	4 3

```
 5493      2485
 5057         9
 4946     -----
-----     22365
15496     -----
-----
```

```
9)28479
  3164—3
8)33621
  4202—5
8)33661
  4207—5
```

13. (*a*)

```
6)35423
7)5903—5
   843—2
```

complete remainder 17.

(*b*)

```
8)143854
7)17981—6
5)2568—5
   513—3
```

14. 7 p m.

15. 102 women. 16. 16 miles to an inch. 17. 32, 24. 18. 932 8 tons, 2 furlongs. 19. 24 posts. 20. 8 min. 19 sec. 21. (*a*) 8 as. (*b*) Rs. 2-10-0. (*c*) Re. 0-6-3. 22. A. Re 1-2-0, C Rs. 2-14-0. 23. 4 as. 10 p. 24. $172\frac{4}{5}$ 25. 3 hrs 15 min., 3 hrs 10 min. 26. 88 ft. per second. 27. 2-15 p m. 28. Rs 187-8-0. 29. 2/3; 3/8 30. £54,000 31. 5035 32. 8 crores. 33. 1/1200. 34. 1/2 35. 480 yds; 3 min 14 sec 36. 28 words. 37. 3916 posts 38. $2 \times 2 \times 2 \times 2 \times 31$; 1, 2, 4, 8, 31, 62, 124, 248. 39. 93 millions of miles. 40. 13 lb. 41. 21 pieces, $4^1/_2$ ft. 42. 15 times. 43. The motor by $13^1/_3$ yds. per minute 44. 19 crores of rupees. 45. 595 ft 2 in.

46. (*a*)

```
7)48647
  6949—4
```

(*b*)

```
 4508          4008
   12            12
-----   or    -----
54096         48096
-----         -----
```

(*c*)

```
 3284
    8
-----
26272
-----
```

(*d*)

```
 7064          7014          7089          7039
    4             4             4             4
-----   or    -----   or    -----   or    -----
28256         28056         28356         28156
-----         -----         -----         -----
```

48. On 11th May 1926, on 11th May 1929. 49. Sat, Wed, Tues, Sat. 50. (*a*) Friday. (*b*) Friday. (*c*) Wednesday. (*d*) Saturday.